Evaluation of P-Δ Dynamic Effects in Steel Frames with Eccentric Braced System by Second Order Dynamic Analysis

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SUMMARY
Evaluation of P-Δ effects is an important factor in structural stability control. In the conventional first order analysis of structures, equations of equilibrium are obtained based on the undeformed state of the structures, while in the second order analysis, acting loads must first be transferred into a deformed state and then obtain the second order forces, moments, and additional displacements. In this study, a computer program was developed to analyze two dimensional structures by the first and the second order dynamic analysis. The numerical solution method used in this program is based on the Jennings numerical method. By this program, P-Δ Dynamic effects were evaluated for two types of steel structures with different heights. One type is a moment frame system and the other is an eccentric braced frame system. The first and the second order dynamic analysis were performed in these structures using the time history of the 1940 El-Centro earthquake. Results of the analyses show about 10% difference in absolute values of the first and the second order dynamic responses. This difference reveals the importance and the necessity of performing a second order dynamic analysis in the calculation of P-Δ effects.

Keywords: P-Δ Dynamic Effects; Second Order Analysis; Eccentric Braced Frames; Moment Resisting Frames.

1. INTRODUCTION

One of the most important issues in the stability of structures is the secondary effects of gravity loads on lateral displacement due to lateral forces. Second order dynamic analysis of the structures under dynamic loading is actually the application of P-Δ effect in the structural analysis. Internal forces of structural members and displacements are obtained through the first order analysis of the structure under the simultaneously effects of gravity and lateral loads. Results of the first order analysis are like to those of the principle of superposition of forces and are separate at each loading case. Interaction between the effects of gravity and lateral loading is not considered in the first order analysis.

In the second order dynamic analysis, for certain values of frequency, lateral vibration is induced in the columns and makes them become instable. This problem was first solved by Belajev (1924). In addition, he also recognized this fact that the maximum dynamic compression force may be much more than the Euler critical load, but column still remains stable.

Many researchers have studied the second order static and dynamic analysis of structures. For instance, Chen and Lui (1986 and 1991) and Bernal (1987) carried out detailed studies on the stability of structures against lateral forces by applying P-Δ effect. They also performed studies on the coefficients of non-linear dynamic amplification under the same effect. Smith also compared the response of conventional and tall structures by applying the P-Δ effect.

2. THEORETICAL DESCRIPTION OF SECOND ORDER ANALYSIS

Developing a geometric stiffness matrix is the base for any exact second order analysis. If a change in axial forces is dynamic, then the stiffness matrix is obtained through the combination of stiffness matrices of members at any moment. This matrix will be different from those obtained a moment before or a moment after. Since the axial force of the member changes under earthquake loading, its stiffness matrix also changes with respect to time. If stiffness matrices of members are transferred to global coordinates by rotation and transformation matrices, then the total stiffness matrix of the system can be calculated by their sum. Having the first order axial forces of the members, coefficients of the second order matrices are calculated at each moment. In this study, results of the final analysis are obtained by two repetitions. In the first repetition, axial forces obtained from the first order analysis are considered and in the second repetition, axial forces obtained from the first repetition are taken into account.

Equation of motion of the system under the dynamic loading which considers changes in axial forces is as follow:

$$[M]\{\ddot{u}(t)\}+[C(t)]\{\dot{u}(t)\}+[K(t)]\{u(t)\} =\{r\}\{P(t)\} \tag{1}$$

where, [M] is the mass matrix of the system, [C] is the damping matrix, [K(t)] is the stiffness matrix, {u(t)} is the displacement vector, {r} is the connecting vector of dynamic forces, and {p(t)} is the vector of dynamic forces. In solving this equation, it should be noted that the stiffness matrix of the system is not constant and continuously changes with respect to time. Variable stiffness matrix is obtained by assembling of the second order stiffness matrices of members. The second order stiffness matrix of the members which is variable with respect to time is

$$Ki(t)=\frac{Ei.Ii}{Li}\cdot\begin{bmatrix} +\frac{Ai}{Ii} & 0 & 0 & -\frac{Ai}{Ii} & 0 & 0 \\ 0 & \frac{+12}{Li^2}\Phi1(t) & -\frac{6}{Li}\Phi2(t) & 0 & \frac{-12}{Li^2}\Phi1(t) & \frac{-6}{Li}\Phi2(t) \\ 0 & \frac{-6}{Li}\Phi2(t) & 4\Phi3(t) & 0 & \frac{+6}{Li}\Phi2(t) & 2\Phi4(t) \\ -\frac{Ai}{Ii} & 0 & 0 & +\frac{Ai}{Ii} & 0 & 0 \\ 0 & \frac{-12}{Li^2}\Phi1(t) & \frac{+6}{Li}\Phi2(t) & 0 & \frac{+12}{Li^2}\Phi1(t) & \frac{+6}{Li}\Phi2(t) \\ 0 & -\frac{6}{Li}\Phi2(t) & 2\Phi4(t) & 0 & \frac{+6}{Li}\Phi2(t) & 4\Phi3(t) \end{bmatrix} \tag{2}$$

where, Ei is the modulus of elasticity of the i^{th} member, Ii is the moment of inertia of the i^{th} member, Li is the length of the i^{th} member and φ1(t)i, φ2(t)i, φ3(t)i, φ4(t)i are the second order coefficients of the i^{th} member, respectively (Chen's coefficients). According to definition, their expansions are

$$\Phi1_{i,j} := \frac{1+\sum_{x=1}^{nn}\frac{\left(\pi^2\cdot\frac{P1_{i,j}}{PE_i}\right)^x}{(2x+1)!}}{12\cdot\left[\frac{1}{12}+\sum_{x=1}^{nn}\frac{2(x+1)\cdot\left(\pi^2\cdot\frac{P1_{i,j}}{PE_i}\right)^x}{(2x+4)!}\right]} \qquad \Phi2_{i,j} := \frac{\frac{1}{2}+\sum_{x=1}^{nn}\frac{\left(\pi^2\cdot\frac{P1_{i,j}}{PE_i}\right)^x}{(2x+2)!}}{6\cdot\left[\frac{1}{12}+\sum_{x=1}^{nn}\frac{2(x+1)\cdot\left(\pi^2\cdot\frac{P1_{i,j}}{PE_i}\right)^x}{(2x+4)!}\right]}$$

$$\Phi 3_{i,j} := \frac{\frac{1}{3} + \sum_{x=1}^{nn} \frac{2(x+1)\cdot\left(\pi^2 \cdot \frac{P1_{i,j}}{PE_i}\right)^x}{(2x+3)!}}{4\cdot\left[\frac{1}{12} + \sum_{x=1}^{nn} \frac{2(x+1)\cdot\left(\pi^2 \cdot \frac{P1_{i,j}}{PE_i}\right)^x}{(2x+4)!}\right]} \qquad \Phi 4_{i,j} := \frac{\frac{1}{6} + \sum_{x=1}^{nn} \frac{\left(\pi^2 \cdot \frac{P1_{i,j}}{PE_i}\right)^x}{(2x+3)!}}{2\cdot\left[\frac{1}{12} + \sum_{x=1}^{nn} \frac{2(x+1)\cdot\left(\pi^2 \cdot \frac{P1_{i,j}}{PE_i}\right)^x}{(2x+4)!}\right]} \tag{3}$$

where, Pi is the axial force of the i^{th} member with respect to time and PEi is the Euler buckling force of the i^{th} member.

Considering that the damping matrix is proportional to the mass and the stiffness matrices, it can be assumed that there is a certain stiffness matrix in each time step. Then, the Eigen values and the modal contribution can be calculated by the stiffness matrix. This type of analysis is considered in this study.

3. JENNINGS NUMERICAL METHOD BASED ON LINEAR INTERPOLATION

Jennings numerical method is the most suitable and programmable modal numerical method to calculate the response of linear systems. This method is based on the exact solution of the differential equation of motion by interpolation of input data at each time step. Differential equation of motion in the time interval between t_i to t_{i+1} (i^{th} stage) in terms of temporal parameter τ and assuming linear input variations is explained as

$$m\ddot{y}(\tau) + c\dot{y}(\tau) + ky(\tau) = x_i + \alpha\tau \tag{4}$$

Applying the initial conditions and solving the homogeneous differential equation of motion, displacement and velocity in the i^{th} stage in terms of τ are equal to

$$y(\tau) = C_0 + C_1\tau + C_2 e^{-\xi\omega_n\tau} \cos \omega_D\tau + C_3 e^{-\xi\omega_n\tau} \sin \omega_D$$

$$\dot{y}(\tau) = C_1 + (\omega_n C_3 - \xi\omega_n C_2)\, e^{-\xi\omega_0\tau} \cos \omega_n\tau - (\omega_D C_2 + \xi\omega_n C_3) e^{-\xi\omega_0\tau} \sin \omega_D\tau \tag{5}$$

Then, the displacement and velocity of the i^{th} stage can be obtained by separation of the above coefficients using the following equation:

$$\binom{y}{\dot{y}}_{i+1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \binom{y}{\dot{y}}_i + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \binom{x_i}{x_{i+1}} \tag{6}$$

At any time step, new values of the frequencies for all modes are put in the Jennings's equations and are solved simultaneously. For this purpose, a computer program was developed to perform the first and the second order analysis for both static and dynamic methods.

4. SAMPLE STRUCTURES AND COMPARISON OF THEIR RESPONSES

Two types of 2-span steel structures with three, five, and seven stories high were considered. One of the structural types is a Moment Resisting Frame (MRF) and the other is an Eccentrically Braced Frame (EBF) (Λ-Braced). The supports in both types are fixed and the lengths of the spans are 4.5 meters in both X and Y directions. Two distributed loads (700 kg/m^2 as dead load and 200 kg/m^2 as live load) were applied on the floors of both types. The 1940 El-Centro earthquake accelerograms were applied on all sample structures. As an example, the responses of the horizontal and vertical displacements and rotation of the degrees of freedom of the seven storey building are shown in Figs 1 and 2, respectively (all units are in centimeter).

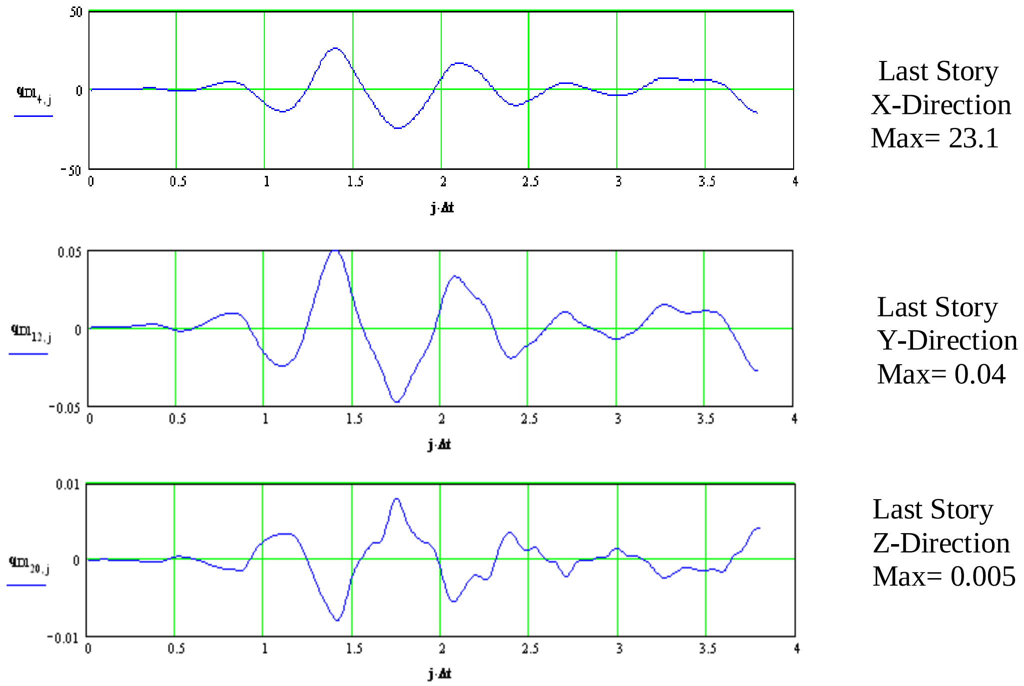


Figure 1. Horizontal and vertical responses and rotation of degrees of freedom in the seven storey building with MRF system.

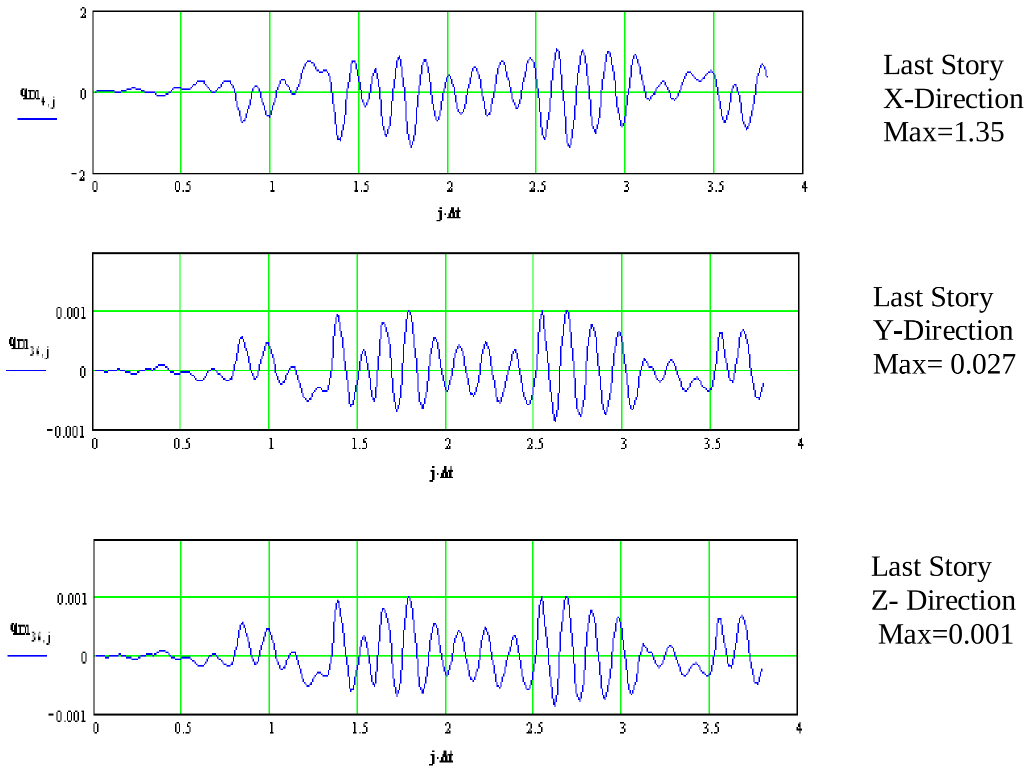


Figure 2. Horizontal and vertical responses and rotation of degrees of freedom in the seven storey building with EBF system.

5. COMPARISON OF RESULTS OF P-Δ DYNAMIC EFFECTS

Table 1 shows a quantitative comparison of the results of the analyses of P-Δ dynamic effects which includes the difference between the displacements of the first and second order dynamic analyses. It should be note that the second order analysis is done in two repetitions. In the first repetition, the obtained axial forces from the first order dynamic analysis is considered, and axial forces obtained from the first repetition is taken as a basis for the second repetition. Although the results of the second repetition are considered as final, but results of both repetitions are appeared in Table 1 to show the convergence quality of both analyses.

Table 1. Percent of difference of displacements in nodes (DOF) by the first and the second order dynamic analyses.

Type of Structure	Repetition	Horizontal DOF		Vertical DOF		Rotational DOF	
3 story MRF	First	0.8	1.5	0	0	0	0
	Second	0.5	0.9	0	0	0	0
5 story MRF	First	4	0.7	0	1.2	1.9	0
	Second	6.4	2.4	0	1.2	2.3	0
7 story MRF	First	0.9	2.1	0	2	13	0
	Second	11.5	0.5	0	2	13	0
3 story EBF	First	3.5	0.4	7.5	0	0	0
	Second	3.5	0.1	7.5	0	0	0
5 story EBF	First	2.7	0.4	3.1	2.1	0	0
	Second	2.6	0.21	3.1	2.1	0	0
7 story EBF	First	5.2	0.4	4.1	2.2	6.6	0
	Second	5	0.2	4.3	2.2	6.7	0

5.1. Comparison of Rotation of Nodes in MRF Structures

Significant difference in the rotation of nodes, particularly in the lower stories is an issue that should be taken into consideration, because other than creating secondary moments in the nodes, it causes a complete change in the condition of plastic hinges and ultimately, failure mechanism in the structure. According to Table 1, a significant difference in rotational transformation of the lower stories, particularly in the MRF system leads to significant support moments in the lower nodes which are naturally not considered in the first order dynamic analysis. For instance, the difference between the results of the first and second order dynamic analyses for nodes in rotation of the lowest storey in the seven storey structure with MRF system is about 13% (Fig. 3).

5.2. Comparison of Moments of Columns in MRF Structures

Significant difference in the moments of members is also an important point that should be considered, because other than creating secondary moments, it finally leads to develop mechanism in the members. This increase in moments, especially in columns is due to the change in the stiffness of members in the process of the second order dynamic analysis. For instance, difference between the results of the first and second order dynamic analyses for the moments in the lowest storey's column in the seven storey structure with the MRF system is 10.5% (Fig. 4).

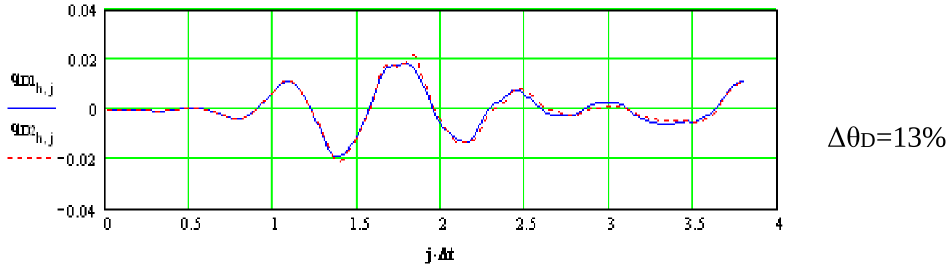


Figure 3. Difference of rotations between the responses of the lowest storey node in the seven storey building with MRF system.

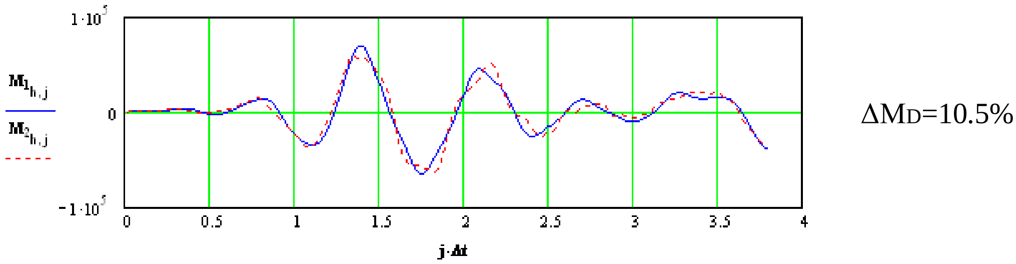


Figure 4. Difference of moments between the responses of the lowest storey column in the seven storey building with MRF system.

5.3. Comparison of Shears of Linked Beams in EBF Structures

Shears in the linked beams of the EBF structures should also be paid attention. Increase of shear in the linked beams is not included in the first order analysis. Some differences are observed between the shear values in the linked beams which are due to the change in the stiffness of these members in the process of the second order dynamic analysis. As an example, the difference of shear values in the linked beams of the second floor in the five story structure with EBF system is 9.6% (Fig. 5).

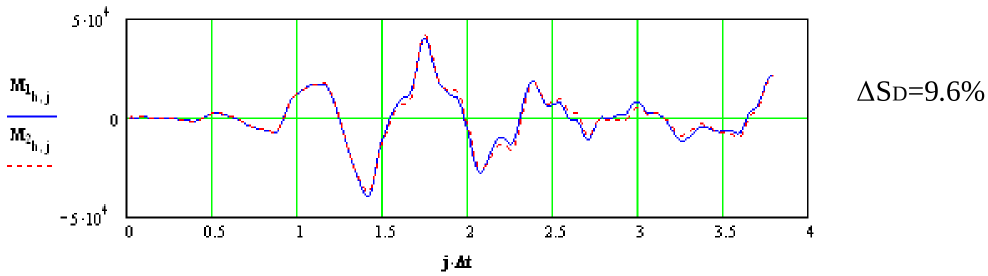
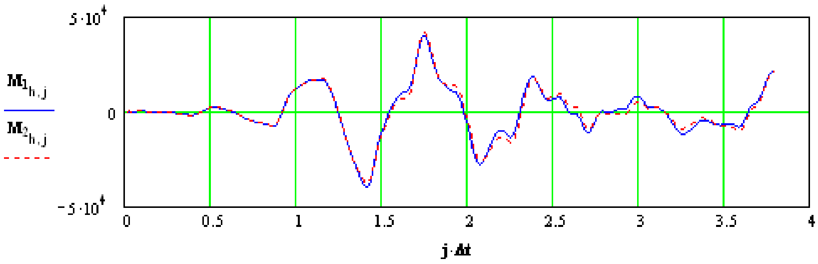


Figure 5. Difference of shears between the responses of the lowest storey beam in the five storey building with EBF system.

5.4. Comparison of Moments of Beams in MRF Structures

Another important factor that should be considered is the moments in beams. Results of the analyses reveal that the difference in the moments of beams, particularly in the MRF structures needs attention. Increase of moment in beams is not considered in the first order analysis. It is obvious that if the applied loads on nodes increase, the moments in beams will also increase by the same proportion. Some differences are observed between the moments of beams in the first and second order dynamic analyses which are due to the change in the stiffness of members in the process of the second order dynamic analysis. For instance, difference between the results of the first and second order dynamic analyses for the moments in the highest storey's beam in the five storey structure with the MRF system is 7.3% (Fig. 6).

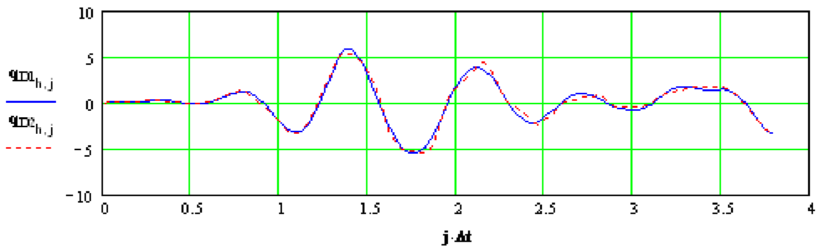


ΔMD=7.3%

Figure 6. Difference of moments between the responses of the highest storey beam in the five storey building with MRF system.

5.5. Comparison of Axial Forces of Columns in MRF Structures

Increase in the axial forces of columns is also an important aspect that should be considered, because it can finally produce plastic hinges in the columns which may lead to collapse of the structure. These changes in the axial forces, especially in columns, change the second order coefficients in the process of the second order dynamic analysis. For instance, difference between the results of the first and second order dynamic analyses for the axial forces in the lowest storey's column in the five storey structure with the MRF system is 6.9% (Fig. 7).

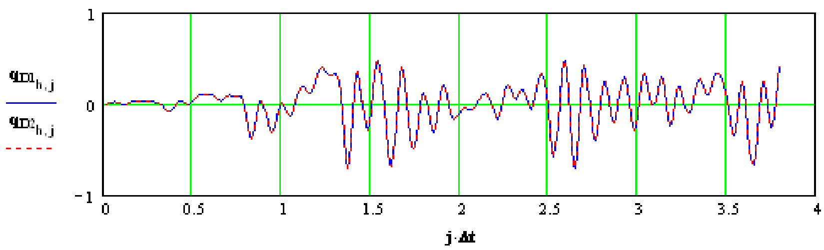


ΔPD=6.9%

Figure 7. Difference of axial forces between the responses of the first and second order dynamic analyses of the lowest storey column in the five storey building with MRF system.

5.6. Comparison of Axial Forces of Columns in EBF Structures

Difference in the axial forces in columns in EBF structures is also an important issue that may lead to failure mechanism in the columns. Changes of axial forces in EBF structures are less than those in MRF structures. For instance, difference between the results of the first and second order dynamic analyses for the axial forces in the lowest storey's column in the five storey structure with the EBF system is 5.2% (Fig. 8).



ΔPD=5.2%

Figure 8. Difference of axial force between the responses of the lowest storey column in the five storey building with EBF system.

6. CONCLUSIONS

In this study, dynamic effects of P-Δ in steel structures with MRF and EBF systems were evaluated by the second order dynamic analysis. For this purpose, a computer program was developed and different analysis methods such as the first order and the second order dynamic analyses were studied on sample structures. Results of the study show that the convergence between the two repetitions in EBF structures is faster than MRF structures.

Also, results of analyses show an average difference of about 10% in the absolute values of responses in the first and second order dynamic analysis. This difference emphasizes the importance of performing the second order dynamic analysis in calculation of P-Δ effects.

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