# Collaborative Optimization Design of Bridge System Using Two Decomposition Methods Considering Aseismic Requirements

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**SUMMARY:**
In order to execute the optimization design to a bridge system involving large design variables and multiple design standards including aseismic requirements, a bi-level optimization algorithm Collaborative Optimization(CO) algorithm is introduced to decompose the large and complex system into some relatively small and simple disciplines or subsystems which are often highly coupled by design variables. In this way, two decomposition methods are proposed: disciplinary-oriented decomposition and component-oriented decomposition. For disciplinary-oriented decomposition, bridge system design is decomposed into three disciplines: carrying capacity discipline, E1 earthquake discipline and E2 earthquake discipline. For component-oriented decomposition, bridge system design is decomposed into three subsystems: superstructure subsystem, bearing subsystem and substructure subsystem. In order to examine the efficiency of CO and to compare the optimization performance of the two proposed decomposition methods, mono-discipline optimization algorithm and the proposed methods are implemented to an example of a multi-span continuous girder bridge optimization design. The design results obtained from different methods are presented and discussed.

*Keywords: Collaborative Optimization, Disciplinary-oriented decomposition, Component-oriented decomposition*

## 1. GENERAL INSTRUCTIONS

Many optimization algorithms have been developed for structure design aiming at making the structure economical and reasonable. The results of application to structures optimization design have confirmed the effectiveness of published optimization algorithms both in numerical experiments and in practice cases. However, most of these published methods are concerned with the optimization of structural member or structural measure which has few design variables. These optimization problems can always be settled by adopting mono-discipline algorithm such as gradient-based algorithms. For a complex structural system, especially for a bridge system design, a large number of design variables are involved during the optimization design process. The increase of design variable numbers induces the considerable leap in complexity of optimization model when solved using mono-discipline optimization algorithms. It makes computation cost increased sharply or could be worse, the optimization process traps in purposeless successive iterations unable to reach a reasonable solution. In order to deal with this kind of problems, reasonable idea is put forward: divide the large and complex system into some relatively small and simple disciplines or disciplines which are often highly coupled by design variables and easier to be solved by applying appropriate optimization algorithm. This is the main idea of multi-level optimization methods.

As a bi-level optimization method, Collaborative Optimization(CO) method has been applied to structure optimization design in order to deal with the high complexity optimization problems. Balling[1] applied CO to a three-dimensional reinforced concrete frames taking concrete-section dimensions and the number, diameter, and topology of reinforcing bars as design variables. Balling and Gale[2] applied collaborative optimization to two structures (hub and tower) design problems.

Balling and Rawlings[3] applied CO to a long span bridge concept optimization design, which was the first time CO was applied to bridge optimization design. Huang and Wang[4] decomposed RC structure design process into two mechanics disciplinary: statics discipline and dynamics discipline, verified the feasible of discipline-oriented decomposition CO to RC structure design. Wang and Tang[5] applied CO to bridge system performance-based seismic design, in which bridge system design process is decomposed into three disciplines: carrying capacity discipline, E1 earthquake discipline and E2 earthquake discipline in terms of load types and structural seismic performance levels. Then, Wang and Tang[6] subdivided the bridge system design into three component-oriented subsystems: superstructure subsystem, bearing subsystem and substructure subsystem. By comparing the results with traditional optimization algorithm, CO shows high computing efficiency when achieving the same optimization results.

Note that in above-mentioned documents, all the structures were component-oriented or disciplinary-oriented decomposed and ideal results were obtained. For bridge systems, both decomposition methods can be employed. However, when the aseismic requirements are taken into account in a structure design process, in some cases, nonlinear analysis is needed especially for the structure design in terms of performance-based seismic design criterions. It is a more time-consuming process than linear analysis when some numerical computational tools such as finite element method are used. How to decompose a bridge system with which the optimization process can be more efficient is a problem necessary to study. In this paper, bridge systems are decomposed with two decomposition methods: disciplinary-oriented decomposition and component-oriented decomposition. For disciplinary-oriented decomposition, bridge system design is decomposed into three disciplines: carrying capacity discipline, E1 earthquake discipline and E2 earthquake discipline. For component-oriented decomposition, bridge system design is decomposed into three subsystems: superstructure subsystem, bearing subsystem and substructure subsystem. In order to examine the efficiency of CO and to compare the optimization performance of the two proposed decomposition methods, mono-discipline optimization algorithm and the proposed methods are implemented to an example of a multi-span continuous girder bridge optimization design. The design results obtained from different methods are presented and discussed.

## 2. COLLABORATIVE OPTIMIZATION ARCHITECTURE

As a bi-level optimization strategy, collaborative optimization is an approach to Multi-Disciplinary Optimization(MDO) problems based on the decomposition of the problem along the lines of the constituent disciplines. It seeks to state and solve MDO problems in a way that preserves the autonomy of the disciplinary calculations by eliminating from the system level problem all those design variables local to individual disciplinary subsystems[7].

As shown in Fig. 2.1, in system level, the main function is to find objective values and drive the disciplinary discrepancies below tolerance $\varepsilon$ by interdisciplinary consistency constraints. In this way, disciplinary discrepancies are implicitly contained in the procedure of system level optimization. System level sends its collaborative solutions (shared variables $\tilde{S}$ and interdisciplinary coupling variables $\widetilde{F_{s,1}}, \cdots, \widetilde{F_{s,\iota-1}}, \widetilde{F_{s,\iota+1}}, \cdots, \widetilde{F_{s,n}}$) to disciplines, at the same time it receives solutions $\widetilde{S_\iota}$, $\widetilde{F_\iota}$ of $i$-th disciplinary minimization optimization problem for the given value of the system level variables $\tilde{S}$ and $\widetilde{F_{s,1}}, \cdots, \widetilde{F_{s,\iota-1}}, \widetilde{F_{s,\iota+1}}, \cdots, \widetilde{F_{s,n}}$. In this way, system level coordinates coupling variables among disciplines until achieving a balance and reaching the optimum values for the coupling variables.

In discipline level, discrepancy function serves as objective function. Such a introduction of disciplinary minimization problem is a distinctive characteristic of CO. after receiving shared variables $\tilde{S}$ and interdisciplinary coupling variables $\widetilde{F_{s,1}}, \cdots, \widetilde{F_{s,\iota-1}}, \widetilde{F_{s,\iota+1}}, \cdots, \widetilde{F_{s,n}}$, it begins to solve the disciplinary optimization problem independently. It should be note that the system level variables which have received serve either as parameters or objectives that discipline optimization try to match. Here $L_i$ is a list of local design variable for $i$-th discipline and $F_i$ is computed via the $i$-th disciplinary analysis. $g_i$ is the disciplinary optimization constraints which are formulated in terms of disciplinary demands. When the solutions of disciplinary optimization problem obtained, the solutions $\widetilde{S_i}$, $\widetilde{F_i}$ are sent to the system level for the next iteration.

Note that the discipline analyses may be strong coupled for each other due to the same design variables $S$ and analytical solutions of other disciplines $F_j$ ($j \neq i$). The introduction of system level variables $S$ and its copy $S_i$ in discipline relax such coupling among disciplines. When a point ($S$, $F_i$) is feasible for the system level problem and realizable for all the constituent disciplines, this point is seemed as optimal objective value.

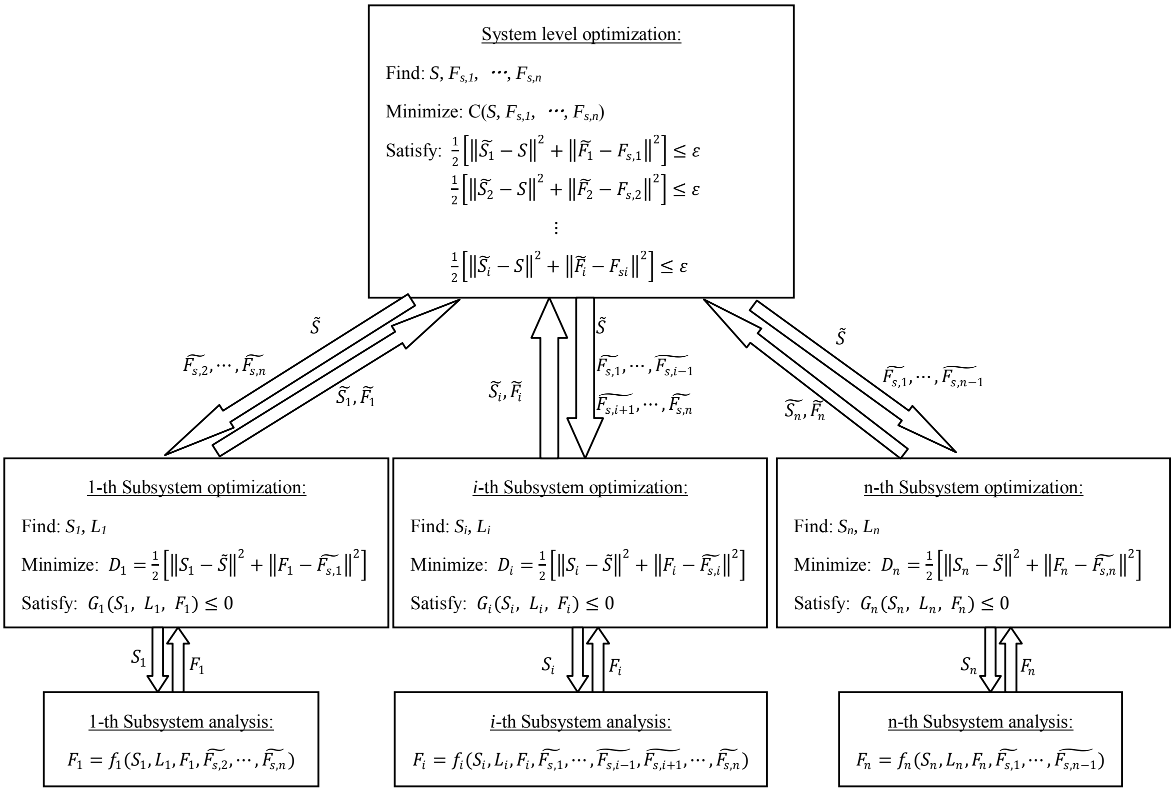


**Figure 2.1.** Collaborative optimization architecture

## 3. STATEMENT OF CASE BRIDGE

In this study, a reinforcement concrete box girder bridge is selected as optimization object with following features: The width of the bridge is 15m to accommodate 4 lanes of traffic. Box girder is supported on the capbeam and 30m long per span. Plate rubber bearings are adopted. 7m high piers are connected monolithically to a concrete pile cap that is supported by nine piles. Girders, capbeams and double row concrete reinforcement circular piers all adopt C40 concrete, reinforced with HRB335 longitudinal bars and R235 hooping bars. Pile caps and piles adopt C30 concrete and reinforced with HRB335 longitudinal bars. It should be point out that this bridge is assumed located on 8th intensity seismic region and 3th site class as defined in Ref. [8].

Reinforcement concrete box girder(Fig. 3.1), pile-pier-capbeam frame(Fig. 3.2) and rubber bearings are design objectives in this study. Design variables are listed in Tab. 3.1.

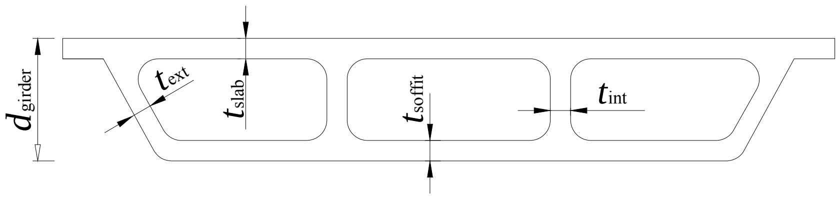


**Figure 3.1.** Reinforcement concrete girder cross section

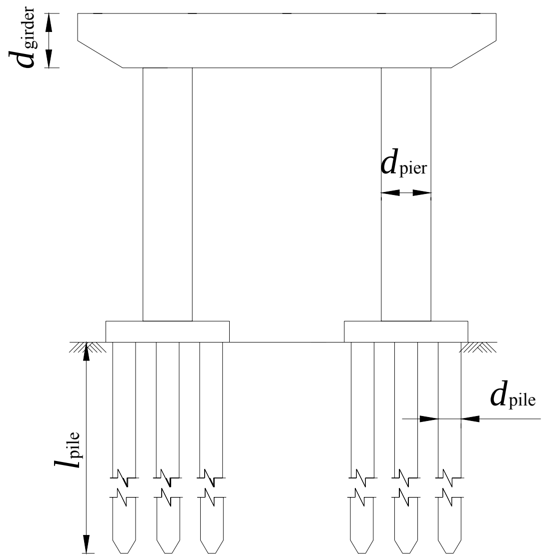


**Figure 3.2.** Reinforcement concrete pile-pier-capbeam frame

**Table 3.1.** Design Variables

| Number | Notation | Description |
|---|---|---|
| 1 | $d_{girder}$ | girder section depth |
| 2 | $t_{slab}$ | slab thickness |
| 3 | $t_{soffit}$ | soffit thickness |
| 4 | $t_{int}$ | interior web thickness |
| 5 | $t_{ext}$ | exterior web thickness |
| 6 | $A_{slablong}$ | slab longitudinal reinforcement area |
| 7 | $A_{soffitlong}$ | soffit longitudinal reinforcement area |
| 8 | $A_{intlong}$ | internal web longitudinal reinforcement area |
| 9 | $A_{extlong}$ | exterior longitudinal reinforcement area |
| 10 | $r_{beamstir}$ | beam stirrups ratio |
| 11 | $t_{bearing}$ | bearing thickness |
| 12 | $l_{bearing}$ | bearing side length |
| 13 | $d_{cap}$ | capbeam section depth |
| 14 | $b_{cap}$ | capbeam section breadth |
| 15 | $A_{caplong}$ | capbeam longitudinal reinforcement area |
| 16 | $r_{capstir}$ | capbeam stirrups ratio |
| 17 | $d_{pier}$ | pier section diameter |
| 18 | $A_{pierlong}$ | pier longitudinal reinforcement area |
| 19 | $r_{pierstir}$ | pier stirrups ratio |
| 20 | $d_{pile}$ | pile diameter |
| 21 | $A_{pilelong}$ | pile longitudinal reinforcement area |
| 22 | $r_{pilestir}$ | pile stirrups ratio |

## 4. MONO-DISCIPLINARY OPTIMIZATION MODEL

### 4.1. Objective Function

For bridge system, objective functions are always expressed as the total direct construction cost related to the total weight or volume of members of structure. In this study, the formula of objective function for bridge structural system optimization design can be stated as shown in Eqn. 4.1:

$$C_0 = C_c + C_s = \sum_i^N w_c b_i\, h_i L_i + \sum_i^N w_s (A_{s1,i} + A_{s2,i}) L_i \quad (4.1)$$

Where $C_c$ and $C_s$ are the cost of concrete and the cost of reinforcing steel, respectively. $w_c$, $w_s$ are the unit cost coefficients of each material. In this study, unit cost is assumed to be \$75/m$^3$ for concrete, \$4,100/m$^3$ for reinforcing bars. $b_i$ and $h_i$ are the section dimensions of member $i$, $L_i$ is its length and finally $A_{s1,i}$, $A_{s2,i}$ are the section area of stirrups and the longitude reinforcement, respectively.

### 4.2. Constraint Functions

In this study, constraint functions are formulated in terms of corresponding codes[8]-[10] demands respectively. In every constraint function, dead loads and corresponding live loads or seismic loads combination are considered and act on the structure. Constraints of this optimization problem are listed in Tab. 4.1.

**Table 4.1.** Constraints for bridge optimization

| Functions | Description | Variable Number |
|---|---|---|
| G1,G2 | combined strength at location of maximum positive and negative moment of girder respectively | 1~9 |
| G3 | shear strength in the webs of girder | 1~9 |
| G4 | transverse flexural strength in the slab of girder | 1~9 |
| G5 | maximum deflection of girder | 1~9 |
| G6 | axial compression stability of bearings | 11,12 |
| G7 | vertical compression deformation of bearings | 11,12 |
| G8 | anti-sliding stability of bearings | 11,12 |
| G9 | flexural capacity of normal cross section of capbeam | 13~15 |
| G10 | shear capacity of oblique cross section of capbeam | 13~16 |
| G11 | axial compression capacity of piers | 17,18 |
| G12 | eccentric compression capacity of normal cross section of piers under E1 seismic actions of piers | 17,18 |
| G13 | displacement capacity under E2 seismic actions of piers | 17,18,19 |
| G14 | shear capacity oblique cross section of piers | 17,19 |
| G15 | ultimate plastic rotation capacity of plastic hinge zone under E2 seismic actions of piers | 17,18,19 |
| G16 | axial compression capacity of piles | 20 |
| G17 | eccentric compression capacity of normal cross section of piles | 20,21 |
| G18 | shear capacity oblique cross section of piles | 20,22 |

It should be note that, in this optimization problem, the essential difference from other optimization problems which just contain carrying capacity constraints is the constraint functions G13 and G15. They are special aseismic constraints for resisting E2 seismic loads in terms of the performance-based seismic design requirements. Considering the present research situation of performance-based design, displacement based seismic design concept is adopted in Ref. [8] to formulate constraints G13 and G15 as shown in Eqn. 4.2 and Eqn. 4.3:

$$\mathrm{G13} = \Delta_d - \Delta_u \quad (4.2)$$

$$\mathrm{G15} = \theta_p - \theta_u \quad (4.3)$$

Where, $\Delta_d$ is the displacement demand taken along the local principal axis of piers under E2 seismic loads, $\Delta_u$ is the displacement capacity taken along the local principal axis corresponding to $\Delta_d$ of piers. $\theta_p$ is the plastic rotation of potential plastic hinge zone under E2 seismic loads, $\theta_u$ is the ultimate plastic rotation capacity of plastic hinge zone corresponding to $\theta_p$ of piers. Since $\Delta_d$, $\Delta_u$, $\theta_p$ and $\theta_u$ are all the nonlinear responses about seismic loads, nonlinear numerical model of pier-capbeam frame is built with finite element method in this study. $\Delta_u$ and $\theta_u$ are evaluated by inelastic quasi-static "pushover" analysis until the frame reaches its limit of structural stability. $\Delta_d$ and $\theta_p$ are evaluated by nonlinear static analysis. This two evaluation processes are the most time-consuming parts in the optimization process and the computational cost of this two evaluation processes are sharply higher than any other evaluations. Hence, method which involves least evaluation number will be the most efficient method for this bridge optimization problem.

## 5. DECOMPOSITION FOR BRIDGE SYSTEM

In this study, based on CO method, the bridge design optimization process is decomposed into double-level nonlinear optimization problem: system level and disciplinary level. Like the mono-disciplinary optimization problem illustrated in Section 3.1, objective functions in system level are all expressed as the total direct construction cost for easy comparison. Constraints in system level are all interdisciplinary consistency constraints with share variables and coupling variables. For disciplinary level, two decomposition methods are presented as below:

### 5.1. Disciplinary-oriented Decomposition

For disciplinary-oriented decomposition method, bridge optimization design process is decomposed into three disciplines: carrying capacity discipline, E1 earthquake discipline and E2 earthquake discipline. The constraints contained in every discipline are listed in Tab. 5.1.

**Table 5.1**. Constraints in disciplines

| Disciplinary Name | Constraints | Variable Numbers |
|---|---|---|
| Carrying capacity | G1~G7, G9~G11 , G16 | 1~9, 11,12, 13~18, 20 |
| E1 earthquake | G12 | 17,18 |
| E2 earthquake | G8,G13,G14,G15,G17,G18 | 11,12, 17~22 |

Considering the coupling among the three disciplines, variable 11, 12, 17, 18, 20 are selected as share variables; for carrying capacity discipline, mass of superstructure, mass of capbeam and pile length are selected as coupling variables during optimization design process.

### 5.2. Component-oriented Decomposition

For component-oriented decomposition method, bridge design optimization is decomposed into three subsystems: superstructure subsystem, bearing subsystem and substructure subsystem. The constraints contained in every subsystem are listed in Tab. 5.2.

For superstructure subsystem, total mass are taken as coupling variables; for bearing subsystem, vertical reaction force, maximum of concentrated reaction force in horizontal directions are taken as coupling variables; for substructure subsystem, vertical reaction force, maximum of concentrated reaction force in horizontal directions are taken as coupling variables.

**Table 5.2.** Constraints in subsystems

| Disciplinary Name | Constraints | Variable Numbers |
|---|---|---|
| Superstructure subsystem | G1~G5 | 1~9 |
| Bearing subsystem | G6~G8 | 11,12 |
| Substructure subsystem | G9~G18 | 12~22 |

### 5.3. Qualitative Comparison of Two Decomposition Methods

For these two decomposition methods, the former can greatly meets the division of labour and gives full play to the professional skill of specialists of various disciplines during the design procedure of bridge. Designers export at one discipline can easily control the optimization process of their discipline especially when concurrent computation is applicable among different departments for one design problem. However, there is large number of share variables move to system level. This will induce more iterations compared with the same optimization problem system level have fewer number of share variables. The later take every component as a subsystem. This measure makes the design variables in one subsystem differ from that in others because all the design variables are component dependent. Design variables in this decomposition method are all local variables. Subsystems are coupled via coupled state variables such as mass, reaction force or moment. Comparing with disciplinary-oriented decomposition method, component-oriented decomposition method can induce fewer variable numbers in system level, whereas the coupled state variables involved in system level are functions with respect to design variables explicitly or implicitly. Hence, the optimization efficiency adopting component-oriented decomposition method is relied on the design variable numbers which state variable related to and complexity between the state variable and design variables.

## 6. RESULTS COMPARISON

### 6.1. Optimizer Selection

Assuming all the variables are real continuous variables, for system level and three disciplines, all the optimization procedures are performed by a gradient-based strategy: Sequential Quadratic Programming-NLPQL. Note that NLPQL is an optimization algorithm gradient is needed in every step of iteration. In this study, gradient is replaced with difference coefficient for constraint functions which is evaluated by finite element method.

It should be point out that in carrying capacity discipline, the disciplinary optimization process can be subdivided into four sequential optimization problems: girder optimization, capbeam optimization, pier optimization and pile optimization. This is mainly because: for carrying capacity discipline, bridge system just suffers constant vertical loads transferred from top down. Correspondingly, the optimization design sequence just needs to follow the order.

### 6.2. Optimization Results

In order to examine the efficiency of CO and to compare the optimization performance of the two proposed decomposition methods, mono-discipline optimization algorithm and the proposed methods are implemented to the same bridge design problem. Variables to both strategies are initialized with same values from a conservative design. Optimum results from the three methods are listed in Tab. 6.1.

As shown in Tab. 6.1, CUP time rate is the rate of CUP computational time of corresponding method to that of Mono-discipline method. Evaluation Number Rate is the rate of evaluation number of nonlinear finite element analysis in corresponding method to that of Mono-discipline method. Note that for the same optimization problem, the objective value from the mono-discipline design optimization method is very close to the value from the CO(both disciplinary-oriented decomposition and component-oriented decomposition), whereas the computation cost of CO is significantly less than the execution time of mono-discipline design optimization. This is mainly because for CO, rather than mono-discipline design optimization, there are fewer variables for every discipline or subsystem. It makes the subsystem optimization process easier to convergence than a multi-variables optimization problem.

**Table 6.1**. Results of optimization

| Item | Mono-discipline | Disciplinary-oriented decomposition | Component-oriented decomposition |
|---|---|---|---|
| Starting cost (per span) | $46,478 | $46,478 | $46,478 |
| Optimum cost(per span) | $42,628 | $41,817 | $42,257 |
| CUP time rate | 100% | 69% | 46% |
| Evaluation Number Rate | 100% | 56% | 42% |

We can see from the results, no matter computation cost or evaluation number, that of component-oriented decomposition method are all better than that of disciplinary-oriented decomposition. It means that for this kind of optimization problems, the predominant influence is the evaluation number of the nonlinear finite element analysis, whereas the calculation in optimizer is not the main control factor to computation efficiency. Hence, the component-oriented decomposition method with which fewest evaluation number of nonlinear finite element analysis have to performe is the best selection for this kind of bridge optimization problems.

## CONCLUSION

Bi-level optimization method collaborative optimization is introduced to deal with the Bridge optimization design problem considering aseismic requirements. Two decomposition methods are proposed: disciplinary-oriented decomposition and component-oriented decomposition. For disciplinary-oriented decomposition, bridge system design is decomposed into three disciplines: carrying capacity discipline, E1 earthquake discipline and E2 earthquake discipline. For component-oriented decomposition, bridge system design is decomposed into three subsystems: superstructure subsystem, bearing subsystem and substructure subsystem. In order to study the efficiency of the decomposition methods, these two methods and mono-discipline optimization algorithm are applied to an example of multi-span continuous girder bridge optimization design problem. All the optimization procedures are performed by a gradient-based strategy: Sequential Quadratic Programming-NLPQL. It is observed that for optimization problems such as bridge seismic design, the computation cost of CO is significantly less than the execution time of mono-discipline design optimization when achieving a similar optimization results. The predominant influence of efficiency for this kind of optimization problems is the evaluation number of the nonlinear finite element analysis. The efficiency of component-oriented decomposition method is better than that of disciplinary-oriented decomposition method for bridge optimization problem considering aseismic requirements.

Note that, in this study, variables are assumed to be real and continuous, whereas design variables are discrete due to the requirements of industry regulations and the availability of members in standard sizes. This kind of requirements forces the optimization processes into discrete optimization problem. Sequentially, optimization strategy which can handle discrete variables has to been adopted. In order to verify efficiency, more study is needed.


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