Influence of cracked inertia and moment-curvature curve idealization on pushover analysis

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SUMMARY:
The pushover analysis is commonly used to analyze seismic capacity of existing bridges. The Eurocode 8 method was used, based on ductility approach. One key step of the method is the idealization of the moment-curvature curve with a bilinear curve. With three examples of piers, we showed that this first slope of the idealized curve may be highly depending on the cracking inertia, according to the section shape. The second parameter that influences much the results is the yield curvature. As a consequence, the results of the pushover analysis may be strongly variable, and the capacity of the structure over or under estimated.

Key words : pushover analysis, moment-curvature curve idealization, yield curvature, cracked inertia

1. INTRODUCTION

Using three examples of a circular section, a rectangular section and a hollow core one, a pushover analysis was done to evaluate the influence of key parameters on the pushover results. An important step of the pushover analysis is the determination of the moment-curvature curve, and its idealization. The influence of idealization choices for the moment-curvature curves is directly linked to pushover results. As a consequence, the influence of critical parameters as the cracking inertia and the yield curvature has been studied, all along the pushover analysis.

2. PUSHOVER PRINCIPLES

The pushover analysis consists in applying to the structure an increasing horizontal force, in order to see the successive plastifications of the different elements of the structure. Real behavior of sections and materials are considered.

As a consequence, the different steps of the analysis are the following :

- Determination of the non-linear constitutive laws of materials
- Determination of the moment-curvature curves for all critical sections by integration of the material laws on the section's width
- Determination of the force-displacement curves of the pier by integration of the moment-curvature law on the pier height. This curve is also named capacity curve of the pier.
- For each point of the capacity curve, determine all the equivalent characteristics of the pier (rigidity, period, damping) and find the acceleration that permits the intersection at this displacement of the capacity curve and the acceleration-displacement spectrum.

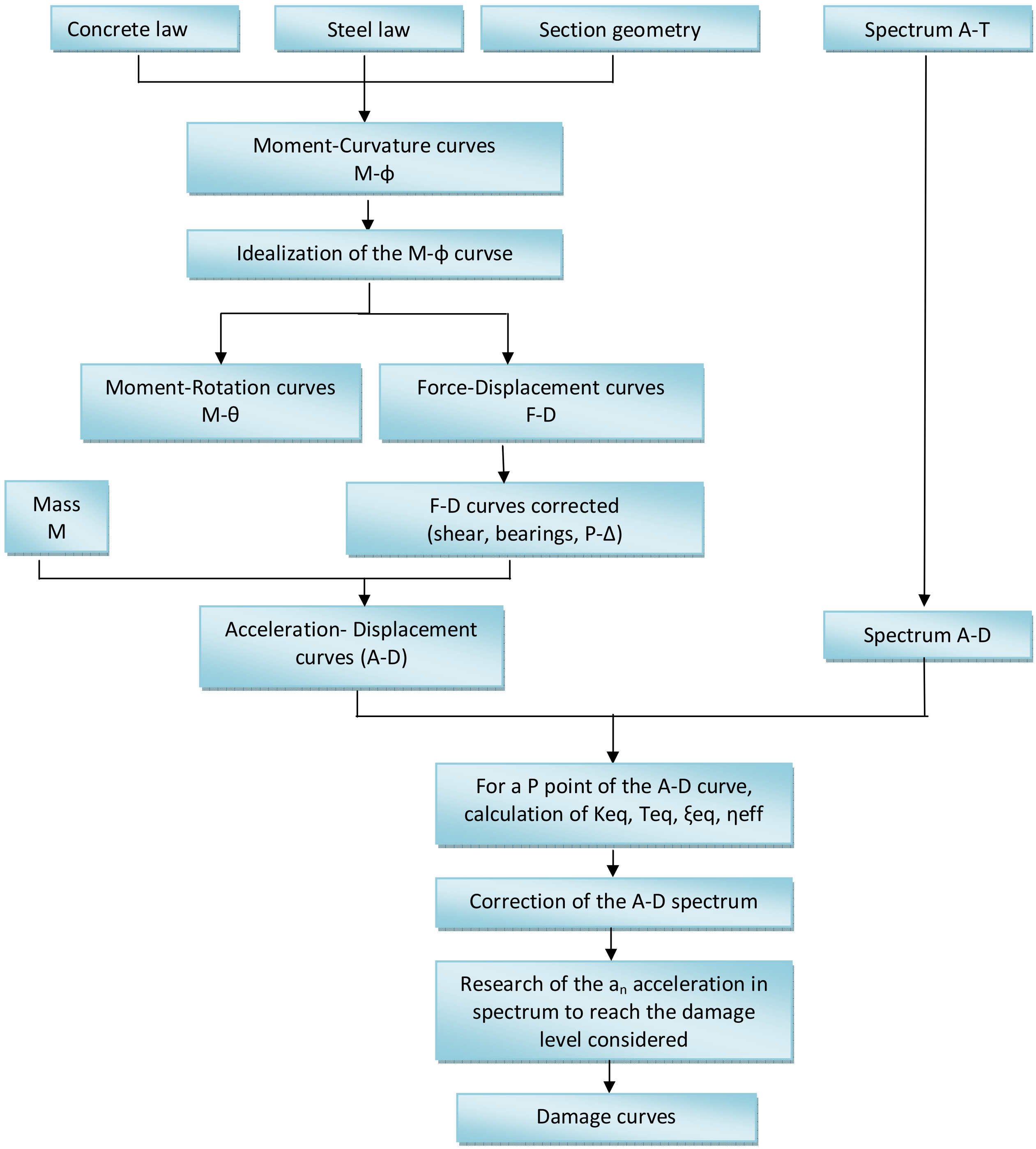

Figure 1. Pushover analysis developed in hand calculation

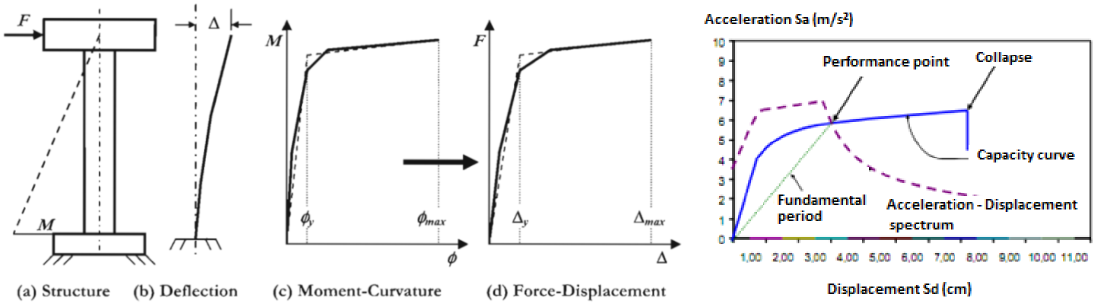

Figure 2. Principle of each step of the pushover analysis

3. MOMENT-CURVATURE PRINCIPLES

3.1. Moment-curvature curves

To analyze the behavior of the pier, it is necessary to have the section behavior of the critical sections of the pier, that is to say, the potential plastic hinge sections. The moment-curvature curve is calculated from the constitutive material laws of concrete and steel reinforcement.

This moment-curvature law is obtained by an iterative calculation of sections, increasing progressively the level of deformation in the section. At each step, the point (M-Φ) can be defined, using the compression height of the section

$$\varphi = \varepsilon c/c = (\varepsilon c + \varepsilon s)/ds \tag{3.1}$$

ds width of the section from the tension reinforcement, c: height of the compression section, εc and εs, deformation of the concrete and steel.

The moment value corresponding for each step is obtained by integration on the gross section of the stresses in the materials for the level of deformation.

$$M = \int \Phi c(\varepsilon(x)) b(x)\, x dx + \sum_1^n \Phi s(\varepsilon(x)i)\, xiAsi \tag{3.2}$$

Φc : curvature in concrete, Φs : curvature in reinforcement,

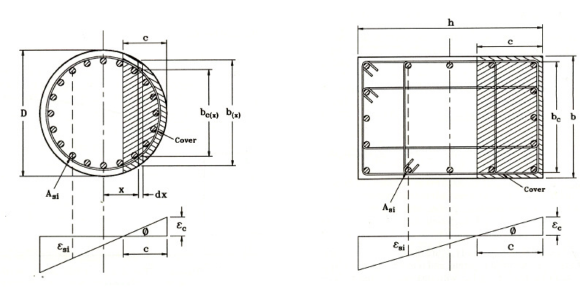

Figure 3. Behavior and parameters of the section analysis

The moment-curvature curve, which represents the concrete ductility, has three inflection points:

- The concrete decompression
- The cracking of tensioned concrete
- The yield of tension steel rebars or/ and of the compressed concrete
- The collapse of the section, reached by crushing of compressed concrete or by

breaking of tensioned steel rebars

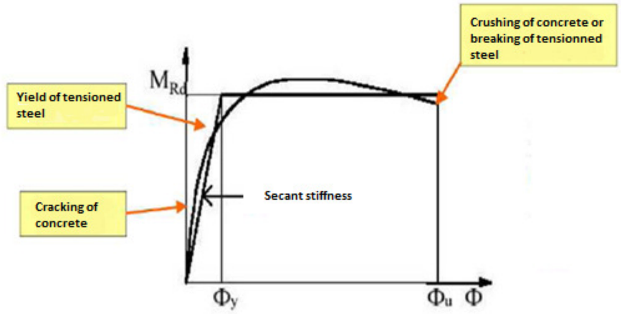

Figure 4. Moment-Curvature curve inflection points

3.2. Idealization of the moment-curvature curves

In order to simplify calculations, it is common to idealize the moment-curvature curves. There are two principles methods to idealize the moment-curvature curves. The first method, described in appendix E of the Eurocode 8-2, consists in keeping the equality of the areas after the point of the first plastification of the steel. Another way consists in idealizing the curve considering the initial stiffness. This method is very conservative, because it over-estimates the stiffness of the structure.
This idealization is directly linked with the inertia considered, which is the cracked inertia in the first method and the uncraked inertia in the second method.
The two methods are presented in the following figure.

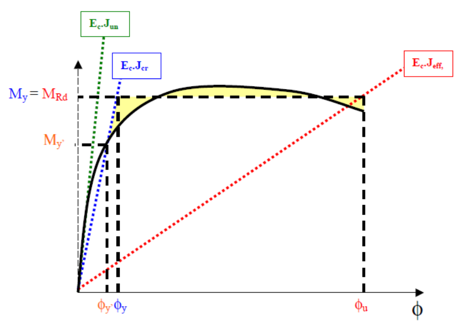

Figure 5. Idealization of the moment-curvature curve

Stiffness before cracking : $E_c.J_{un}$ (uncracked inertia)
Yield limit (before first plastification) : $\phi_{y'}$; $M_{y'}$
Theorical yield limit corresponding to the bilinearisation of the curve : ϕ_y ; M_y
Ultimate limit corresponding to the collapse : ϕ_u ; M_{Rd}
Equivalent cracked stiffness : $E_c.J_{cr} = M_{y'} / \phi_{y'} = M_y / \phi_y$, (cracked inertia)
Equivalent effective ultimate stiffness : $E_c.J_{eff,u} = M_{Rd} / \phi_u$ (plastic inertia)
Moreover to idealize moment-curvature curve, in order to simplify calculations, simplified formulae are given in Eurocode 8 and literature to evaluate the yield curvature, and the plastic moment is taken equal to the yield moment. To simplify, yield moment can be taken equal to the maximum moment.

Table 3.1. Formulae for Φy calculation

Section	Formulae for Φy
Circular concrete column	$2.25\varepsilon y/D$ (D : diameter)
Rectangular concrete column	$2.10\varepsilon y/hc$ (hc : section depth)
Rectangular concrete wall	$2.00\varepsilon y/lw$ (lw : section depth)

4. EXAMPLES

In order to analyze the influence of the inertia, the yield curvature and the idealization of the moment-curvature curve, three examples are considered.

The first example is a circular pier of 1.75m of diameter, 9m height and 2.7% of longitudinal steel reinforcement. The second example is a square pier, of 2m*2m, 9m height and 1.6% of reinforcement. The last example is a hollow core bridge pier, of 6.8m * 2.5m with a hollow core of 5.8m * 1.9m, that is to say a thickness of walls of 0.5m and 0.3m. The longitudinal reinforcement is about 1.2%.

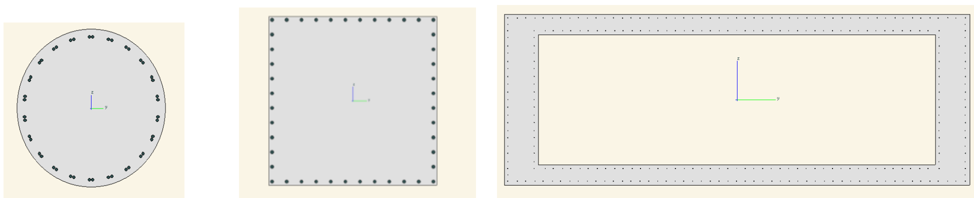

Figure 6. Section view

5. RESULTS

5.1. Moment-curvature curves

The following figure presents the results of the different idealizations.

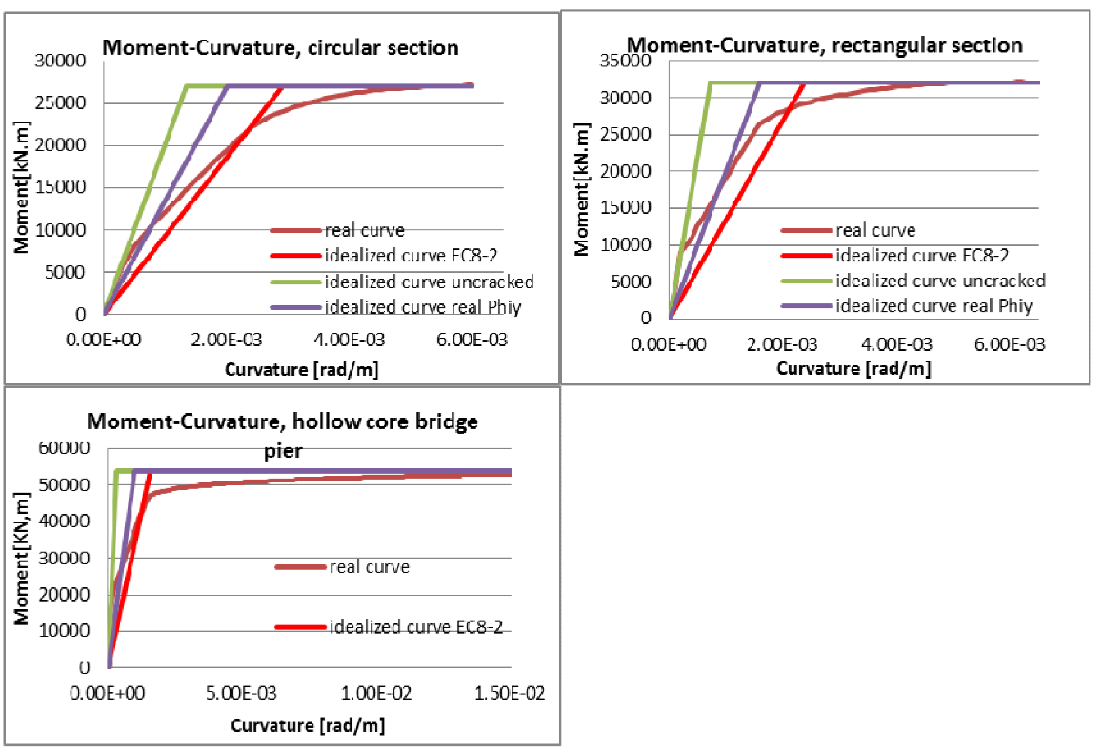

Figure 7. Moment-Curvature curves

Table 5.1. *Φy* calculation

		Eurocode 8-2	Uncraked	Real	Difference
Circular section	*Φy*	0.0029	0.0013	0.0020	54%
Rectangular section	*Φy*	0.0024	0.0007	0.0016	70%
Hollow core section	*Φy*	0.0016	0.0003	0.0010	78%

Regards to the moment-curvature curves and the *Φy* results, we can note a significant variation in the results. Moreover, those variations are more important for hollow core piers.
The difference between the methods of idealization is a little overestimated because of the hypothesis that the yield moment is equal to the maximum moment. Nevertheless, differences stay significant.

5.2. Functioning point

The following figure presents the results of the different idealizations for the functioning point, for the unitary spectrum.

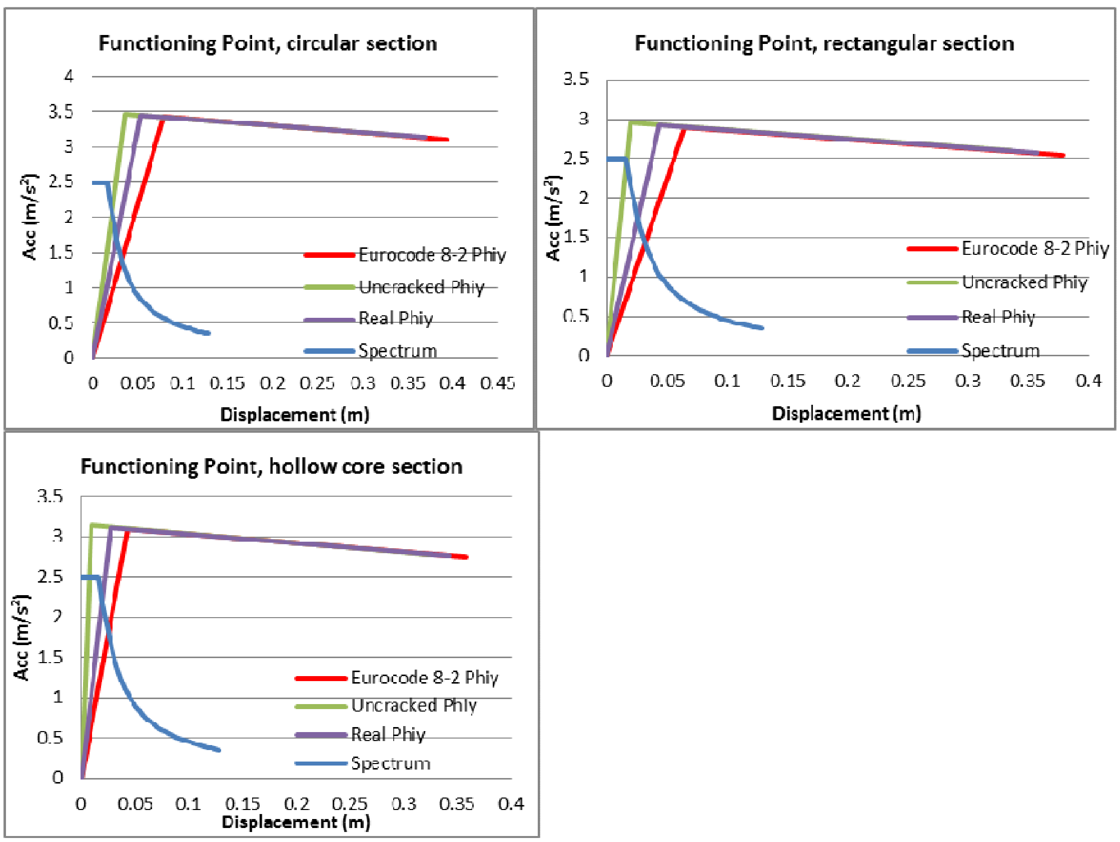

Figure 8. Functioning point curves

Regards to the functioning point curves, we can note a significant variation in the results. However, the difference will be quasi null for a real ductile behavior (plastic behavior). The most important result is that limit for plastic behavior is significantly different. As a consequence, for structure which has a limited ductility, according to the method, calculations show that plastification occurs or not…
The difference between the methods of idealization is a little overestimated because of the hypothesis that the yield moment is equal to the maximum moment. Nevertheless, differences stay significant.

5.3. Damage curves

The following figure presents the results of the different idealizations for the damage curve, which represents the displacement of the structure under the ultimate displacement in function of the acceleration.

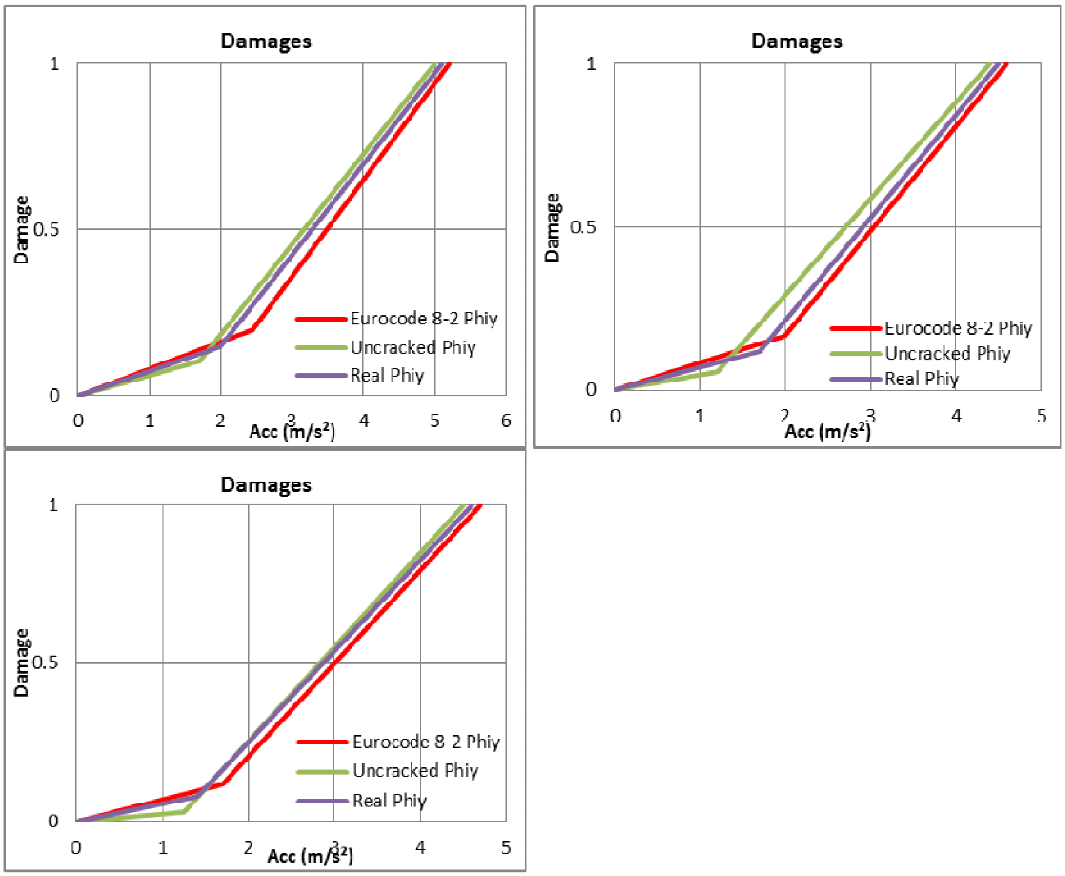

Figure 9. Damage curves

Regards to the damage curves, we can also note a significant variation in the results. We confirm that the difference will be quasi null for a real ductile behavior (plastic behavior) and the limit for plastic behavior is significantly different.

5.4. Numerical results

Table 5.2. Numerical results

			Eurocode 8-2	Uncraked	Real	Difference
Circular section	Functioning point Unitary spectrum	*an (m/s²)*	1.48	2.00	1.66	26%
		d (m)	0.034	0.015	0.027	56%
		T (s)	0.95	0.64	0.79	33%
	Yield	*an (m/s²)*	2.45	1.70	2.00	31%
		Teq (s)	0.95	0.54	0.80	43%
		Icr (m^4)	0.31	0.67	0.45	54%
	Ultimate	*an (m/s²)*	5.20	5.00	5.10	4%
		Teq (s)	2.23	2.10	2.16	6%
		Icr (m^4)	0.15	0.15	0.15	0%
Rectangular section	Functioning point Unitary spectrum	*an (m/s²)*	1.48	2.70	1.83	45%
		d (m)	0.030	0.015	0.030	50%
		T (s)	0.93	0.51	0.76	46%
	Yield	*an (m/s²)*	2.00	1.20	1.70	40%
		Teq (s)	0.89	0.47	0.80	48%
		Icr (m^4)	0.45	1.50	0.67	70%
	Ultimate	*an (m/s²)*	4.60	4.40	4.50	4%
		Teq (s)	2.42	2.25	2.34	7%
		Icr (m^4)	0.13	0.13	0.13	0%
Hollow core section	Functioning point Unitary spectrum	*an (m/s²)*	1.66	2.70	1.83	39%
		d (m)	0.027	0.015	0.030	50%
		T (s)	0.74	0.34	0.58	53%
	Yield	*an (m/s²)*	1.70	1.25	1.40	26%
		Teq (s)	0.80	0.47	0.80	42%
		Icr (m^4)	1.14	5.16	1.79	78%
	Ultimate	*an (m/s²)*	4.70	4.50	4.60	4%
		Teq (s)	2.27	2.14	2.21	5%
		Icr (m^4)	0.09	0.09	0.09	0%

The numerical results confirm the real difference of results for yield behavior, with different plastification limits, whatever the shape of the section. Moreover, those results confirm that for all sections, behavior is quasi the same after plastification.
At least, the cracking inertia is an important parameter, because it determines the slope of the yield branch of the moment-curvature curve. As a consequence, it is important to have a precise evaluation of the cracking behavior of the section. In fact, rectangular and especially hollow core rectangular section is cracking a lot and then results have a lot of variations.

6. CONCLUSION

This study clarifies the role of the yield curvature and the cracked inertia on the pushover results. In fact, it has been shown that yield curvature can vary until 70 % regards to the criterion selected

(Eurocode 8-2, uncracked stiffness or real first yield of rebars). The yield curvature depends directly on the cracking inertia and the major parameter of idealization of the moment curvature curves. As a consequence, with cracked inertia considered, those differences of idealization lead to important differences on functioning point results for a defined spectrum for low level of ductility. Moreover, this can lead to uncertainty of around 30% about yield acceleration. As a consequence, conclusion about plastification of limited ductile structure can be erroneous. On the contrary; those parameters have little influence on ductile structure, all sections having close results for ultimate accelerations.
An interesting point to develop in a future study can be the influence of the idealization of moment-curvature curves on the dissipating energy and the influence on pushover results.

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