Seismodynamics of an Underground Pipeline

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SUMMARY:
Starting from famous and key works of Sakurai – Takahashi, Toki – Takada, and Ilyushin –Rashidov it was suggested a variety of approaches and models in order to analyze earthquake response of underground tubular structures. Some of these approaches deal with given ground motion, which do not vary by reason of pipeline presence. In others, the influence of structure on motion in surrounded medium is taken into account but only limiting steady states are investigated. The present paper develops a new methodology and here a model is constructed for the study of connected non-stationary motion and vibrations of ground – pipeline system when seismic wave propagates along the pipeline. Both subsonic and supersonic cases are considered and it is shown that only in supersonic case the resonance phenomenon may occur. The solutions obtained enable us to evaluate dynamic stresses in pipeline during earthquake.

Keywords: Underground pipeline, seismic waves

1. INTRODUCTION

Dynamic response of buried pipelines is a subject of considerable interest for earthquake engineers. Buried tubular structures and pipelines can be modeled as thick-wall cylinders (rods) or cylindrical shells. The estimation of peak displacements and stress induced in such structures during earthquakes is the problem of primary practical importance.

Simple analytic procedure for assessing the pipe response due to wave propagation was developed by Sakurai, A. and Takahashi, T. (1969) taking the constraint force as proportional to the “relative” displacement (that is, to difference between displacements in incident wave and in pipe in direction of the structural axis). The coefficient of this proportionality is called by the coefficient of interaction. The Sakurai and Takahashi approach has since been used or extended by a number of authors (e.g., Ilyushin, A. A. and Rashidov, T., 1971). In doing so, the coefficient of interaction was defined either experimentally (for different types of soils) or from additional assumptions. The extensive survey of these works may be found in monographs by Rashidov, T. (1973) and by O´Rourke, M. J. and Liu, X. (1999). More rigorous approach when soil, surrounding the pipeline, is treated as an infinite elastic medium was proposed by Toki K. and Takada S. (1974). Here the ground motion (axial and radial displacements in cylindrical coordinates) is defined from exact equations of elastodynamics. But this treatment is restricted to the case of sinusoidal excitation.

2. ANALYTICAL METHOD FOR STUDY OF COUPLED PIPELINE-GROUND MOTIONS

The straight pipeline is considered to be an infinitely long thick-wall cylinder (rod) having outer radius a and inner radius b. The motion of pipeline and surrounding elastic medium (soil) are caused by propagation of a longitudinal seismic wave in direction of the pipeline axis, denoted as Oz in cylindrical coordinates (r, θ, z). Namely, it is assumed that at distance $r = R$, far enough from the pipe, displacements $\boldsymbol{u}(u, v, w)$ in the medium are equal to ones in incident longitudinal wave, that is,

$$u|_{r=R} = 0, \qquad v|_{r=R} = 0, \quad w|_{r=R} = w_0(c_1 t - z) \tag{2.1}$$

Here u, v, w mean the components of displacement vector along the coordinate axes r, θ, z, respectively, and $a < R \leq R_0$, where R_0 is the burial depth of pipeline. Let w_0 be a function that vanishes when the argument $Z \equiv c_1 t - z < 0$. Then wave (2.1) defines a plane wave having the front Γ is perpendicular to the Oz – axis, which travels in positive direction of Oz with the velocity of *P*- waves $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$ (λ, μ, ρ are the Lame constants and density of the elastic soil). At the initial instant $t = 0$ the front Γ passes through the point $z = 0$. Boundary conditions at the contact surface of pipeline with ground are taken as follows:

$$u|_{r=a} = 0, \qquad v|_{r=a} = 0, \quad w|_{r=a} = U(z, t) \tag{2.2}$$

where $U(z, t)$ stands for the axial displacement of pipeline, which is defined from the equation of longitudinal vibrations of pipeline considered as a rod. Physically conditions (2.2) mean that the transversal motion of tubular structure (pipeline) is negligible and no slip on the contact surface of structure with ground.

Under conditions (2.1), (2.2) we come to the case of axial symmetry in which $v \equiv 0$ and functions u and w are independent from angular coordinate θ. Then the equations of motion of elastic medium in cylindrical coordinates reduce to the next two equations for radial (u) and axial (w) displacements

$$\begin{gathered}
(\lambda + 2\mu)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru)}{\partial r}\right) + \mu\frac{\partial^2 u}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 w}{\partial z \partial r} = \rho\frac{\partial^2 u}{\partial t^2} \\
\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + (\lambda + 2\mu)\frac{\partial^2 w}{\partial z^2} + \frac{\lambda + \mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial z}\right) = \rho\frac{\partial^2 w}{\partial t^2}
\end{gathered} \tag{2.3}$$

Now, taking into account the first condition in (2.2), assume that in the second equation of (2.3) the term, involving u (last term on the left-hand side), can be ignored. That is true when the strains ε_{rr} and $\varepsilon_{\theta\theta}$ are much less than the strain ε_{zz} in direction of wave propagation. After which, the cited equation transforms to the form

$$\left(\frac{c_2}{c_1}\right)^2\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2} = \frac{1}{c_1^2}\frac{\partial^2 w}{\partial t^2} \tag{2.4}$$

where $c_2 = \sqrt{(\mu/\rho)}$ is the velocity of *S*- waves.

By virtue of boundary conditions (2.2) the surface traction, which acts on the contact surface of pipeline with ground, is calculated by formula

$$\sigma_{rz}|_{r=a} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\Big|_{r=a} = \left(\frac{\partial w}{\partial r}\right)\Big|_{r=a} \tag{2.5}$$

As is seen from expression (2.5), for determination of the interaction force between structure and medium we need not know the radial displacement u. So, there is no necessity to solve the first equation in (2.3).

Next, must be noted that the problem has just been formulated submit the solution in which the unknown functions $w(r, z, t)$ and $U(z, t)$ depend from variable $Z = c_1 t - z$ instead of dependence from t and z in separate way. Since Z is the distance from wave front, such solution assigns a steady state of wave propagation in pipeline-ground system. Under these circumstances Eqn. 2.5 and boundary conditions (2.1), (2.2) for function w can be rewritten as follows:

$$D_r^2 w(r,Z) + r^{-1} D_r w(r,Z) = 0, \qquad D_r \equiv \frac{d}{dr} \tag{2.6}$$

$$w(r,Z)|_{r=R} = w_0(Z), \quad w(r,Z)|_{r=a} = U(Z) \tag{2.7}$$

Eqn. 2.6 is an ordinary differential equation involving derivatives with respect to r in which the dependence w of Z may be considered as dependence of the parameter. The solution of boundary value problem (2.6), (2.7) may be easily obtained and given by

$$w(r,Z) = \frac{\ln\left(\frac{r}{a}\right)}{\ln\left(\frac{R}{a}\right)}[w_0(Z) - U(Z)] + U(Z) \tag{2.8}$$

To derive the equation of pipeline vibrations it is necessary to calculate the shear stress σ_{rz} on the contact surface $r = a$ proceeding from expression (2.5). Choose the element of pipe of unit length. Then, according to (2.5) and (2.8), the resulting force affect at its surface is

$$f(Z) = \int_0^{2\pi} (-\sigma_{rz})|_{r=a} \cdot a\, d\theta = \frac{2\pi\mu}{\ln\left(\frac{R}{a}\right)}[w_0(Z) - U(Z)] \tag{2.9}$$

Surface force (2.9) divided into the volume of the pipeline element of unit length, that is,

$$\frac{f(Z)}{V} = \frac{f(Z)}{\pi(a^2 - b^2)}$$

is taken as a body force per unit volume of pipeline. In view of the above reasoning we come to the next differential equation describing the forced longitudinal vibrations of pipeline

$$\frac{d^2U}{dZ^2} \pm p^2 U(Z) = \pm p^2 w_0(Z) \tag{2.10}$$

in which

$$p = \left(\frac{2\mu}{E'|1 - M^2|(a^2 - b^2)\ln(R/a)}\right)^{1/2}, \quad M \equiv \frac{c_1}{c} \tag{2.11}$$

where $c = \sqrt{(E'/\rho')}$ is the velocity of wave propagation in pipeline, and E' and ρ' denote Young´s modulus and density of the pipe material. Besides, the upper signs in Eqn. 2.10 stand for supersonic case $M > 1$ (M is called the Mach number), whereas the lower signs stand for subsonic case $M < 1$. As will be seen from further considerations, behavior of the solution of our problem and its physical pattern depend heavily on whether the Much number more or less than unity.

3. SOLUTIONS FOR SUPERSONIC AND SUBSONIC REGIMS

First of all, note that if $M > 1$ (supersonic case) then in differential Eqn. 2.10 it has to be understood $Z \geq 0$; this means that both the medium and the pipeline are disturbed behind the front of incident wave only. Whereas, if $M < 1$ (subsonic case) in this equation the variable Z allows all values within the interval $(-\infty, +\infty)$ and disturbances in the medium and pipeline exist behind the wave front Γ as well as ahead of it. In each case adding conditions must be formulated to extract a unique solution of Eqn. 2.10. Proceeding from what has been said, in the first case we seek the solution of

Eqn. 2.10 that satisfies the (initial) conditions $U = dU/dZ = 0$ at the wave front $Z = 0$ since the part of pipeline ahead of wave front remains at rest. But in the subsonic case ($M < 1$) it is reasonable to require that the vibrations of pipe are bounded (in amplitude) at infinity (as $Z \to \pm\infty$). Solutions of Eqn. 2.10, satisfying the restrictions mentioned above, are easily found and given by the next expressions: if $M > 1$

$$U(Z) = p \int_{0-}^{Z} w_0(Z') \, sin\, p(Z - Z')dZ' \qquad (3.1)$$

and, if $M < 1$

$$U(Z) = \frac{p}{2}\left(e^{pZ} \int_{Z}^{\infty} w_0(Z')H(Z')e^{-pZ'}dZ' + H(Z)e^{-pZ} \int_{0-}^{Z} w_0(Z')e^{pZ'}dZ'\right) \qquad (3.2)$$

wherein $H(Z)$ is the Heaviside´s unit function, which takes the values 1 and 0 when the argument is positive or negative, respectively.

Consider now particular case when the law of ground motion is sinusoidal, namely,

$$w_0 = \begin{cases} A_0 \sin \omega_1 Z, & Z > 0 \\ 0, & Z < 0 \end{cases} \qquad 3.3)$$

Then solution (3.1), (3.2) takes the form: for $M > 1$

$$U^{\circ}(Z) = \frac{A_0 p}{p^2 - \omega_1^2}(p\, sin\omega_1 Z - \omega_1 \sin pZ) \qquad (3.4)$$

and for $M < 1$

$$U^{\circ}(Z) = \begin{cases} \dfrac{A_0 p}{p^2 + \omega_1^2}\left(p \sin \omega_1 Z + \dfrac{\omega_1}{2} e^{-pZ}\right), & Z > 0 \\ \dfrac{A_0 p}{2} \dfrac{\omega_1}{p^2 + \omega_1^2} e^{pZ}, & Z < 0 \end{cases} \qquad (3.5)$$

On the base of (3.4), (3.6) may be calculated axial (dilatation – compression) stress arising in the pipeline. This gives that in the case $M > 1$

$$S = -E' \frac{\partial U^{\circ}}{\partial Z} = -\frac{E' A_0 \omega_1}{1 - (\omega_1/p)^2}(\cos \omega_1 Z - cos\, pZ) \qquad (3.6)$$

and in the case $M < 1$

$$S = \begin{cases} -E' A_0 \dfrac{\omega_1}{1 + (\omega_1/p)^2}\left(\cos \omega_1 Z - \dfrac{1}{2} e^{-pZ}\right), & Z > 0 \\ -\dfrac{E' A_0}{2} \dfrac{\omega_1}{1 + (\omega_1/p)^2} e^{pZ}, & Z < 0 \end{cases} \qquad (3.7)$$

4. DISCUSSION AND CONCLUSIONS

From the solutions obtained above it follows that in supersonic case only the resonance phenomenon may occur. Indeed, longitudinal displacement (3.5) and axial stress (3.7) of pipe for subsonic regime stay finite for all values of argument Z and for any ω_1 and p. While in supersonic case the same functions (formulae (3.4), (3.6)) increase by linear law with a rise in distance Z when $p \cong \omega_1$. The quantity pc_1 of dimension sec^{-1}, where p is defined by formula (2.11), will be called a frequency of coupled vibrations of pipeline. So, the resonance phenomenon occurs in supersonic case when the frequency of coupled vibrations of pipeline is equal or close to the frequency of vibrations in incident wave.

For actual earthquakes the wave-length of seismic waves is about several hundred meters. Take, as an example, the value $200\ m$; then $\omega_1 \simeq 1/30\ m^{-1}$ but p has, as a rule, the order $O(1)$. Values of p, computed by formula (2.11), for steel (A), iron (B), and concrete (C, D) pipelines are listed in Table 4.1. The cylinder pipelines had the next geometrical characteristics: pipes A and B of diameter $2a = 20\ cm$ and wall thickness $h \equiv a - b = 0{,}5\ cm$, pipes C and D of diameter $60\ cm$ and wall thickness $1\ cm$ and $2\ cm$, respectively. In each case the burial depth of pipeline was taken as $1{,}5\ m$. The data on mechanical properties of ground were taken from the work of Chadwick, P., Cox, A. D. and Hopkins, H.G. (1964). From Table 4.1 it is seen that in all situations examined $\omega_1 \ll p$, what permits to simplify essentially the solutions obtained in previous section. And when this estimate is valid, (3.4) implies a good approximation $U^{\circ}(Z) \simeq w_0(Z)$.

Table 4.1. Values of p (m^{-1}) in Dependence on Pipe Material and Sort of Ground

Ground\Pipe	A	B	C	D
Soft soil	1,19	1,61	2,09	1,49
Clay	1,65	2,23	2,90	2,07

If displacements in pipe were the same as in incident seismic wave, the axial stress in pipeline would define by expression

$$S_0 = -A_0 E' \omega_1 \cos \omega_1 Z$$

The ratio

$$n = \frac{\max S}{\max S_0} = \frac{\max S}{|A_0| E' \omega_1}$$

will be called the coefficient of dynamic overstress. Results of the previous section give that

$$n = \frac{2}{1 - (\omega_1/p)^2}$$

for supersonic case $(M > 1)$ and

$$n = \frac{1}{1 + (\omega_1/p)^2}\left(1 + e^{-\pi p/\omega_1}\right)$$

for subsonic case $(M < 1)$. The last value of the dynamic coefficient n is somewhat greater than the value given by Sakurai, A. and Takahashi, T. (1969).

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