Estimation of probability distribution of the residual displacement of bilinear SDOF systems after earthquake ground motions

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SUMMARY:
This paper discusses the effect that the ground motion and vibrational system characteristics has on the probability distributions of the residual displacement after earthquake ground motions, by considering the time history of plastic deformations as a random process. In this paper, a hypothesis of "random walk" is used, in which the magnitude of every single occurrence of plastic deformation is constant and its direction random. Through response analysis, it is shown that this theory is able to explain the probability distribution of the residual displacement in isotropic hardening bilinear SDOF systems. Since this hypothesis is not able to model the behavior of kinematic hardening systems where the secondary stiffness deviates from zero. The hypothesis is modified so that the plastic energy input at each occurrence of yielding is constant. With this modification, good estimates of the standard deviation of the residual displacements for kinematic hardening systems can be obtained.

Keywords: Random walk process, Post-earthquake residual displacement, Probability distribution

1. INTRODUCTION

The final aim of this research is to establish a method to estimate the residual displacement of a building after an earthquake. The post-earthquake residual displacement is considered as one of the possible indexes for representing the repairability of a damaged building, and has been used to propose several structural design scheme. (Christpoulos et al.(2003), Iwata et al. (2005), etc.) However, the idea of repairability design is not popular at this moment, because the value of the residual displacement has a large dispersion and is difficult to estimate quantitatively in a building after an earthquake.

Much effort has been made in order to quantify residual displacement. MacRae and Kawashima (1997) and Kawashima et al. (1998) have tried to evaluate the residual displacement of a bilinear single-degree-of-freedom(SDOF) system through a residual displacement spectrum. This spectrum represents the relationship between the natural frequency of the system and the residual displacement ratio which is a ratio of the residual displacement to the maximum displacement during an earthquake. They concluded that the spectrum is mostly affected by the secondary stiffness of the system, and as a result proposed a residual displacement design spectrum as a function of this parameter.

Akiyama and Takahashi (1996) have investigated the residual displacement of multi-story bilinear systems through response analysis, and proposed an empirical formula to evaluate the maximum and average residual displacement in terms of the secondary stiffness. This study also claims that the secondary stiffness has a significant effect on the residual displacement. However, the residual displacement obtained from the response analysis has a large dispersion and further study is required to improve the precision of the formula.

Ogawa et al. (1996) have proposed a model in which the seismic input energy of an SDOF system is divided into its positive and negative loading directions. This model is able to quantitatively explain the effect of the secondary stiffness and can be used to estimate the residual displacement. However, they mention that the accuracy may be affected by system or the input ground motion characteristics.

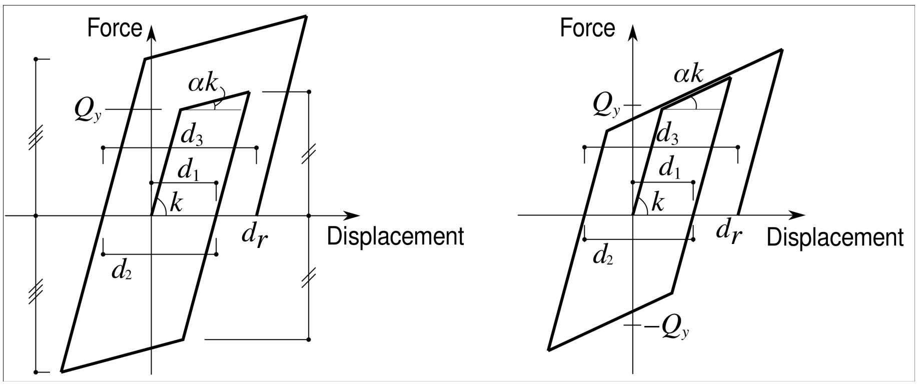
In recent years, many other researchers have conducted numerical studies on the prediction of the residual displacement. Generally speaking the results state that the residual displacement has a large dispersion and is greatly affected by the secondary stiffness, but are unclear about the effect of other parameters other than the secondary stiffness. Thus, further study is required to better estimate the residual displacement.

This paper introduces a theoretical probability distribution for distribution of residual displacements based on the hypothesis of a random walk where at each occurrence of yielding(walk) the plastic deformation is constant. Though this hypothesis is quite simple, it can explain the dynamic response analysis results of isotropic hardening bilinear SDOF system. It will be shown that this hypothesis can also be applied to a system with a non-zero secondary stiffness, by assuming that at each occurrence of yielding the plastic energy input is constant.

2. THEORETICAL PDF AND DEVIATION OF RESIDUAL DISPLACEMENT

2.1 Hysteretic characteristics

This paper discusses bilinear systems with isotropic and kinematic hardening, as shown in Fig. 1.



(a) Isotropic hardening (b) Kinematic hardening

Figure 1. Hysteretic characteristics

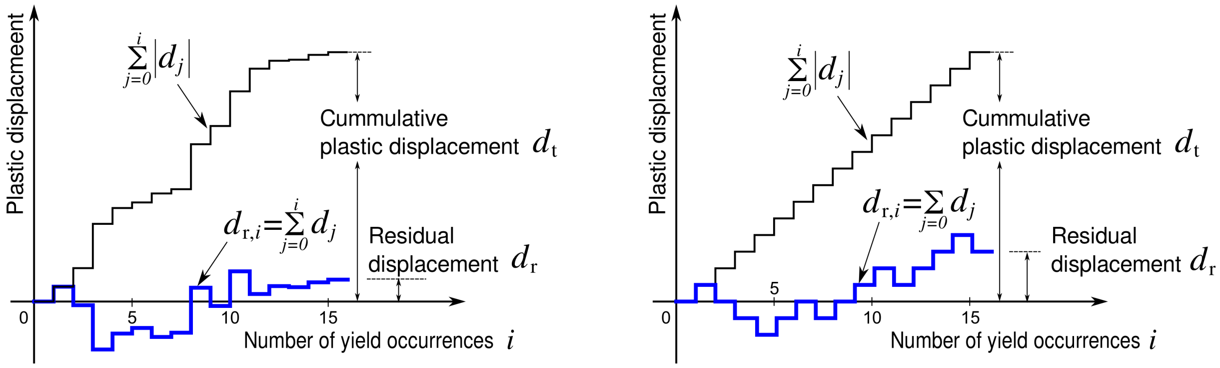
In this paper, the plastic deformation at the *i*-th occurrence of yielding is represented as d_i. The residual displacement d_{r} and the cumulative plastic displacement d_{t} are represented as,

$$d_{\mathrm{r}}=\sum_{i=1}^{n} d_i, \qquad d_{\mathrm{t}}=\sum_{i=1}^{n} |d_i| \qquad (1)$$

The normalized values of the plastic deformation at each occurrence of yielding, $\overline{d_i}$, and the residual displacement, $\overline{d_{\mathrm{r}}}$, are defined as,

$$\overline{d_i}=\frac{d_i}{d_t}, \qquad \overline{d_r}=\frac{d_r}{d_t} \qquad (2)$$

Fig. 2(a) shows a conceptual diagram of time histories of the residual displacement and the cumulative plastic displacement. The cumulative plastic displacement increases monotonically, while the residual displacement increases and decreases according to the time-history characteristics of the input ground motions.



(a) Example of response analysis result

(b) Random walk hypothesis with constant plastic displacement

Figure 2. Time history of cumulative and residual plastic displacement

2.2 "Random walk" hypothesis with constant plastic displacement

To model the behavior of the time history of the residual displacement, it is assumed to be a "random walk", in which the magnitude of every single plastic displacement is constant and its direction is completely random. Under this assumption, as shown in Fig. 2(b), the normalized plastic deformation at the *i*-th occurrence of yielding, $\overline{d}_i$, should be either $+1/n$ or $-1/n$, each with a 1/2 probability of occurring. Here *n* represents the number of occurrences of yielding during an earthquake. This assumption leads to an estimation that the probability distribution of the post-earthquake residual displacement, $\overline{d}_{\mathrm{r}}$, is a binomial distribution with a standard deviation of $1/\sqrt{n}$.

2.3 "Random walk" hypothesis with constant energy input

In the case of kinematic hardening, the positive and negative yielding loads are different and the positive and negative plastic displacements at each occurrence of yielding may not be similar in magnitude. In this section, instead of assuming that the magnitude of the plastic displacement is the same, the plastic energy input is. The direction of the deformation is still assumed to be the same. Fig. 3 explains the detailed behavior of the plastic deformation at the *i*-th occurrence of yielding under this assumption. The system is assumed to have an elastic energy $E_{\mathrm{e},i}$ before the *i*-th plastic deformation, as shown in Fig. 3(a). In Fig. 3(b), from an energy input of E_{p1}, the system plastically deforms to absorb the energy. As a result, the residual displacement at this occurrence of yielding is considered $d_{\mathrm{r},i}$ as shown in Fig. 3(c), and the elastic energy $E_{\mathrm{e},i+1}$ will be used in the next yielding step.

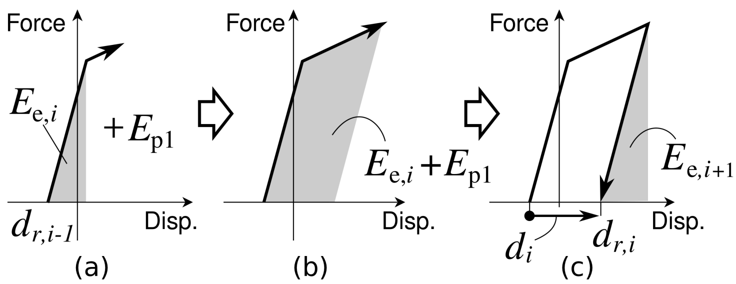


Figure 3. Assumed behavior during a single plastic deformation

In contrast to the random walk hypothesis with constant plastic displacement, the derivation of an exact closed-form formula to estimate the standard deviation of the residual displacement under this hypothesis is extremely difficult. However, it is possible to obtain the probability distribution and its standard deviation numerically through a Monte Carlo simulation using the procedure shown in Fig. 4.

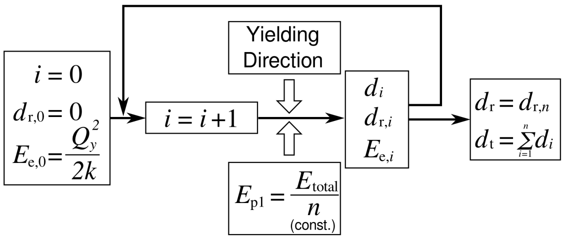


Figure 4. Procedure to compute the residual displacement under the hypothesis of constant energy input

3. RESPONSE ANALYSIS METHOD

3.1 Analysis model

A bilinear SDOF model with isotropic and kinematic hardening, as shown in Figure 1(a) and (b), are used in the response analysis. The parameters of the model are the natural period, the yielding load, and the secondary stiffness ratio α . The natural period is varied from 0.1 s to 4 s at a 0.02 s interval by changing the initial stiffness. The yielding load, Q_y , is determined as $Q_y = D_s Q_e$, where Q_e is the maximum restoring force of the elastic model, and for D_s , values of 0.2, 0.4, 0.6, 0.8 are used. For the secondary stiffness ratio α , 7 values (0, ± 0.02 , ± 0.05 , ± 0.1) are used.

3.2 Input ground motions

In this paper, ground motions of the 2011 Tohoku Earthquake obtained from KiK-net X) are used. From various locations, 166 records which have a maximum acceleration larger than 20 gal have been selected. These have been selected irrespective of the measured direction.

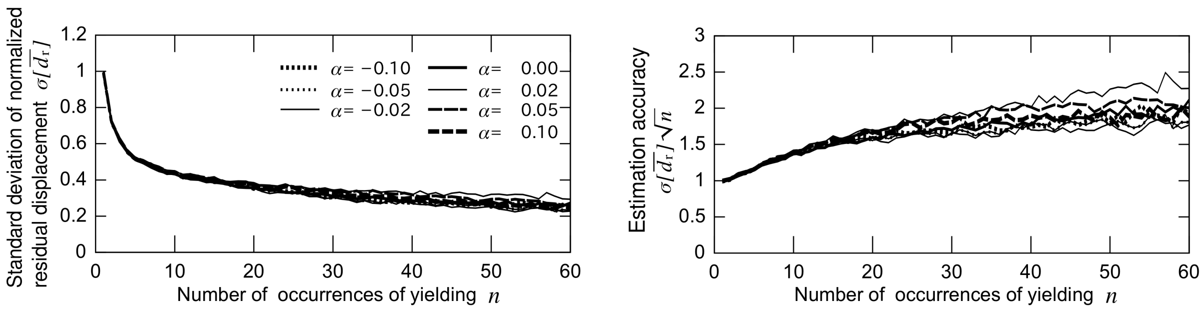
4. RESPONSE ANALYSIS RESULTS

4.1 Standard deviation of residual displacement of isotropic bilinear system

Figure 5(a) shows the relationship between the number of occurrences of yielding, *n,* and the standard deviation of the normalized displacement, $\sigma[\overline{d}_r]$, for the isotropic hardening bilinear system. The value of $\sigma[\overline{d}_r]$ corresponding to each *n* has been calculated as the standard deviation of an ensemble of $\overline{d}_r$, which all yield n times, selected from the set of the response analysis results obtained with various parameters and ground motions. Different line types represent different α .

Figure 5(b) shows the accuracy of the hypothesis. The y-axis is $\sigma[\overline{d}_r] \times \sqrt{n}$. When this value equals 1.0, the random walk hypothesis agrees with the analysis result.

As shown in Figure 5(a) and 5(b), the curves of the analytical results with various α 's are almost identical. This means that the effect of α on the accuracy of the hypothesis is negligible.

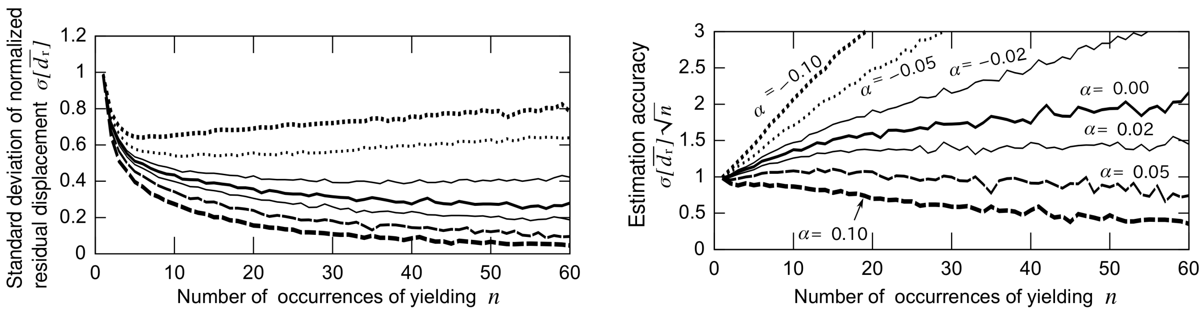


(a) Standard deviation of the residual displacement (b) Accuracy of the estimate under the hypothesis

Figure 5. Analysis results with isotropic hardening model

4.2 Standard deviation of the residual displacement of a kinematic bilinear system

Fig. 6(a) shows the relationship between the number of occurrences of yielding, *n,* and the standard deviation of normalized displacement, $\sigma[\overline{d}_{\mathrm{r}}]$, and 6(b) the estimation accuracy, $\sigma[\overline{d}_{\mathrm{r}}]\times\sqrt{n}$, with respect to *n*.



(a) Standard deviation of the residual displacement (b) Accuracy of the estimate under the hypothesis

Figure 6. Analysis results with kinematic hardening model

$\sigma[\overline{d}_{\mathrm{r}}]$ is strongly affected by α, and the value of $\sigma[\overline{d}_{\mathrm{r}}]\times\sqrt{n}$ is deviates from 1.0 as α differs from 0.0. This means that the random walk hypothesis with constant plastic displacement does not agree with the analysis results for the bilinear model with kinematic hardening.

4.3 Validity of the random walk hypothesis with constant energy input.

Here, the Monte Carlo simulations under the hypothesis with constant energy input and the response analysis results are compared.

From a single vibrational system and ground motion record pair, the response analysis computes one value of $\overline{d}_{\mathrm{r}}$. On the contorary, the Monte Carlo simulation computes the theoretical probability distribution of the residual displacement, $\overline{d_{\mathrm{r,\,simulated}}}$, and therefore its standard deviation, $\sigma[\overline{d_{\mathrm{r,simulated}}}]$. Theoretically, the value $\overline{d}_r/\sigma[\overline{d_{\mathrm{r,simulated}}}]$ should follow a standard normal distribution, so its standard deviation, $\sigma[\overline{d}_r/\sigma[\overline{d_{\mathrm{r,simulated}}}]]$, should be 1.0. Therefore, if $\sigma[\overline{d}_r/\sigma[\overline{d_{\mathrm{r,simulated}}}]]$ is close to 1.0, the response analysis results follow the hypothesis.

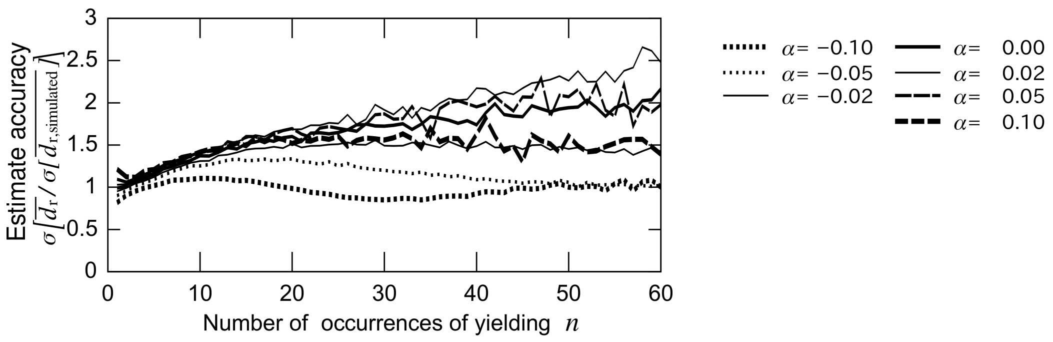


Figure 7. Accuracy of estimation by Monte Carlo simulation in case of kinematic hardening model

Fig. 7 shows the standard deviation of $\overline{d_r}/\sigma[\overline{d_{r,simulated}}]$ under various α, with respect to the number of yielding occurrences, *n*. The curve of $\alpha=0$ in Fig. 7 is identical to the corresponding one in Fig.6(b), since $\sigma[\overline{d_{r,simulated}}]=\sqrt{n}$ for this case.

Compared to Fig.6(b), all the curves in Fig.7 are much closer to the curve with $\alpha=0$. This means that the hypothesis can eliminate the effect of α. As a result, they are also closer to 1.0, which means the hypothesis can give better estimates compared to Fig.6(b). For the case of $\alpha=0$, these two hypothesis are completely the same and therefore the lines with $\alpha=0$ in Fig.6(b) and Fig.7 are the same. The precision in the case of $\alpha=0$ is affected by the vibrational system and ground motion characteristics, as discussed in Iyama (2012), and larger than 1.0 especially where *n* is larger in this study. However, in many cases, the value of *n* is less than 10 and Fig.7 shows that within this range the estimate accuracy of the simulation is less than 1.5 irrespective of α.

5. CONCLUSION

In this paper, the occurrences of plastic deformation is considered as a simple one dimensional "random walk" problem, where the amount of plastic deformation at each step is constant and its direction random. This assumption theoretically leads to a relationship stating that the standard deviation of the probabilistic distribution of residual deformations is inversely proportional to the square root of the number of occurrences of yielding. This analytical relationship is capable of explaining the trend observed in the numerical results from dynamic response analysis of isotropic hardening bilinear systems. However, for kinematic hardening systems, the standard deviation of the distribution of residual deformations estimated from this equation has large errors when the post-yield stiffness differs from zero, because the assumption does not consider its effect.

In order to incorporate the effect of the post-yield stiffness, the energy absorbed from every single plastic deformation is assumed constant. With this further assumption, an analytical relationship between the standard deviation of the residual displacement distribution and the number of yielding occurrences can no longer be obtained. Therefore, a Monte Carlo simulation is used to estimate this relationship numerically. Comparison between the results from the Monte Carlo simulations and the dynamic response analysis show that the additional assumption is able to greatly improve the accuracy of the estimation.

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