DEFORMATION CAPACITY OF STRUCTURAL MASONRY: A REVIEW OF THEORETICAL RESEARCH

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SUMMARY
A research project on the deformation capacity of unreinforced masonry is underway at the Institute of Structural Engineering of ETH Zurich. The development of the basic building blocks for the displacement-based design of masonry structures is the objective of the present research project, which should be seen as a first step in an initiative to investigate the limits of the deformation capacity of structural masonry. This paper presents a summary review of previous theoretical studies on the deformation capacity of structural masonry. This review is the first phase of a three year long research program, launched by the authors, whose objective is highlighted above. The review includes existing models for in-plane response of URM walls. The reviewed models are discussed and a set of conclusions is given. Special attention is devoted to the ability of the reviewed models to predict the deformation capacity of structural masonry.

Keywords: Deformation capacity, in-plane response, mechanical model, structural masonry, URM

1. INTRODUCTION

Masonry structures and materials represent one of the oldest building concepts available. Masonry construction is a traditional, widely used, extremely flexible and economical construction method with considerable potential for future developments. However, possibly due to the substantial empirical knowledge collected through several centuries of utilization of masonry as a structural material, the need for establishing a more modern basis for the design of masonry structures has not been appreciated in the same manner as for concrete structures. As a result, conventional masonry design practice is over conservative, particularly in regard to the assessment of seismic resistance. Hence, the potential of masonry has not yet been fully exploited and there is a clear need for better utilization. For example, while current codes of practice extremely narrow the possibility of construction with unreinforced masonry, mainly because of prescription of over conservative values for the behaviour factor, recent studies show that the performance of structurally-designed low-rise URM buildings should be considered adequate for the ordinary buildings category even in regions with appreciable seismic hazard. These studies also show that unreinforced masonry is still very competitive choice for two- or three-story residential buildings (Magenes et al. 2009 and Lourenco et al. 2009).

Based on the positive experience gained during the recent past in developing the basis for the displacement-based design of concrete structures, it appears that the most feasible approach to enhance the rationality for the design of masonry structures is to apply the same basis in the analysis of masonry structures. A more consistent representation of the material resistance as well as of the (seismic) loads leads to more economical designs in general, and especially for masonry structures where the safety margin, i.e. safety factor at present, is mostly based on experience rather than quantified engineering modelling. The development of the basic building blocks for such an approach is the objective of the present research project, which should be seen as a first step in an initiative to investigate the limits of the deformation capacity of masonry structures.

In masonry structures subjected to seismic actions, if local brittle failure modes, e.g. out-of-plane failure, are prevented by providing proper connections between intersecting walls and also between walls and diaphragms, a rather ductile global behaviour which is governed by the in-plane response of walls can develop. Hence, the investigation of the deformation capacity of masonry structures should be initiated from studying the in-plane behaviour of masonry walls and their constitutive elements, i.e. piers and spandrels (in case of perforated walls). Even though spandrels have a significant effect on the in-plane response of URM masonry walls, it seems, however, that the deformation capacity of masonry walls is mainly identified with the deformation capacity of piers. A substantial amount of experimental and theoretical work has been carried out to investigate the in-plane response of masonry piers. This paper presents a summary review of previous theoretical studies on the seismic response of masonry piers, with special attention to the ultimate deformation capacity parameter. It is worth noting that very little experimental and theoretical research has been carried out so far on the response of masonry spandrels, and only recently they have been subjected to full-scale in-plane cyclic testing (Magenes and Penna 2011). Clearly, to better understand the deformation capacity of structural masonry, there is a need for a thorough investigation of the in-plane behaviour of masonry spandrels.

2. IN-PLANE BEHAVIOUR OF URM PIERS

The in-plane behaviour of masonry walls subjected to horizontal and vertical forces has been investigated in various test programs. As indicated by experiments, the in-plane response of masonry walls depends mainly on their failure mechanisms. In case of low vertical load and/or poor quality mortar, seismic loads cause shearing of the wall in two parts and sliding of the upper part on the other part. The mechanism is called sliding shear failure. The in-plane response of masonry walls failing in sliding shear mode is very stable and close to an elastic-perfectly plastic response with high energy dissipation and displacement capacity. In case of sliding shear failure mode, the displacement capacity is theoretically unlimited. However, for practical applications it should be limited since the shear walls normally interact with other building elements.

The diagonal shear mode takes place where the principal tensile stress exceeds the tensile strength of masonry. Peak resistance is governed by the formation and development of diagonal cracks. In case of the diagonal shear mode, the typical response of masonry walls is characterized by rapid strength and stiffness degradation, moderate energy dissipation and limited displacement capacity. Although diagonal shear mechanism which often governs the in-plane response of masonry walls subjected to seismic loads has limited deformation capacity, a classification of such mechanism as plainly brittle would lead to significant underestimation of the seismic capacity of the masonry buildings. Hence, a moderate ductility, or better, a non-negligible nonlinear behaviour and deformation capacity has to be recognised for the diagonal shear failure mode (Magenes and Penna 2011).

The rocking-flexural failure usually takes place in case of high moment/shear ratio, i.e. slender walls. As horizontal load increases, bed joints crack in tension, and shear is carried by the compressed masonry. Final failure is obtained by crushing of the compressed corner. In general, in-plane response of masonry walls failing in rocking-flexural mode is almost nonlinear elastic with very moderate hysteretic energy dissipation and negligible strength degradation. Regarding the displacement capacity, very large displacements can be obtained, especially when the axial load is low compared to the compressive strength of masonry. Actually, if no other failure mechanisms occur, the displacement, which can be attained in a rocking response, can be limited only by the second order (P-Δ) effects associated with overturning (Magenes and Calvi 1997).

3. COMPUTATINAL STRATEGIES FOR STRUCTURAL MASONRY

A substantial amount of theoretical work has been invested in modelling structural masonry. Simple models are based on the linear theory of elasticity and its application to structural masonry. Regarding the serviceability limit state, i.e. when investigating the behaviour of masonry subjected to load levels up to 40-50% of the ultimate load, the applicability of the linear theory of elasticity is beyond dispute. However, when approaching higher load levels nonlinear modelling is generally required. Hereby, both geometrical and material nonlinearities must be taken into account. In general, there are three sources of nonlinearity in the in-plane response of URM piers: geometrical nonlinearity due to the evolutionary partialization of cross-sections as cracking spreads within the panel, material nonlinearity in the elastic range and material nonlinearity in the plastic range (Augenti and Parisi 2009a). Hence, a reliable model must be able to consider all the above-mentioned sources of nonlinearity in a proper way. Since only very few closed-form solutions for nonlinear problems are available, numerical solution methods must be applied. Such solutions are usually obtained by means of Finite Element Method (FEM) procedures. However, the main problem when applying FEM is related to the modelling of the material. Since masonry is composed of two components, i.e. masonry units and mortar, and is highly anisotropic and nonlinear, modelling the physical reality is very demanding.

In general, three different approaches are found in literature for modelling seismic response of URM structures: micro-modelling, macro-modelling and macro-element discretization. In micro-modelling strategy, the different components, i.e. the units, mortar, and the unit-mortar interface are distinctly represented. In detailed micro-models, masonry units and mortar joints are represented by continuum elements, whereas the masonry unit-mortar interface is represented by discontinuous elements (Figure 3.1.a). The detailed micro-modelling requires great computational effort. This drawback is partially overcome by the simplified micro-models. In the simplified micro-modelling strategy, masonry units are represented by continuum elements whilst the mortar joints and masonry unit-mortar interface are lumped into discontinuous elements (Figure 3.1.b). The micro-modelling approaches are suitable for small structural elements with particular interest in strongly heterogeneous states of stress and strain. The primary aim is to closely represent masonry based on the knowledge of the properties of each constituent and the interface (Roca et al. 2010). In macro-modelling strategy, masonry is treated as a fictitious homogeneous orthotropic continuum with different tensile and compressive strengths as well as different inelastic properties along the material axes (Figure 3.1.c). In particular, FE meshes are simpler since they do not have to accurately describe the internal structure of masonry and the finite elements can have dimensions greater than the single brick units (Roca et al. 2010).

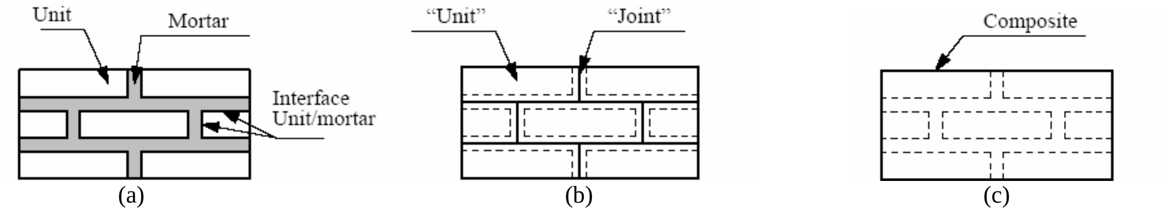


Figure 3.1. Detailed (a) and simplified (b) micro-modelling; macro-modelling (c) (Lourenco 1996)

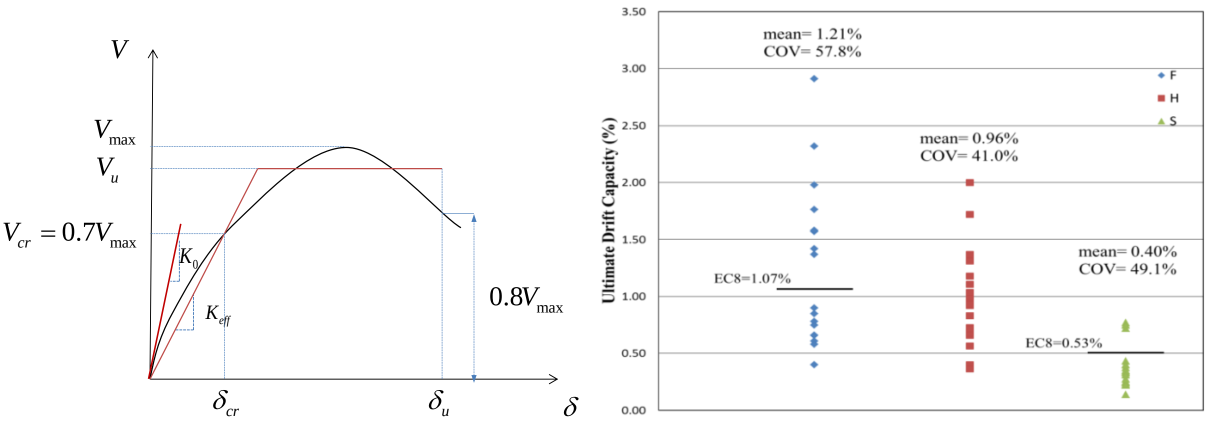
Despite significant progress has been made in the field of micro- and macro-modelling strategies, e.g. development of the so-called homogenized modelling, see e.g. Lourenco et al. (2007), still these approaches are not suitable for analysis of whole buildings in everyday engineering practice. This is because a considerable number of material parameters are needed as input for a meaningful analysis using these approaches, and these parameters are usually unavailable. Furthermore, the current micro and macro models have a limited range of validity and also require significant computational resources and high expertise. In addition, due to the great difficulty in the formulation of robust numerical algorithms representing satisfactorily the inelastic behaviour of masonry, micro and macro analyses of masonry structures are often limited to the structural pre-peak regime (Maruccio 2010 and Xu et al. 2011). However, the importance of the post-peak response is clear in order to evaluate the deformation capacity and to assess the structural safety. A recent comprehensive review on the micro- and macro-modelling approaches for the masonry structures can be found in Roca et al. (2010).

As a consequence, several methods based on macro-element discretization have been developed, particularly in Italy. In this approach, each panel in the structure, i.e. piers and spandrels, is modelled by using a single element. Such elements called macro-elements are based on the simplification of both the material behaviour and the stress field within the panel. These elements seem the most appropriate for design and assessment of masonry buildings because of the simplicity of modelling, the straightforward interpretation of the results, particularly in terms of collapse mechanisms, and the accuracy demonstrated in different validations (Lourenco et al. 2009 and Grande et al. 2011). The use of macro-elements for the nonlinear analysis of masonry structures has been introduced in several guidelines, e.g. FEMA 356 (ATC 2000) and Eurocode 8 (CEN-EN 1998-3 2005), with particular reference to the use of pushover analysis method. In the following section, a review on the macro-elements for seismic analysis of unreinforced masonry piers is presented with emphasis on their ability to predict the ultimate displacement capacity.

4. MACRO-ELEMENTS FOR URM PIERS

4.1. One-dimensional macro-elements

The simplest one-dimensional macro-elements are single shear springs which represent experimental resistance envelopes of URM piers with idealised bilinear (linear-elastic, perfect plastic) relationships. In order to determine the idealized bilinear envelope curve, after construction of the experimental hysteretic envelope, three parameters must be identified: the effective stiffness (K_{eff}), the ultimate shear strength (V_u) and the ultimate displacement capacity (δ_u). The effective stiffness is usually calculated from a secant of the cyclic envelope at $0.7V_{max}$, where V_{max} is the maximal lateral load obtained from the test. The definition of the ultimate displacement capacity is somehow subjective. However, it is usually defined as the displacement corresponding to the strength degradation of 20%. The ultimate shear strength is obtained by equating the areas under the experimental and bilinear envelopes, see Figure 4.1.a (Tomazevic 1999).



(a) (b)

Figure 4.1. Bilinear idealization of hysteretic envelope (a); Experimental drift capacity of URM walls (b)

In general, the effective stiffness is a very complex parameter and difficult to determine. For practical applications it is usually taken as the elastic stiffness, K_0 (see Figure 4.1.a), which is calculated based on the elastic beam theory incorporating shear deformation, or as 50 percent of the elastic stiffness, see e.g. Tomazevic (1999), Eurocode 8 and FEMA 356. It should be noted here that, in general, the determination of the effective stiffness is subject to variation and the data obtained from tests exhibits a large scatter. Comparison with experimental results shows that the effective stiffness varies between 40 percent and 80 percent of the elastic stiffness, and it depends strongly on the pre-compression level (Bosiljkov et al. 2005). Concerning the ultimate shear strength, it has been found by evaluating the results of several tests that the choice of $V_u=0.9V_{max}$ is appropriate for the ultimate shear strength (Tomazevic 1999). The maximum shear resistance of URM piers, V_{max}, can be determined according to the formulations provided by the codes of practice.

Different recommendations could be found in the literature for the ultimate displacement capacity parameter which are basically based on statistical analysis of the results of the past experiments. Unfortunately, the proposed values are not always readily applicable since the data obtained from tests exhibits a rather large scatter. A review on the technical literature on experimental research on the deformation capacity of structural masonry conducted by the authors indicated that the mean values of the ultimate drift capacity (the ultimate deformation capacity divided by the height of the specimen) for 71 shear tests performed on full-scale unreinforced masonry shear walls made of clay bricks were 1.21%, 0.96% and 0.40% for flexural, hybrid and shear failure modes, respectively, whereas the ultimate drift capacity provided by Annex 3 of Eurocode 8 is 0.53% for unreinforced masonry walls with the diagonal shear failure mode and 1.07% for walls failing in the rocking-flexural for the limit state of Near Collapse (NC). The analysis also pointed up that the values of the ultimate drift capacity were too scattered so that the corresponding values of COV for walls with flexural, hybrid and shear failure modes were 57.8%, 41.0% and 49.1%, respectively, see Figure 4.1.b (Salmanpour et al. 2012).

In general, the deformation capacity of structural masonry is influenced not only by the failure mechanism but by many other factors such as constituent materials, geometry, pre-compression level, etc. Due to inhomogeneous experimental data and a lack of reliable mechanical models, we are still not able to properly take into account the influence of all factors affecting the deformation capacity of structural masonry. Obviously, to get a clearer picture on the problem, in addition to conducting more tests, we need to develop reliable mechanical models to describe the load-deformation behaviour of structural masonry.

Among the methods using bilinear shear springs, the POR method (Tomazevic 1978) and the FEMA 356 method are well-known and extensively used. The POR method is an equivalent static, simplified nonlinear assessment method which assumes that the failure occurs only in the piers without any damage of spandrels. This method which is historically the first seismic assessment method for structural masonry is based on the story mechanism approach. The procedure consists of a separate interstory shear-displacement curve for each story, where each masonry pier is typically modelled by a linear elastic-perfectly plastic shear spring with limited ductility. The FEMA 356 method also employs nonlinear shear springs to model the force-displacement response of individual piers. The spandrels of URM walls are only considered to affect the boundary conditions of the piers, i.e. fixed-fixed or cantilever. The force–displacement relationship for each pier is defined based on the governing failure mode which is taken as the failure mode of least lateral resistance. In conclusion, the macro-elements which are based on bilinear idealization of experimental resistance envelope are not reliable, particularly regarding the prediction of the ultimate displacement capacity, due to large scatter in available experimental data.

Magenes and Della Fontana (1998) and Magenes et al. (2000) proposed an improvement to the POR method based on the so-called equivalent frame idealization. In the proposed method, termed SAM (Simplified Analysis of Masonry buildings), both the spandrels and the piers are modelled as beam-column elements with shear deformation, while their intersections are modelled by means of rigid offsets at the ends of the pier and spandrel elements. To describe the nonlinear response of piers and spandrels, the SAM method employs several plastic hinges which are located by the user to account for the possible failure modes. Typically these plastic hinges are placed at both ends and at the mid-span of the beam-column elements to capture the flexural and the shear failure modes, respectively. This approach can be easily implemented by the conventional commercial programs, e.g. SAP2000 (Pasticier et al. 2008). However, since the properties of the plastic hinges are mainly based on the available experimental data, as discussed before, the model suffers from the lack of reliability of the results, particularly in terms of the displacement capacity.

Figure 4.2.a shows the macro-elements proposed by Chen et al. (2008) for URM piers and spandrels. The proposed approach is supposed to be used in conjunction with FEMA 356. This approach, which provides rotational, shear, and axial springs in series, was first introduced for the analysis of reinforced concrete (RC) shear walls (Kabeyasawa et al. 1982 and James and Kunnath 1994). The elements developed for RC shear walls were improved for modelling of URM piers by the addition of two shear springs at the top and bottom of the macro-element to account for bed joint sliding deformation in these regions. For modelling spandrels constructed of running bond masonry, these sliding springs are not needed because the interlocking of units prevents sliding along head joints. As shown in Figure

4.2.a, these macro-elements have three degrees of freedom (DOF) at each end and thus can be used within the equivalent frame idealization strategy for the analysis of perforated URM walls.

Regarding the pier element, the axial spring has linear elastic response in compression and no resistance to tensile forces beyond the tensile strength of masonry. The properties of the flexural springs are based on the moment-curvature response of the top and bottom sections of the pier, which are established through the use of a fibre model. The moment-rotation properties of the rotational springs are then obtained by integrating the curvature along the height of the pier. The adopted force-displacement relationships for the shear springs corresponding to the bed joint sliding and diagonal tension behaviour are shown in Figures 4.2.b and 4.2.c.

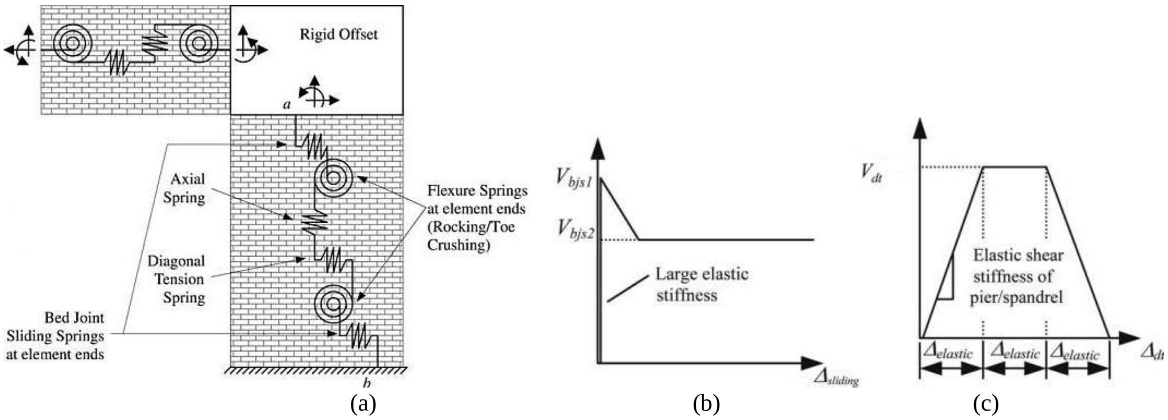


Figure 4.2. Macro-elements for URM piers and spandrels (a); adopted force-displacement relationships for bed joint sliding (b) and diagonal tension behaviour (c) (Chen et al. 2008)

Figure 4.3 provides a comparison between the results of the macro-element simulation and experimentally obtained hysteresis response for the different failure modes. It can be seen that in case of rocking-flexural and shear sliding failure modes, the simulation results are almost satisfactory, but in case of diagonal shear failure, the simulation is highly erroneous. The error arises mainly from the adopted force-displacement response for diagonal tension spring and also from the disability of the element to model the geometrical nonlinearity.

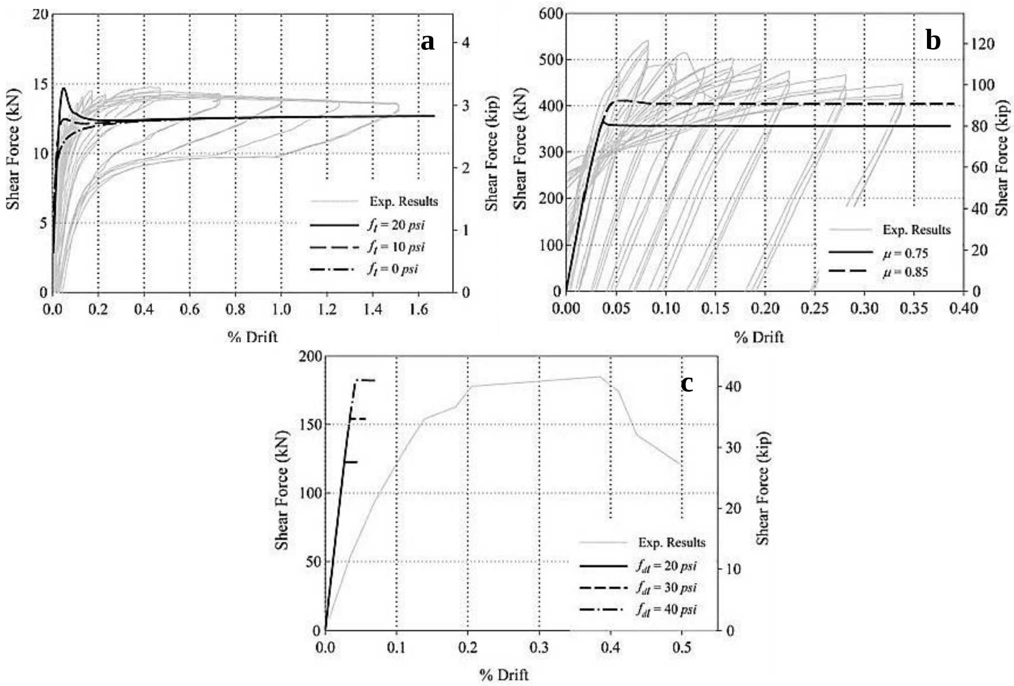


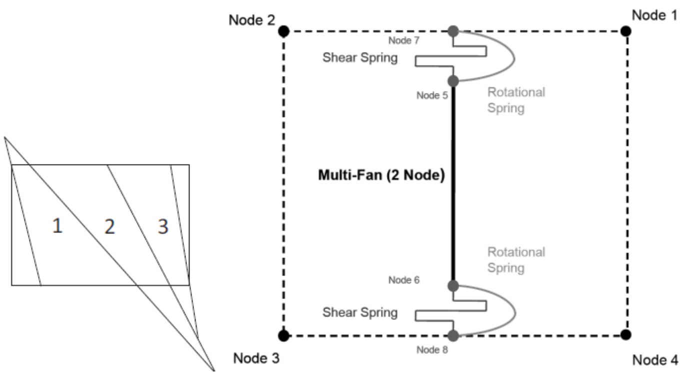
Figure 4.3. Comparison between the macro-element simulation and experimental results for flexural (a), bed joint sliding (b) and diagonal tension (c) failure modes (Chen et al. 2008)

4.2. Two-dimensional macro-elements

There are some limitations in the use of one-dimensional macro-elements, namely due to inaccurate simulation of the interaction between piers and spandrels, and due to the weak modelling of the geometrical nonlinearity of the panels (Marques and Lourenco 2011). Two-dimensional macro-elements cannot be applied within the equivalent frame idealization framework and require more computational efforts compared to one-dimensional macro-elements. However, they offer a more accurate simulation of the nonlinear response of the masonry piers and spandrels, particularly in the pre-peak regime. This is because unlike one-dimensional elements, these elements are able to simulate the propagation of the tensile cracks along the height of pier, i.e. geometrical nonlinearity.

The “no-tension multi-fan panel element” was developed by Braga and Liberatore (1990) based on the idea that the stress field of a masonry panel with free edges follows a multi-fan pattern (Figure 4.4.a). In addition, it is assumed that the upper and lower faces of the panel are rigid, and that there is no interaction in the circumferential direction between the infinitesimal fans. The material behaviour is assumed linear elastic in compression and non-reacting in tension. There is a very good agreement between simulation results obtained using multi-fan element and experimental results up to a certain level of lateral displacement, but at the higher displacements, the accuracy of simulation decreases rapidly because of the adopted elastic constitutive law (Liberatore et al. 1996).

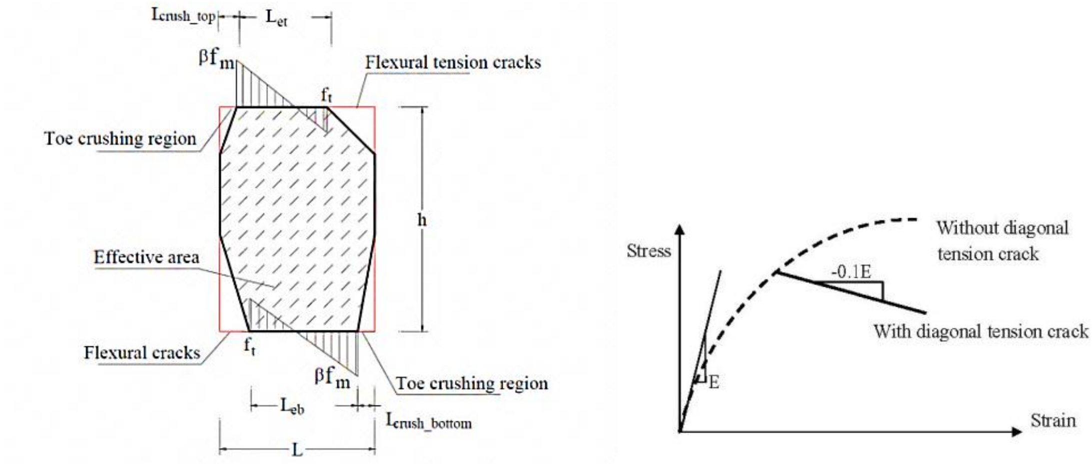
A modification to the multi-fan element proposed by Maruccio (2010). The updated multi-fan element, introduces zero-length springs to the multi-fan element to add failure mechanisms in the constitutive law, see Figure 4.4.b. In fact, the behaviour in the elastic stage is defined by a set of radial stress fields in the panel, while the springs are required to define failure mechanisms and the inelastic response. The properties of these springs are based on past component tests. Therefore, as discussed previously in detail, the simulation results could not be considered reliable in terms of the displacement capacity. The multi-fan model seems to have a considerable potential for further development.



(a) (b)

Figure 4.4. Multi-fan element (a); modified multi-fan element (b) (Maruccio 2010)

Yi et al. (2005) proposed a macro-element, named “Effective pier model”, to describe the nonlinear behaviour of an individual URM pier subjected to external forces. The model describes the effective area of the pier by eliminating the flexural tensile cracks and the toe crashing regions, see Figure 4.5.a. Regarding the toe crushing, the model assumes that the compressive strength of masonry immediately drop to zero after the ultimate compressive strength of the masonry has been exceeded. Although this assumption is questionable, it greatly simplifies the problem and results in a conservative strength estimate (Yi et al. 2005). However, more sophisticated stress-strain relationships which account for the nonlinear stress-strain behaviour of masonry can be employed if needed, see e.g. the macro-element model developed by Augenti and Parisi (2009b). In order to address the bed joint shear sliding, the model employs the Mohr-Coulomb friction model. Regarding the diagonal tension, a smeared crack technique is employed. It is assumed that the effective area of the pier remains continuous even after the development of diagonal cracks. To model the potential rapid and unstable propagation of these cracks, the effective tangent elastic modulus of masonry is assumed to be a negative value. since no test data are available for the softening behaviour of URM piers after the diagonal cracking, the tangent modulus of URM piers with diagonal tension crack is set equal to *-0.1E*, where *E* is the initial elastic modulus of masonry (Figure 4.5.b).



(a) (b)

Figure 4.5. Effective pier model (a); principal compressive stress-strain relationship of masonry before and after diagonal tension cracks (b)

According to this model, when a URM pier experiences a reduction of the cross section due to either tensile or compressive failure, the remaining part of the pier will typically be inclined at some angle. Hence, after the cracking, a portion of the lateral force will be resisted through axial deformation, and the lateral force which causes shear and flexural deformation will be $V - P\tan(\theta)$, where θ is the angle between the central axis of the pier and the vertical line, and V and P are applied lateral and vertical loads, respectively. The proposed model only considers the lateral deformation of the pier induced by flexure and shear:

$$\Delta = \frac{V - P\tan(\theta)}{K}, \qquad K = 1/\left[\frac{4\gamma h^3}{EL^3 t} + \frac{h}{GLt}\right] \tag{4.1}$$

where, Δ is the lateral deformation; K is the lateral stiffness of the pier; γ is a coefficient that describes the boundary conditions of the pier (γ is equal to 0.83 for double fixed, and 3.33 for cantilever boundary conditions); E is the elastic modulus of masonry, and G is the shear modulus of masonry which is taken as *0.4E*. In order to consider the nonlinear compressive stress-strain behaviour of the masonry, the model uses the relationship proposed by Naraine and Sinha (1989).

Figure 4.6 provides a comparison between the results of the Effective pier model and experimentally obtained hysteresis response for the different failure modes. It can be seen that the effective pier model is able to partially predict the behaviour of masonry piers failing in sliding shear or rocking-flexural modes, but in case of rocking-flexural, there is no good agreement between the model and the experimental results, except at the beginning of the response.

In conclusion, although two-dimensional macro-elements provide better results compared to one-dimensional ones (because of their ability to simulate the geometrical nonlinearity), as shown, they are still not reliable concerning the prediction of the ultimate deformation capacity. It should also be noted that current macro-elements cannot distinguish between the diagonal cracks passing through the units and those running through bed and head joints, while in the former case, the deformation capacity is limited, but in the latter case, there is a considerable displacement capacity -the displacement capacity is actually equal to a certain percentage of the units length, e.g. 50% of units length in a running bond masonry wall. Hence, there is a need for a thorough investigation on the problem already mentioned above.

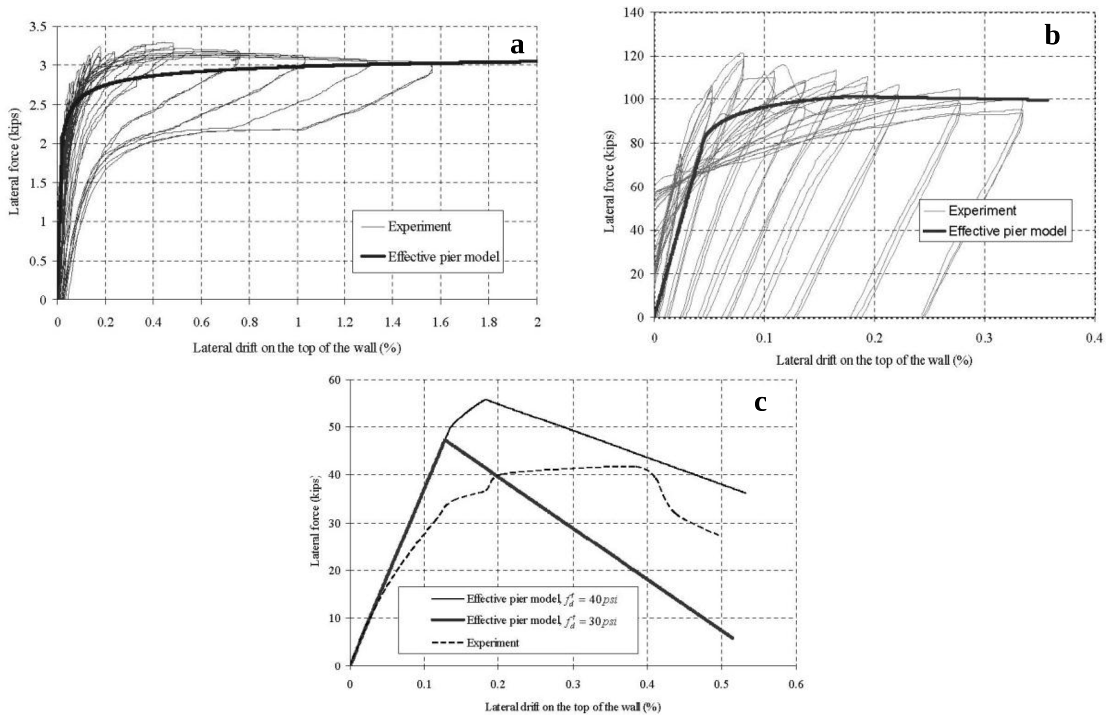


Figure 4.6. Macro-element simulation of the flexural (a), bed joint sliding (b) and diagonal tension (c) response using Effective Pier model

5. CONCLUSIONS

The ultimate deformation capacity is the most important parameter in seismic design and evaluation of structures. Our current state of knowledge about the deformation capacity of structural masonry is limited. The available experimental data are too scattered, and it is not possible to identify a rational value for the deformation capacity of masonry structures based only on such experimental data. Furthermore, there are no reliable analytical models for the force-deformation relationship of structural masonry: refined finite element models suffer from numerical instabilities in post-peak regime, and available structural macro-elements are still so far from being considered accurate enough regarding the deformation capacity parameter, especially in case of the diagonal shear failure mode.

Obviously, to get a clearer picture on the problem, in addition to conducting more tests, we need to develop reliable mechanical models to describe the load-deformation behaviour of structural masonry. This task is being approached within the framework of the current research project. The research project will include several cyclic static shear tests on full-scale, story-high masonry walls as well as developing and introducing new sophisticated mechanical models for structural masonry. A novel approach will be developed and utilized for the purpose of applying experimental evidence collected from our own tests performed for the development of reliable mechanical models.

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