Stiffness-Based Sizing of Bracing Systems for Tall and Slender Buildings

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SUMMARY
A stiffness-based methodology for the preliminary sizing of the braces and support columns of a steel bracing system is discussed. The methodology applies to the case of tall earthquake-resistant buildings, whose dynamic response is significantly influenced by global flexural drifts and higher modes of vibration. The preliminary sizing of the structural members of several versions of a bracing system for a twenty four-story building is carried out. From the evaluation of the dynamic characteristics of the different versions of the bracing system, it is concluded that the proposed methodology results in adequate stiffness-based sizing during the performance-based preliminary design of tall braced buildings.

Keywords: Stiffness-based sizing, Displacement-based design, Bracing system, Tall buildings

1. INTRODUCTION

Structural design of tall buildings under lateral forces is usually governed by drift control. Within this context, it has been considered that the most important property of a structural system is its lateral stiffness and that rigid frame systems alone are not an efficient solution for tall steel buildings. Not surprisingly, steel bracing has been usually provided to this type of buildings to efficiently control their lateral drifts within acceptable limits.

In the last two decades, various sizing methods have been developed for the structural elements of tall buildings. The majority of these methods are evolutionary or iterative in nature, in such a manner that an initial solution is refined in search of an optimal solution that satisfies a series of drift and strength constraints. In many cases, a structural or performance parameter is used to formulate an optimization problem that often requires complicate analytical tools and the careful formulation of extra constraints.

Initially, sizing methods for braced frames aimed at optimizing the weight or a performance parameter within the structural system of a building (Baker 1990, Chan and Grierson 1993). In time, the lateral stiffness of tall buildings was identified as their most relevant structural property; and this lead to several stiffness-based sizing methods aimed at elastic structural systems subjected to constant lateral loading and various strength and drift constraints (Kim et al. 1998, Kameshki and Saka 2001). At some point, researchers went beyond sizing structural elements for a given structural configuration, and proposed performance-based evolutionary methods to establish the optimal topology of bracing systems (Liang et al. 2000, Baldock and Shea 2006). Furthermore, drift-based methodologies that account for the interaction between the dynamic properties of the structural system and ground motion have been proposed (Park and Kwon 2003, Zou and Chan 2005). Complicate mathematics and a large computational effort or excessive simplification of the structural model usually result in difficulty in interpreting the final results of the optimization process and the need for formulating convergence and sizing constraints.

In terms of earthquake-resistance of tall braced frames many issues have not been dealt with appropriately by previous stiffness-based sizing techniques. Firstly, braces do not only provide lateral stiffness to earthquake-resistant buildings, but in many cases, energy dissipating capacity. While in some cases energy dissipation may be provided through viscous dampers, other cases demand yielding devices. It should be noticed within this context that a bracing system essentially behaves as a vertical cantilever truss (Kim et al. 1998). While the columns that support the braces act as the truss chords, the braces, with the aid of the beams, act as web members that carry axially the horizontal shear forces in such a manner that, the total drift of a bracing system can be adequately estimated by summing the *global flexural drift* produced by the axial deformation of the columns that support the braces and the *global shear drift* associated to the axial deformation of the braces (Teran and Coeto 2011). Within this context, it is important to point out that the energy dissipation capacity of the majority of tall earthquake-resistant systems depends exclusively on the global shear behavior of the bracing system. Secondly, drift-based design is a more challenging task than strength design because lateral drift is a system design criterion that requires simultaneous consideration of all structural members of the building. Although some valuable information can be obtained from refined optimization techniques, earthquake engineering requires sound structural systems that incorporate the designer's insight and knowledge. Highly efficient bracing topologies could result in highly unstable earthquake resistance if they do not follow basic rules in terms of redundancy and structural layout. Iterative or evolutionary adjustment of sizes of the structural members of an ill-conceived initial solution may easily lead to inadequate earthquake-resistant systems in such a manner that methodologies need to be developed for the conceptual development of preliminary sound solutions that may lead through few and understandable iterations to the optimum sizing of the structural elements.

This paper presents a simple conceptual methodology for the preliminary stiffness-based sizing of the structural members (braces and their support columns) of a bracing system for tall earthquake-resistant buildings. Based on basic concepts of mechanics and dynamics, the methodology yields sizes for the braces and support columns that promote an adequate structural performance through the explicit control of the lateral displacement demand in the building. The methodology yields highly efficient structural systems that may be used directly to obtain the final design of the building, or used as initial solutions that are able to promote a rational use of analytical schemes aimed at optimizing earthquake-resistant tall steel buildings.

2. DISPLACEMENT-BASED DESIGN

After analyzing the reasons why several recent seismic events have resulted in excessive economic loss, the international community of seismic engineering has concluded that the level of structural and nonstructural damage in a building is a direct consequence of excessive levels of deformation. Innovation in earthquake-resistant design has been directed towards the conception, design and construction of structural systems, either traditional or innovative, that are capable of adequately limiting their level of seismic damage through the explicit control of their lateral deformation during ground motions of different intensity. This has lead to the formulation of displacement-based methodologies for earthquake-resistant design that aim at explicitly controlling the level of lateral deformation in buildings. Countries that lead the worldwide advancement of earthquake-resistant design (such as the United States and Japan) have started changing their design paradigms through the formulation of displacement-based design formats and codes (e.g., Federal Emergency Management Agency 2000).

3. DAMAGE-TOLERANT STRUCTURES

A promising approach to achieve safer and lighter buildings is that of *damage-tolerant structural systems* (Wada et al. 2003, Teran and Coeto 2011). In one such system, structural damage induced by earthquake concentrates in specific structural devices, known as sacrificial elements. Their role is to act as structural fuses that protect the main or gravitational sub-system of the building, as well as the nonstructural sub-system against excessive damage. Because of this, the structural rehabilitation of the earthquake-resistant sub-system after severe ground motion is reduced to substituting the damaged fuses. The use of this type of system in Japan has not only resulted in lighter buildings, but promises large savings in terms of cost and time of structural rehabilitation. Teran-Gilmore and Coeto (2011) have proposed, within the context of displacement-based design and the concept of damage-tolerant structures, a performance-based methodology for the conception and preliminary design of tall earthquake-resistant buildings. This methodology requires that the vertical loads are fully supported by flexible gravitational moment-resisting frames, and that the earthquake-resistance of the building is fully provided by a bracing system. In terms of modeling, the methodology assumes: A) The slabs of the floor system act as rigid diaphragms; B) The total lateral stiffness of the building can be estimated by adding the lateral stiffnesses provided by the gravitational and bracing sub-systems; and C) The drifts due to global shear and flexural modes of the bracing sub-system are independent and produced, respectively, by the axial deformation of its braces and support columns. Under these three assumptions, it is possible to formulate, as Figure 1 illustrates, a simple model that considers that the structural system of the building can be modeled by means of two parallel sub-systems. In turn, the bracing sub-system can be modeled as two sub-systems working in series: one that represents the global shear stiffness provided by the braces, and another one that represents the global flexural stiffness provided by their support columns.

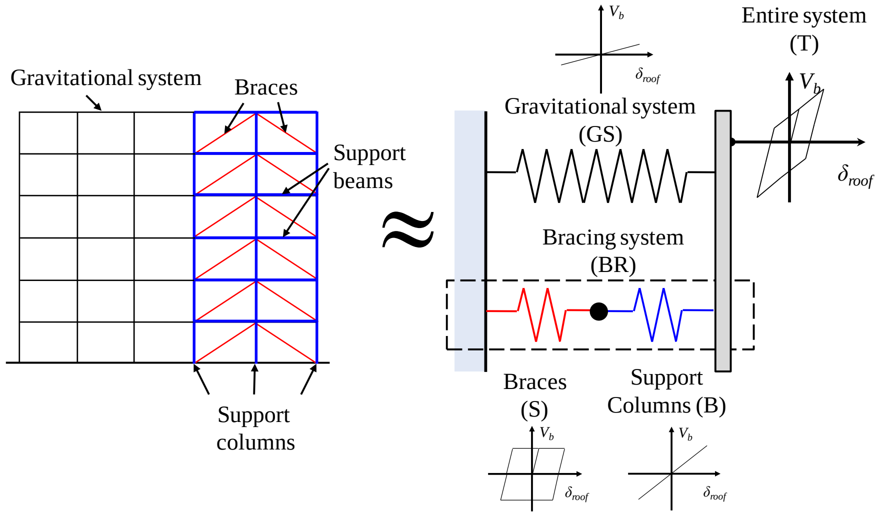

Figure 1. Modeling assumptions for tall buildings stiffened with a bracing system

Teran-Gilmore and Coeto (2011) discuss, within a format that explicitly considers the structural performance of the braces, support columns and the gravitational sub-system, the estimation of the target fundamental period of vibration for tall buildings (T_T). They explain that under the assumption that the gravitational and bracing sub-systems work in parallel, the fundamental period of vibration that defines the stiffness requirements for the bracing sub-system (T_{BR}) can be determined as:

$$\frac{1}{T_{BR}^2} = \frac{1}{T_T^2} - \frac{1}{T_{GS}^2} \tag{1}$$

where T_{GS} is the period the building would have if only the gravitational system would contribute to its lateral stiffness.

4. STIFFNESS-BASED SIZING

As a complement to the methodology discussed by Teran-Gilmore and Coeto (2011), this paper discusses in detail a stiffness-based methodology for the stiffness-based sizing of a bracing system. Within this context, it should be mentioned that although the value of T_{BR} derived from Equation 1 provides information about the local stiffness requirements that should be met by the sizes of the braces and their support columns, it is first necessary to define the relative shear and flexural global lateral stiffnesses required by the bracing sub-system. The methodology proposed by Teran-Gilmore and Coeto solves this issue by defining independent periods for the global shear and flexural drift modes (T_S and T_B, respectively) of the bracing sub-system, which need to satisfy the following relation:

$$T_{BR}^2 = T_S^2 + T_B^2 \tag{2}$$

The stiffness-based sizing of the braces and their support columns should result in that the actual fundamental periods of vibration of the bracing sub-system due to global shear and flexural drift modes are close to T_S and T_B, respectively. This should result in turn in that the overall fundamental period of vibration of the bracing sub-system is close to T_{BR}. Note that a smaller value of T_S with respect to T_B implies a larger lateral stiffness associated to the global shear drift mode relative to that associated to the global flexural drift mode.

An alternative for the sizing of braces and support columns starts by establishing a preliminary distribution through height of lateral forces:

$$F_i = V_b \frac{(w_i h_i)^k}{\sum_{j=1}^{n} (w_j h_j)^k} \tag{3}$$

where V_b is the base shear; w_i and h_i the reactive weight and height with respect to the ground level, respectively, of the slab corresponding to the *ith* level; and n the number of stories. According to FEMA 356 (Federal Emergency Management Agency 2000), k should be equal to 2 for tall buildings.

At some point, some decisions need to be made about the structural materials and overall geometry for the bracing system. These decisions should conform to a solid and conceptual understanding of earthquake-resistance of bracing systems, and should include the definition of the number of braces in each story (N), the total length of each brace (L), and the modulus of elasticity for the braces and their support columns (E_{BR} and E_{COL}, respectively). Once the values of T_{BR}, T_S and T_B, are established for the building (an example on how to determine them can be found in Teran and Coeto 2011), the sizes of braces and support columns can be determined as follows:

1) Establish a lateral force distribution along height, and obtain the corresponding story shear and overturning moment distribution

$$V_i = \sum_{j=i}^{n} F_j \tag{4}$$

$$M_i = \frac{m_B}{m_S} \sum_{j=i}^{n} V_j h_j \tag{5}$$

where F_j is the lateral force established for the *jth* story with Equation 3; h_j, the height of the *jth* story; and m_B and m_S, the masses associated to the fundamental flexural mode of vibration and the fundamental shear mode of vibration, respectively, of the bracing sub-system. Table 1 summarizes values of m_B and m_S that can be used for the stiffness-based sizing of tall buildings that exhibit structural regularity along height. In the table, m is the total reactive mass of the building.

Table 1. Effective masses to be used for shear and flexural drift modes

Stories	m_S/m	m_B/m
1	1	1
2	0.9	0.78
3	0.85	0.71
4	0.84	0.68
5	0.83	0.66
10	0.79	0.63
15+	0.75	0.60

2) Initial areas should be assigned to the braces (A_{BR}^0) and their support columns (A_{COL}^0). At this stage, the only condition that should be satisfied by the areas of the braces is that they exhibit a variation through height that is proportional to the lateral shear distribution along height obtained

from Equation 4. In the case of the columns, their areas should exhibit a variation through height that is proportional to the respective overturning moment distribution obtained from Equation 5.

3) Once the braces have been sized, the drifts of the bracing system due to global shear drift mode can be estimated. For this purpose, it is reasonable to assume that this drift mode is exclusively a consequence of the axial deformation of braces. Within this context, the lateral shear stiffness provided by the braces to the *ith* story can be estimated as:

$$K_{Si} = N_i \frac{E_{BR} A^0_{BRi}}{L_i} \frac{\cos^2 \theta_i}{L_{RFi}} \quad (6)$$

where N_i is the total number of braces located at the *ith* story, A^0_{BRi} is the area initially proposed for each one of these braces, θ_i their inclination angle, L_i their total length (distance that separates the two nodes that delimit the ends of one brace in the analytical model), and L_{RFi} a stiffness adjusting factor that takes into account the zones of larger stiffness at the end of the braces:

$$L_{RF} = \gamma + \eta(1+\gamma) \quad (7)$$

where γ is the ratio of the length of the brace core segment (L_c) to the total brace length L, and η the ratio of the average axial stress in the brace outside the brace core to the stress in the brace core. The lateral drift at the *ith* story can be obtained as:

$$\Delta\delta_{Si} = \frac{V_i}{K_{Si}} \quad (8)$$

and the lateral displacement at the *ith* story as:

$$\delta_{Si} = \sum_{j=1}^{i} \delta_{Sj} \quad (9)$$

4) Once the support columns have been sized, the drifts of the bracing sub-system due to its global flexural drift mode should be estimated. It is reasonable to assume that the bracing sub-system behaves globally like a beam; and thus, that the global flexural drifts are a consequence of the axial deformation of the support columns. Within this context, the global flexural stiffness of the bracing system can be estimated in the *ith* story (I_{Bi}) through the exclusive consideration of the axial areas of the support columns and the distances that separates them.

5) The curvature in the top and bottom ends of the portion of the bracing system located within the *ith* story can be estimated as:

$$\varphi_i^{up} = \left[\frac{M_{i+1}}{E_{COL} I_{Bi}}\right] \quad (10a)$$

$$\varphi_i^{down} = \left[\frac{M_i}{E_{COL} I_{Bi}}\right] \quad (10b)$$

Integrating the curvature diagrams, the rotations at the top and bottom ends of the portion of the bracing system located within the *ith* story can be estimated as:

$$\theta_i^{up} = \left[\frac{2\varphi_i^{up} + \varphi_i^{down}}{6}\right] h_i \quad (11a)$$

$$\theta_i^{down} = \left[\frac{\varphi_i^{up} + 2\varphi_i^{down}}{6}\right] h_i \quad (11b)$$

The total increment in global flexural rotation in the slab located at *ith* level can be established by adding the contributions of the portions of the bracing system located above and below it:

$$\Delta\theta_i^{tot} = \theta_i^{up} + \theta_i^{down} \quad (12)$$

The total rotation at the *ith* level can be established by adding the contributions of all the stories below it:

$$\theta_i^{tot} = \sum_{j=1}^{i} \Delta\theta_i^{tot} \tag{13}$$

Finally, the inter-story drift index in the *ith* story due to global flexural drift mode can be established as:

$$\Delta\delta_{Bi} = \theta_i^{tot} h_i \tag{14}$$

and the lateral displacement in that story as:

$$\delta_{Bi} = \sum_{j=1}^{i} \Delta\delta_{Bj} \tag{15}$$

6) Once the global shear and flexural drifts in the bracing sub-system have been established, initial estimates for T_S and T_B can be estimated as follows:

$$T_S^0 = 2\pi \sqrt{\frac{\sum_{i=1}^{n} w_i \delta_{Si}{}^2}{g \sum_{i=1}^{n} F_i \delta_{Si}}} \tag{16a}$$

$$T_B^0 = 2\pi \sqrt{\frac{\sum_{i=1}^{n} w_i \delta_{Bi}{}^2}{g \sum_{i=1}^{n} F_i \delta_{Bi}}} \tag{16b}$$

where g is the acceleration due to gravity. Note that δ_{Si} and δ_{Bi} are estimated from the lateral force distribution established according to Equation 3, and that for that purpose an arbitrary value can be assigned to the base shear.

7) Once T_S^0 and T_B^0 are estimated, the definitive areas for braces (A_{BR}) and support columns (A_{COL}) can be established as:

$$A_{BR} = A_{BR}^0 \left(\frac{T_S^0}{T_S} \right)^2 \tag{17a}$$

$$A_{COL} = A_{COL}^0 \left(\frac{T_B^0}{T_B} \right)^2 \tag{17b}$$

5. SAMPLE BRACING SYSTEM

To illustrate the application of the methodology, the twenty four-story steel building shown in Figure 2 is considered. Overall, the building has a total height of 114.8 meters. The building has four central bays of 9 meters, two lateral bays of 4.5 meters and seven frames in each one of its principal directions. With the exception of the first three stories and the roof story, whose weights are equal to 1916 and 1355 tons, respectively, the stories of the building exhibit a weight of 1340 tons. As shown, the sample building requires for earthquake-resistance a steel bracing sub-system that includes the two central bays of the three central frames in each direction of analysis. To illustrate the potential of the stiffness-based sizing methodology introduced herein, the four cases summarized in Table 2 are considered for the bracing sub-system. As a restriction, the methodology is applied in a "practical setting", in such a manner that the area of braces and support columns are varied every four stories. Table 3 summarizes the sizes for each brace and support column for all the different versions of the bracing sub-system. The brace sizes shown in the table correspond to L_{RF} = 1.5 (see Equations 6 and 7).

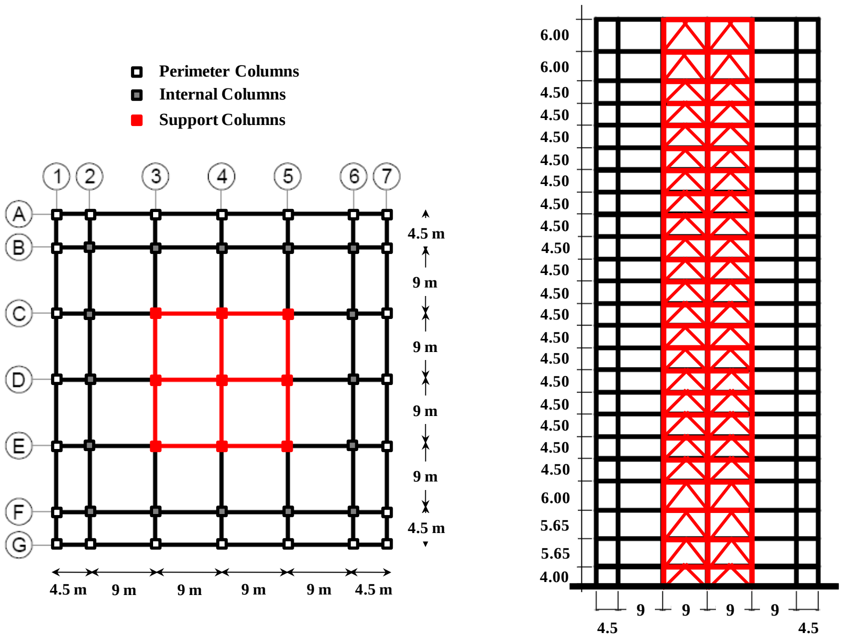

Figure 2. Geometry and structural layout of braced twenty four-story building

Table 2. Cases under consideration for sizing of members of bracing sub-system

Case	T_{BR}	T_S	T_B
1	3.50	1.00	3.35
2	3.50	2.00	2.87
3	3.50	2.48	2.48
4	3.50	3.00	1.80

6. DYNAMIC CHARACTERISTICS OF BRACING SYSTEMS

The fundamental periods of vibration of the four different versions of the bracing sub-system were established using commercial analysis software. The area of the braces was adjusted in the analytical models according to the value of L_{RF} used during the sizing procedure (1.5). Three fundamental periods of vibration were obtained for each version of the bracing sub-system, one associated to its global shear drift mode (T_S), one associated to its global flexural drift mode (T_B), and finally, one associated to the entire system (T_{BR}). T_{BR} was estimated through an analytical model that assigned to the braces and support columns of the bracing sub-system the areas summarized in Table 3. T_S was estimated by modifying this model in such a manner as to have support columns with extremely large axial areas. In case of T_B, the model was modified in such a manner as to have braces with extremely large axial areas. The moment of inertia of the columns for all the analytical models was estimated by considering for them a square transverse section and the areas summarized in Table 3 (these inertias were not modified in the models having columns with extremely large areas).

Table 3. Sizes of braces and support columns obtained from the stiffness-based sizing methodology

Stories	Case 1		Case 2		Case 3		Case 4	
	Braces (m^2)	Columns (m^2)	Braces (m^2)	Columns (m^2)	Braces (m^2)	Columns (m^2)	Braces (m^2)	Columns (m^2)
1-4	0.0537	0.1620	0.0134	0.2210	0.0088	0.2976	0.0060	0.5608
5-8	0.0533	0.1220	0.0133	0.1664	0.0087	0.2240	0.0059	0.4221
9-12	0.0512	0.0887	0.0128	0.1210	0.0084	0.1630	0.0057	0.3071
13-16	0.0461	0.0575	0.0115	0.0784	0.0075	0.1055	0.0051	0.1988
17-20	0.0368	0.0303	0.0092	0.0414	0.0060	0.0557	0.0041	0.1049
21-24	0.0221	0.0103	0.0055	0.0140	0.0036	0.0189	0.0025	0.0355

Table 4 summarizes and compares the target and actual fundamental periods of vibration for the different versions of the bracing sub-system. A smaller value of T_S with respect to T_B implies larger sizes for the braces with respect to those of their support columns. Note that the proposed methodology yields good estimates of T_B for all the different versions of the bracing sub-system. In the case of T_S, the methodology yields better estimates as the target value for this period decreases (i.e, the estimate of T_S for Case 1 is better than that for Case 4). This can be explained by the fact that the methodology neglects the lateral shear stiffness provided by the support columns, and that the moment of inertia of the columns increase with an increase in the value of T_S. Overall, the methodology yields bracing sub-systems that adequately reflect the design requirements in terms of lateral stiffness. Figure 3 shows the fundamental modes of vibration for the four versions of the bracing sub-system. Note the growing influence of the global flexural drift mode as the value of T_S decreases.

Table 4. Target and actual fundamental periods of vibration for all versions of bracing sub-system

Case	Target			Actual			Ratio		
	T_{BR}	T_S	T_B	T_{BR}	T_S	T_B	T_{BR}	T_S	T_B
1	3.5	1.00	3.35	3.60	1.00	3.47	1.03	1.00	1.03
2	3.5	2.00	2.87	3.53	1.95	2.97	1.01	0.98	1.03
3	3.5	2.48	2.48	3.45	2.34	2.56	0.99	0.94	1.04
4	3.5	3.00	1.80	3.23	2.65	1.87	0.92	0.88	1.04

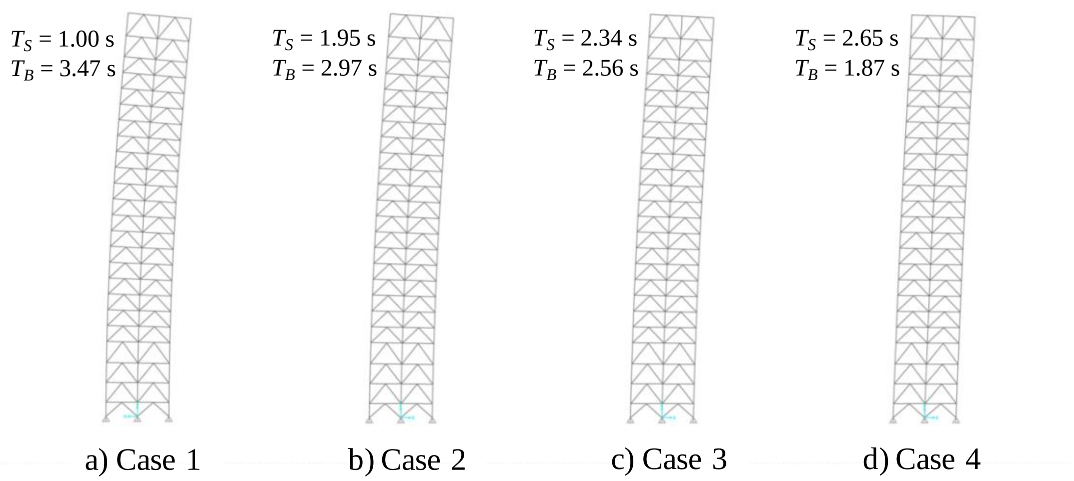

Figure 3. Fundamental mode of vibration for all versions of bracing sub-system

While Figure 4 shows the fundamental modes of vibration associated to the global shear drift mode of all versions the bracing sub-system, Figure 5 does the same for the fundamental modes of vibration associated to their global flexural drift mode. Independently of the relative sizes of braces and support columns, all fundamental modes of vibration due to global shear behavior are practically equal. A similar observation can be made for the fundamental modes of vibration due to global flexural behavior. Figures 5 and 6 provide graphic support for the condition of independence implicit in Figure 1 and Equation 2 for the global shear and flexural drift modes of the bracing sub-system. This implies that a stiffness-based sizing methodology, such as that introduced herein, can establish sizes for the braces and columns through independent numerical formulations.

7. DISCUSSION

The methodology introduced in this paper has been applied successfully to the sizing of the structural members of a bracing sub-system that has the same number of braces (with similar length and structural configuration) in all its stories, and that can be idealized as a global steel cantilever beam. In this respect, the methodology can be easily adapted so that it can be applied to bracing systems with other characteristics. The existence of outrigger trusses in some stories would significantly modify the global flexural behavior of a bracing system. While under these circumstances, the sizing of the braces would follow the same considerations made in this paper; the sizing of their support columns would need to consider the global rotation restrictions induced in the bracing system by the outrigger trusses. In terms of the effects of higher modes, an improved sizing for braces and support columns can be obtained if Equation 3 is adjusted to better reflect a variation along height of lateral forces that explicitly reflects the influence of these modes of vibration.

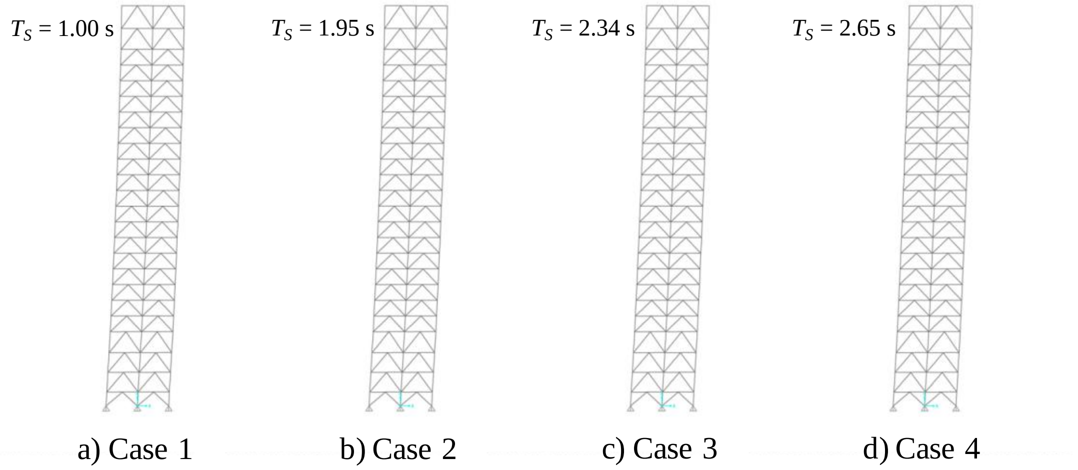

Figure 4. Fundamental mode of vibration due to global shear drift mode for all versions of bracing system

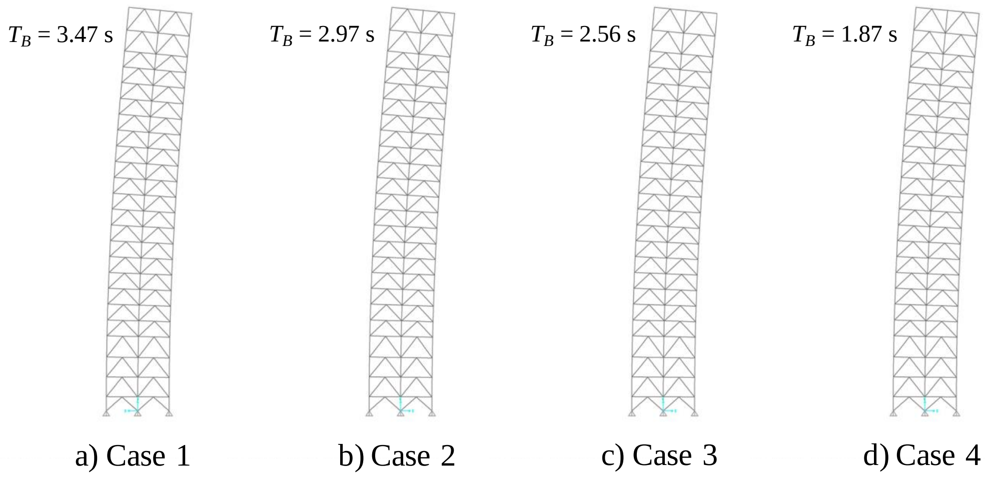

Figure 5. Fundamental mode of vibration due to global flexural drift mode for all versions of bracing system

Table 5 summarizes the weight of the structural members of the four versions of the bracing sub-system. While the weight of the columns was estimated as the product of their area and length times the specific weight of the steel; in the case of the braces this triple product was multiplied by 1.5 to consider the existence of end plate connections. Note that in terms of weight, the most efficient solutions are those in which T_S is similar or slightly less than T_B. Of all the cases summarized in the table, Cases 1 and 4 may be considered inadequate. While in the former case, the axial forces developed by the braces can't be accommodated adequately by the support columns; the latter case requires very large sizes for the columns.

Besides the stiffness considerations made so far in this paper for the sizing of the braces and their support columns, the designer may want to include strength-based considerations. Particularly, it may be decided, within a capacity design context, that it is desirable for the columns to have a larger strength in relative terms, in such a manner that any nonlinear behavior in the bracing sub-system concentrates in the braces.

Finally, it should be mentioned that the sizing derived from the conceptual methodology presented in this paper can be used directly by the structural engineer to establish the final design of the structural system, or can be used as an initial solution for methodologies aimed at optimizing the seismic performance of the building.

Table 5. Weight of different versions of bracing sub-system

Case	Weight (ton)			Relative Weight	Observation
	Braces	Columns	Total		
1	320.94	211.67	532.61	1.44	Weak column/strong brace
2	80.24	288.67	368.91	1.00	
3	52.39	388.71	441.10	1.20	
4	35.66	732.46	768.12	2.08	Inefficient

8. CONCLUSIONS

A simple methodology for the stiffness-based sizing of the structural members of a bracing system for tall earthquake-resistant buildings has been introduced. Based on basic concepts of mechanics and dynamics, and by assuming total independence of the global shear and flexural drift modes of the bracing system, the methodology formulates simple steps that allow for an independent stiffness-based sizing of the braces and their support columns.

The application of the methodology to the sizing of the braces and their support columns for four versions of a twenty four-story bracing system has yielded adequate design in terms of lateral stiffness. Methodologies such as that developed and discussed in this paper constitute themselves in essential and useful tools for the conception and preliminary sizing of structural systems for tall buildings, which can lead to efficient design within the context of a performance-based approach.

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