Multichannel singular spectral analysis of strong motion data record of Chi-Chi earthquake.

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SUMMARY:
MSSA (Multichannel singular spectral analysis) is a modified scheme of the Multivariate Principal Component Analysis (PCA), and it is being applied to many fields, including the spatio-temporal analysis of climatic data. Here we have applied this technique on earthquake strong motion data recorded during 1999 Chi-Chi earthquake as well as on simulated ground motion. The MSSA analysis of time histories recorded at various locations reveal the potentiality of this analysis technique for discriminating, simplifying and identifying the prominent spatio-temporal features of the shaking generated by the earthquake. Though this technique is used for gap filling in spatiotemporal climate data, its application to strong motion data also indicates that a very dense data set is required for a similar gap filling objective ; it can however be useful for identification of spatial variability.

Keywords: Chi-Chi earthquake, Strong motion data, Multichannel Singular Spectral Analysis

1. INTRODUCTION

Spatial variation of ground motion is an important issue for the design of large structures like long span bridges, large dams and pipelines: such structures span over large distances, are founded on variable ground conditions, and significant differential movement may lead to specific behavior and possibly failure. The design of all lifeline facilities should thus deserve a specific care to account for these additional effects. An overview of spatial variation of ground motion both in time as well as in frequency domain can be found in Zerva and Zervas, 2002.

The spatio-temporal analysis of data is a hot issue in climate sciences and various advanced uni- and multivariate techniques of time series data analysis are summarised by Ghil et al., 2002. These analysis techniques are applied in modelling the climate system, in particular for interpolating and filling gaps corresponding to locations or times fro which data are lost or missing. Many of them are based upon Principal Component Analysis (PCA) or are extensions of classical PCA. Some of these techniques involving spatial as well as temporal analysis are being applied on ground motion data for better understanding ground response. For instance, Carniel et al. (2006) used Singular Spectral Analysis SSA for site response studies, SSA being an extension of the basic principal components analysis. Similarly, the Multichannel Singular Spectral Analysis MSSA is another extension of PCA which has been applied for gap filling in climate data (Kondrashov and Ghil, 2006), and for simultaneously filtering and reconstruction of seismic data as well (Oropeza and Sacchi, 2011)

In this paper we apply the Multichannel Singular Spectral Analysis (MSSA) technique on theoretical 1-D, 2-D wave motion, to the actual strong motion data recorded during the main shock of the 1999 Chi-Chi (Taiwan) earthquake, and to results of 3D ground motion simulations along linear arrays for the Euroseistest site in Volvi (Greece). On the basis of these example results, we discuss the potential application of spatio-temporal analysis techniques on ground motion data in view of filling data gaps and identifying dominant trends in spatial variation.

2. METHODOLOGY

The principal component analysis is a multivariate data analysis technique and this method is a used for extracting linear combinations from multivariate data, which capture most of the variations in data. PCA and its extensions are being used in various disciplines to extract the major variability in the data set. Multichannel Singular Spectral analysis is one of the extensions of principal component analysis and is used here for ground motion data analysis. MSSA was proposed theoretically by Broomhead & King (1986) for the analysis on non-linear systems.

According to this method a lagged cross covariance matrix is formed from multichannel time series. If time series data at channel l is $\{x_l(t)$: t=1,.....,N$\}$, $1 \leq l \leq L$. Where N is number of data samples in the time series and L is the number of channels. For each channel lagged matrix is constructed as follow.

$$X_l = \begin{bmatrix} x_l(1) & x_l(2) & & x_l(M) \\ x_l(2) & x_l(3) & & x_l(M+1) \\ .. & .. & .. & .. \\ x_l(N') & x_l(N'+1) & & x_l(N) \end{bmatrix} \tag{1}$$

where M is number of columns (a window spanning over the time series), N' is number of row and N'=N-M

The full augmented multichannel lagged matrix (ML by N') is formed as

$$D = (X_1, X_2, X_3 ....X_L) \tag{2}$$

The grand LM by LM lag covariance matrix **C** is then obtained as

$$C = D^t D \tag{3}$$

Diagonalizing the matrix **C** gives the eigen values λ and corresponding eigen vectors **E**. Each eigen vector is composed of L consecutive M long segments with its elements denoted by E_l(j), j=1,2, ...M. The associated space-time Principal Components (PCs) **A** are single channel time series and these are computed as

$$A(t) = \sum_{j=1}^{M} \sum_{l=1}^{L} x_l(t + j - 1) E_l(j) \tag{4}$$

We can reconstruct time series data from the selected PCs of interest.

3. MSSA OF WAVE MOTION

When propagating through an elastic medium, seismic waves generate an oscillation of the medium particles. This motion can be recorded by seismological instruments in the form of strong motion accelerograms or velocigrams (amongst others), from which the displacement time histories can be computed. Here, we use displacement time histories associated to locations distributed on straight line as well as on horizontal surface. Time histories associated to locations distributed on straight is representing 1-D motion and those associated to spatial distribution on surface are representing 2-D wave motion. A number of combinations of waves with different time period T and wave length (i.e., different propagation velocities) are used for input to MSSA.

3.1. Application to 1-D wave motion

We first consider the simple case of a one dimensional wave channel, with ground motion receivers linearly distributed in space, i.e. arranged along a straight line. We assume a combination of two propagating sine waves, with amplitudes of 2 and 1, frequencies of 1Hz and 2Hz, wave lengths of 100m and 150m, and wave propagations of 100 m/s and 300 m/s, respectively. We apply MSSA to the corresponding theoretical data with M=100, N'=300 (i.e., N=400, the time step being 0.05s) and L=30 (i.e., o total of 30 receivers, equally distributed along a 200 m long line). The results are displayed in Fig. 1a to Fig. 1f in the spatio-temporal and frequency domains. Each principal component is represented both by the associated time series and spectra at six consecutive locations to see the phase shift in a pair of principal component. Moreover, principal components are represented by the particle positions at different times step for the 30 receivers. First 20 eigenvalues are also plotted in the lower right frame, the actual eigenvalue corresponding to the relevant principal component being represented by red circles. These plots indicate that eigenvalues distribute as pair sets, with no or only negligible difference within each eigenvalue pair.

For this simple example, MSSA allows to identify and separate clearly the two main waves, corresponding to the first four eigenvalues and eigenmodes, while the following ones correspond to numerical noise, with much lower amplitudes. The first wave with the dominant amplitude (2) and the 1 Hz frequency is represented by the first and second principal components : the spectra of time series for these two PCs exhibit a clear peak at 1Hz. Differences between PC-1 and PC-2 are visible in the time series exhibiting phase delays : a detailed analysis reveals that these two principal components exhibit indeed a 90 degree phase delay. The spatial plot of principal components (lower part) shows that crests of waves shifts along the distance axis with each time step along timeline, which allows to measure the propagation velocity. The same wave length of 100m is observed for PC-1 and PC-2. The 90 degree phase shift observed in the time series is also visible in the spatial plot.

The second eigenvalue pair is associated with principal components 3 and 4 (Figs 1c and 1d), which is characteristics of the second constituent wave in the input data, with the twice lower amplitude and the 2 Hz frequency. Higher PCs have almost zero amplitude and eigenvalues – as displayed for PC-5 and PC-6 in Figures 1e and 1f, and can thus be considered as numerical, incoherent noise. In a general case however of an input wavefield with a broad spectrum, the eigen values would not decrease suddenly and more "coherent" waves could be identified by MSSA.

We also considered another case (with similar geometry) where the second constituent input wave is a standing wave, i.e., without lateral propagation. In this second case, MSSA also succeeds in separating

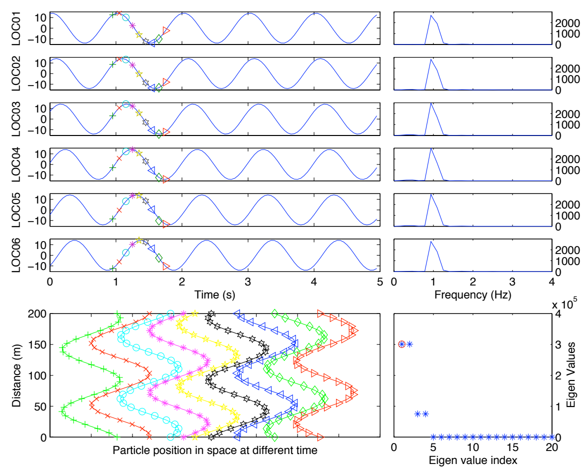

Figure 1a. PC-1, First principal component obtained by MSSA of combination of 1Hz and 2Hz waves

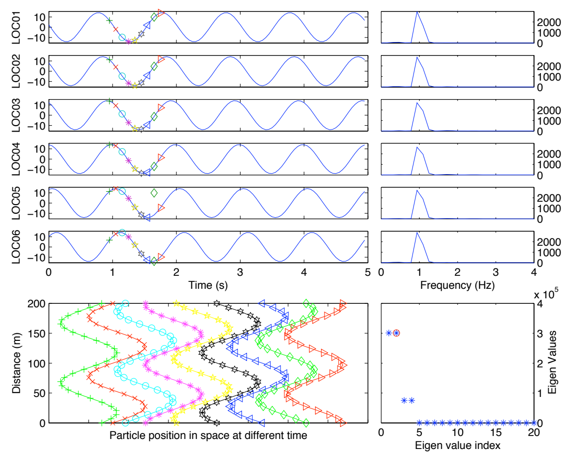

Figure 1b. PC-2, 2nd principal component obtained by MSSA of combination of 1Hz and 2Hz waves

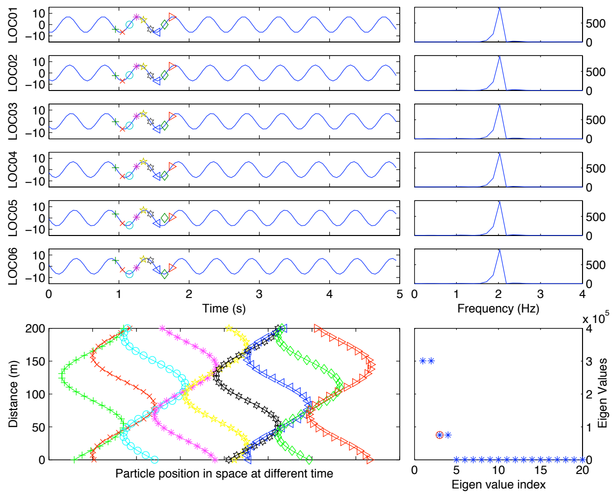

Figure 1c. PC-3, Third principal component obtained by MSSA of combination of 1Hz and 2Hz waves

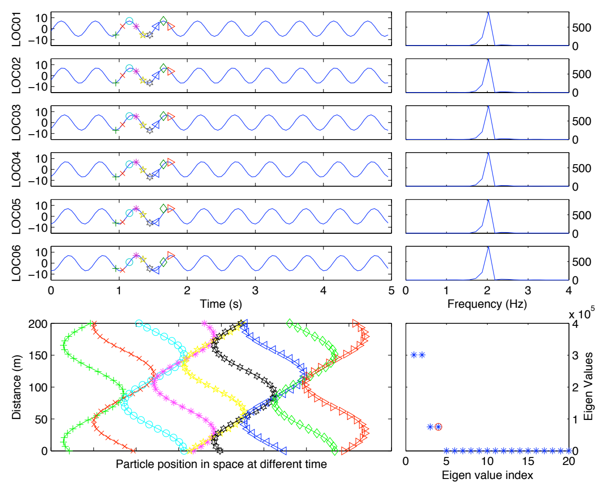

Figure 1d. PC-4 Fourth principal component obtained by MSSA of combination of 1Hz and 2Hz waves

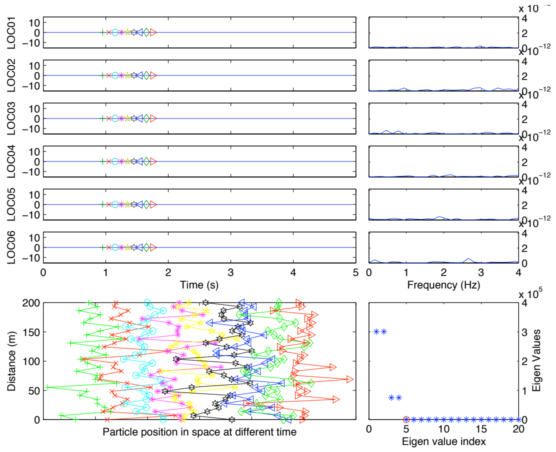

Figure 1e. PC-5 Fifth principal component obtained by MSSA of combination of 1Hz and 2Hz waves

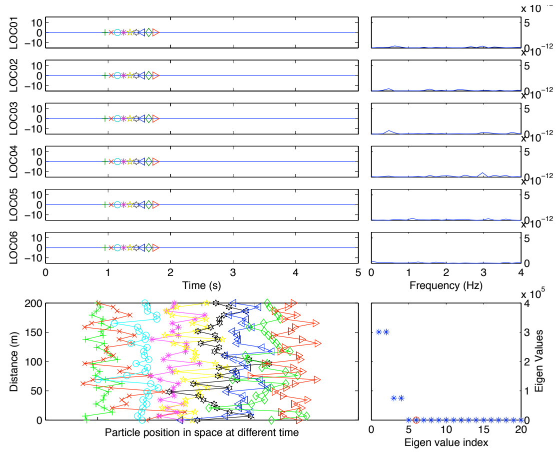

Figure 1f. PC-6 Sixth principal component obtained by MSSA of combination of 1Hz and 2Hz waves

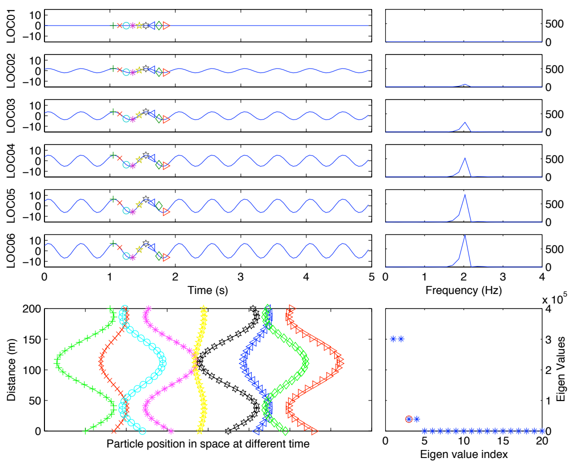

Figure 2a. PC-3, Third principal component obtained by MSSA of combination of 1Hz propagating and 2Hz standing waves

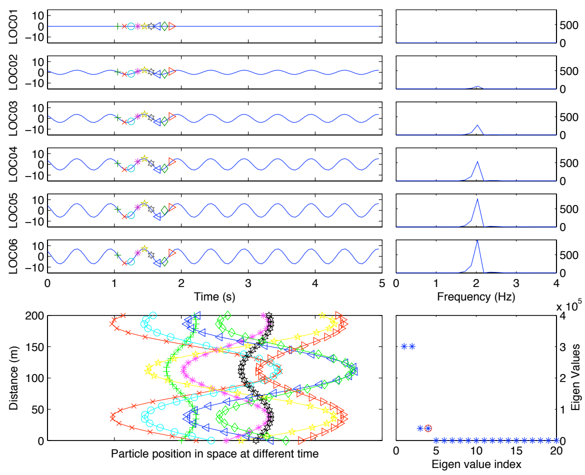

Figure 2b. PC-4, Fourth principal component obtained by MSSA of combination of 1Hz propagating and 2Hz standing waves

the two main components as pairs of principal components. The properties of PC-3 and PC-4 displayed in Figure 2 are then characterized by a receiver dependent amplitude, corresponding to the existence of node points. Would the amplitude of the constituting standing wave be higher than the propagating waves, MSSA would map them in the PC-1 / PC-2 pair, as the eigenvalue order follows the decreasing amplitude order of the input wavefield. Whenever two waves with similar amplitude are contributing to the input wavefield, these two waves are not separated in two different pairs but their combination is represented by single pair of PCs.

3.2. Application to 2-D wave motion

In the reality, the available information most often consist of "2D" data made of time series of ground motion recorded at locations distributed (often highly unevenly) at various points of the Earth's surface. MSSA technique can be applied in a similar way as for the 1D linear array case presented in the previous section. As an example, theoretical time series have been generated over a uniformly distributed grid point (spacing 0.1 degree), to simulate the combination of two sine waves having amplitude 2 and 1, periods of 39 and 78 seconds, outward propagating form a point-source with wavelengths of 0.39 and 0.78 degrees, respectively. The results of the MSSA processing of this combination are shown in Fig. 3 on a geographical background corresponding to northern Taiwan (see next section). AS in the previous 1D-case, MSSA proves successful in identifying the two main constituting waves, represented by two pairs of principal components. The spatial characteristics associated to each input wave can be easily recognized in the contour plots and cross-section along the oblique line on the map. As in the 1D case, the separation of constituent waves is based upon the amplitude, and therefore constituent waves with larger amplitudes are associated with larger eigenvalues and lower principal component order. Also, as in the 1D case, the two components within the same pair are identical with a 90° phase lag.

4. MSSA OF STRONG MOTION DATA RECORD OF 1999 CHI-CHI EARTHQUAKE.

A large set of strong motion data has been obtained during the 1999 Chi-Chi earthquake, and made available, by the Central weather bureau of Taiwan. These data were recorded by a very dense network of strong motion instruments, with however highly uneven spatial distribution as shown on Fig. 4. We have used this data set for MSSA processing. Displacement time histories are used in order to focus on long period waves only, because of computation limitations related to the handling the large size covariance square matrix : the LM size (where M is the data window length and L is the number of receivers) here amounts to 13500, corresponding to 90 locations and 125 s long windows, with a 0.5s time step (N'=100, M=150, N=250). Only high-pass filtered displacement time histories are considered here to eliminate the large permanent displacements in the epicentral area. The data recording points are represented by black crosses on the maps on Figure 4, while the region where no strong motion data was available is masked by grey patch.

The spatial characteristics of the first two constituent waves, corresponding to the first four principal components, are displayed in Fig. 4 together with the distribution of eigenvalues. The first PCs pair (PC-1,2) when observed at different times (not shown here due to space limitation) look like standing waves with high amplitudes in some specific areas. When compared to the geological map of Taiwan shown in Fig. 6, one can notice a correspondence between these "hot spots" and the location of alluvial valleys on the West and East coasts (yellow colour), with node points/lines located near the boundary between alluvial deposits and outcropping rocks.

In theory, once the principal components are derived from MSSA, they could be used to the reconstruction of displacement time series within the "grey areas" where actual recordings are missing. This procedure has however to be done with extreme caution, considering that the exceptionally large density and high quality of strong motion data recorded during the Chichi even may not yet be enough to fill the gap in the central part of the Island, or in other, less densely instrumented areas, depending on the dominant frequencies and wavelengths of each PC.

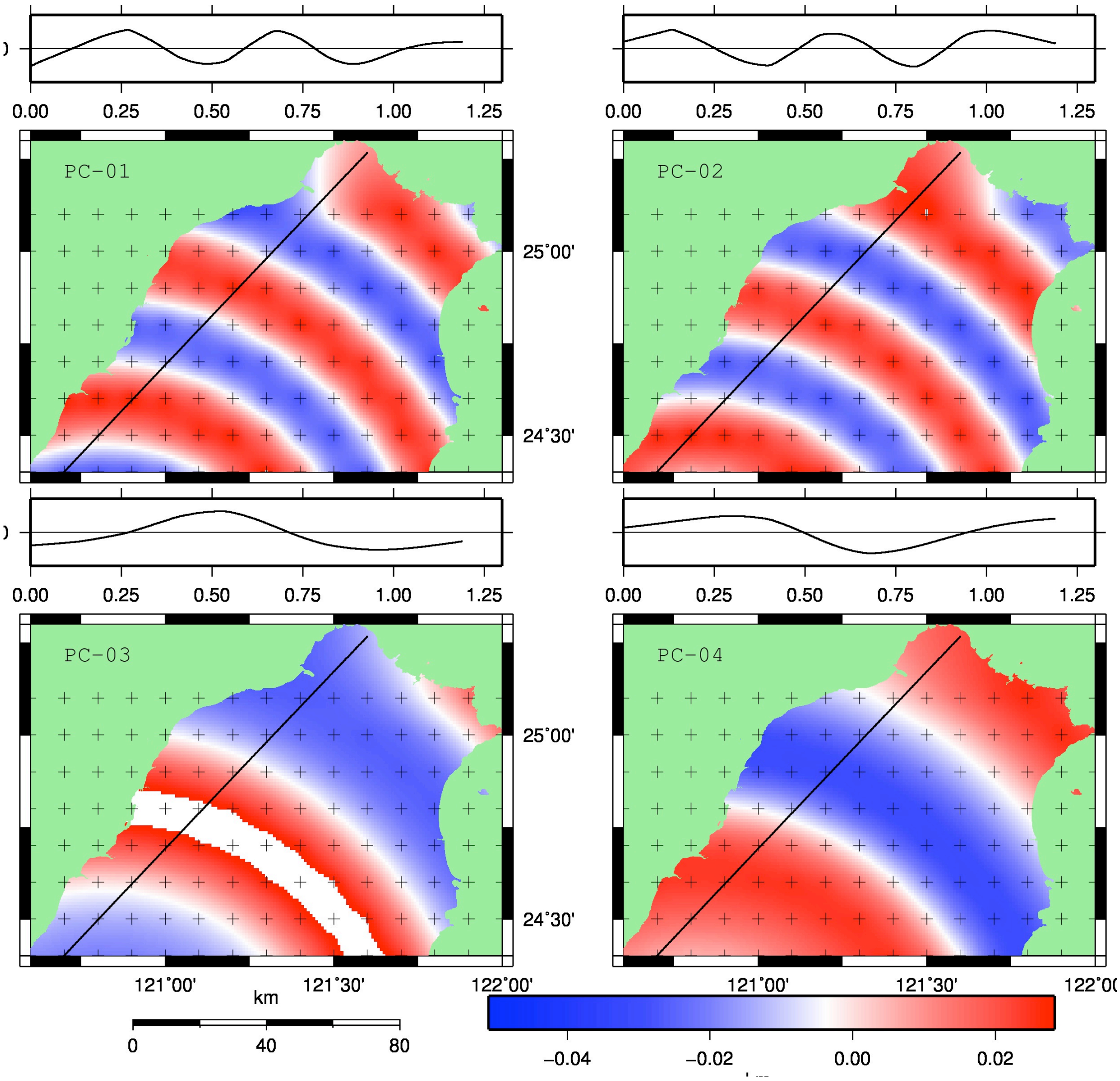

Figure 3. Principal components of simulated two on the ground surface in Northern Taiwan.

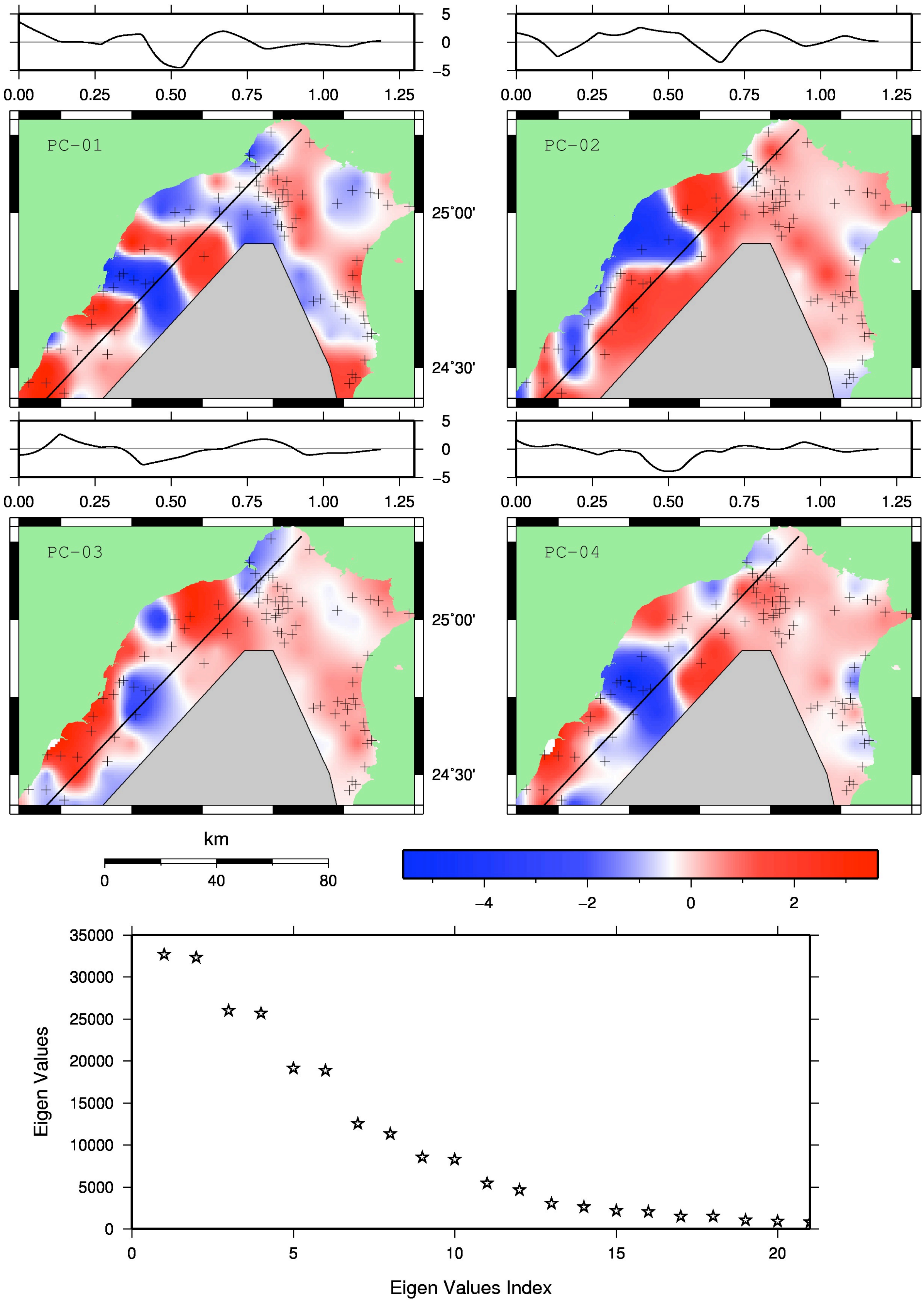

Figure 4. Principal components obtained by MSSA of high-pass filtered strong motion displacement time histories of the Chi-Chi (Taiwan) 1999 event.

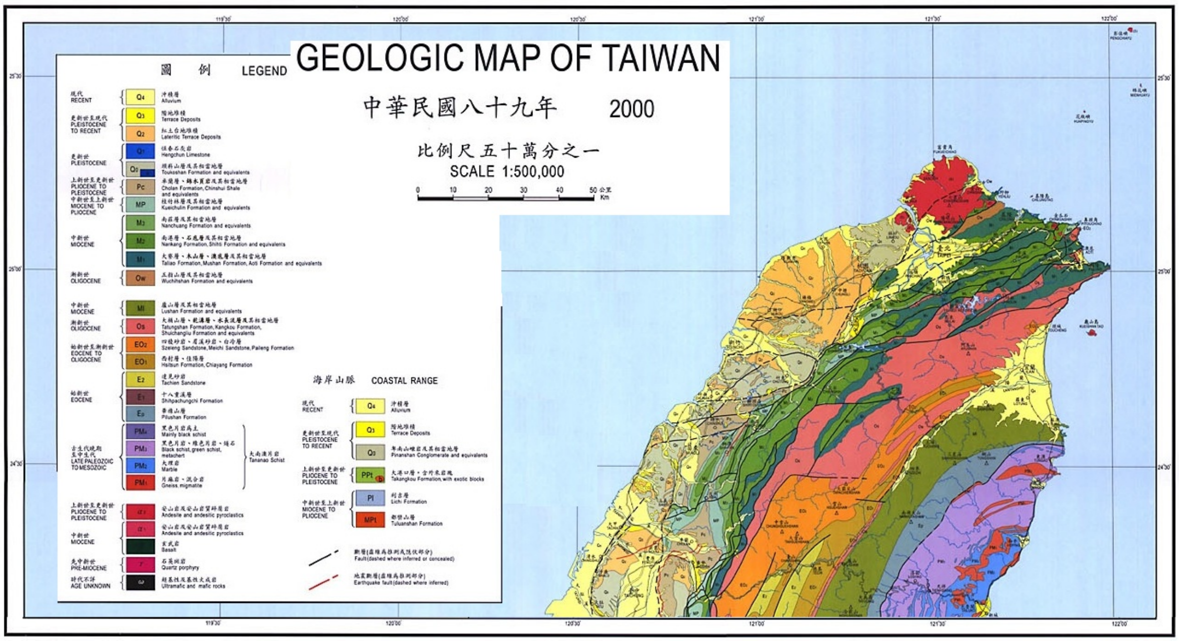

Figure 5. Geological map of Northern Taiwan where light yellow colour represents the alluvial region. (Reproduced from http://eng.wra.gov.tw/public/Data/gh012_p3.htm)

5. MSSA OF SIMULATED GROUND MOTION AT VOLVI SITE IN GREECE.

MSSA has also been applied to the 3D simulation of ground motion in the EUROSEISTEST area in Greece as obtained with the spectral element method SPECFEM (Chaljub et al., 2011). We will present here only the results derived from the analysis of the simulated ground motion at receiver locations linearly distributed across the alluvial valley as shown in Fig. 6. The first four odd principal components (PC # 1, 3, 5 and 7) are shown in Fig. 7, together with the overall distribution of eigenvalues, which exhibit a much broader behaviour than in the previous simple cases. The first one corresponds to rather low frequency waves propagating from both edges to the valley centre: i.e. seismic waves have short durations on rock sites and much longer on the soil sites: spatial distribution at specific time steps do exhibit a very clear lateral propagation towards valley center. A similar feature can indeed be seen for all four PCs in Fig. 7, with however increasing dominant frequency and decreasing wavelength. PC-5 could be interpreted with standing wave like motion in alluvium around 2Hz, and similarly PC-7 with sloshing modes near valley edges. Such patterns should be considered while designing any bridge or pipeline crossing this valley.

5. CONCLUSIONS

Alike all the principal component analysis techniques, MSSA processing of strong motion data has the potentiality to reveal the prominent feature of the spatio-temporal variation of the dataset, as a additional tool complementary to classical array processing techniques. MSSA can also be applied for gap filling and interpolation of strong motion dataset as it is being used in other disciplines. Spatial trend of ground movement on surface across an alluvial valley might be helpful for analysis and design of facilities there, as seen in MSSA of simulated data at Volvi site. However, the present density of strong motion instruments is generally not enough to resolve intermediate to high frequencies of ground motion, corresponding to short wavelengths, which are most often very small compared to the average interstation distance.

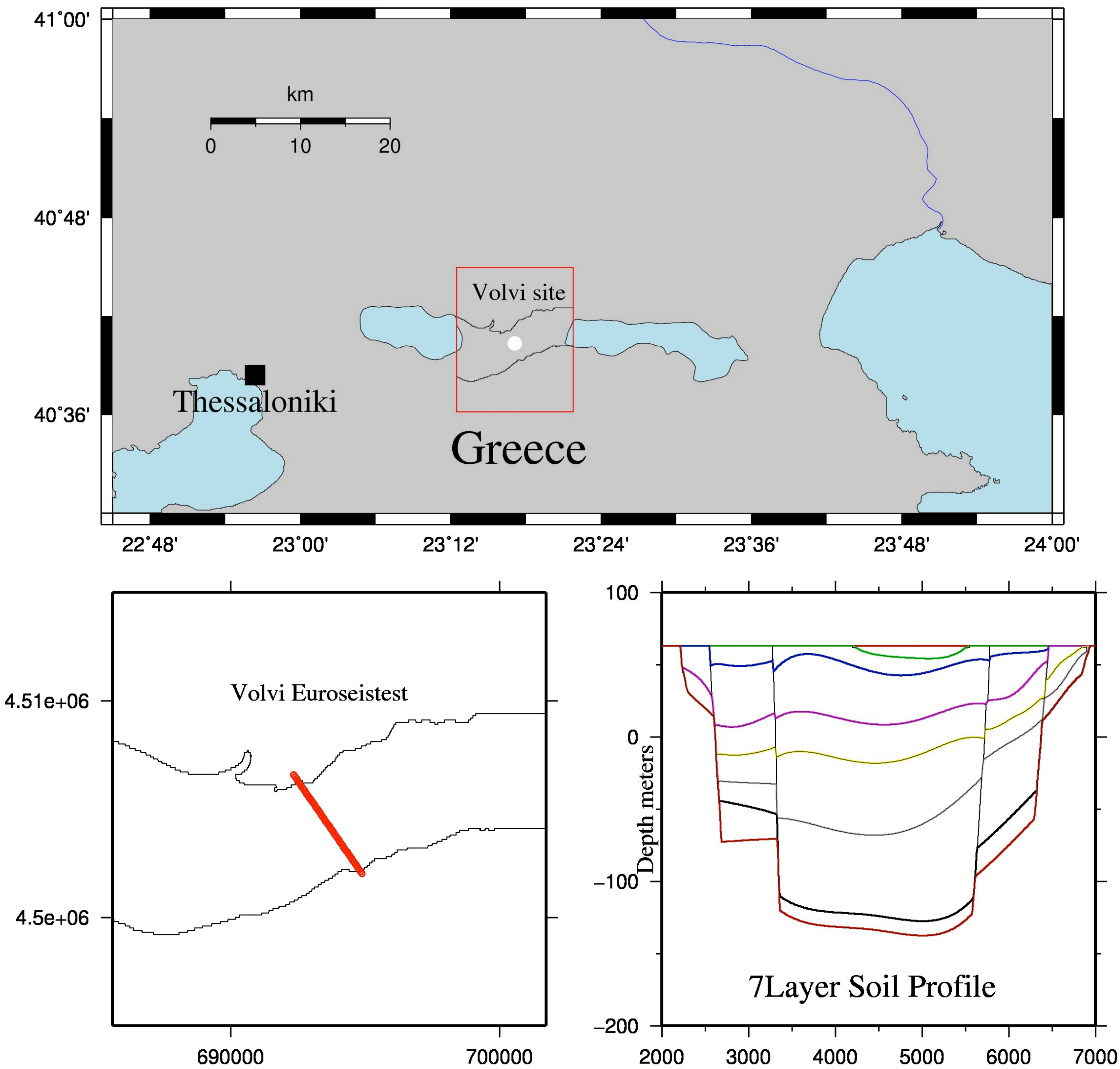

Figure 6. Euroseistest site Volvi located in Greece. The simulated ground motion receivers are evenly distributed along the thick red line, while the corresponding geological cross-section of the alluvial valley is displayed at bottom right.

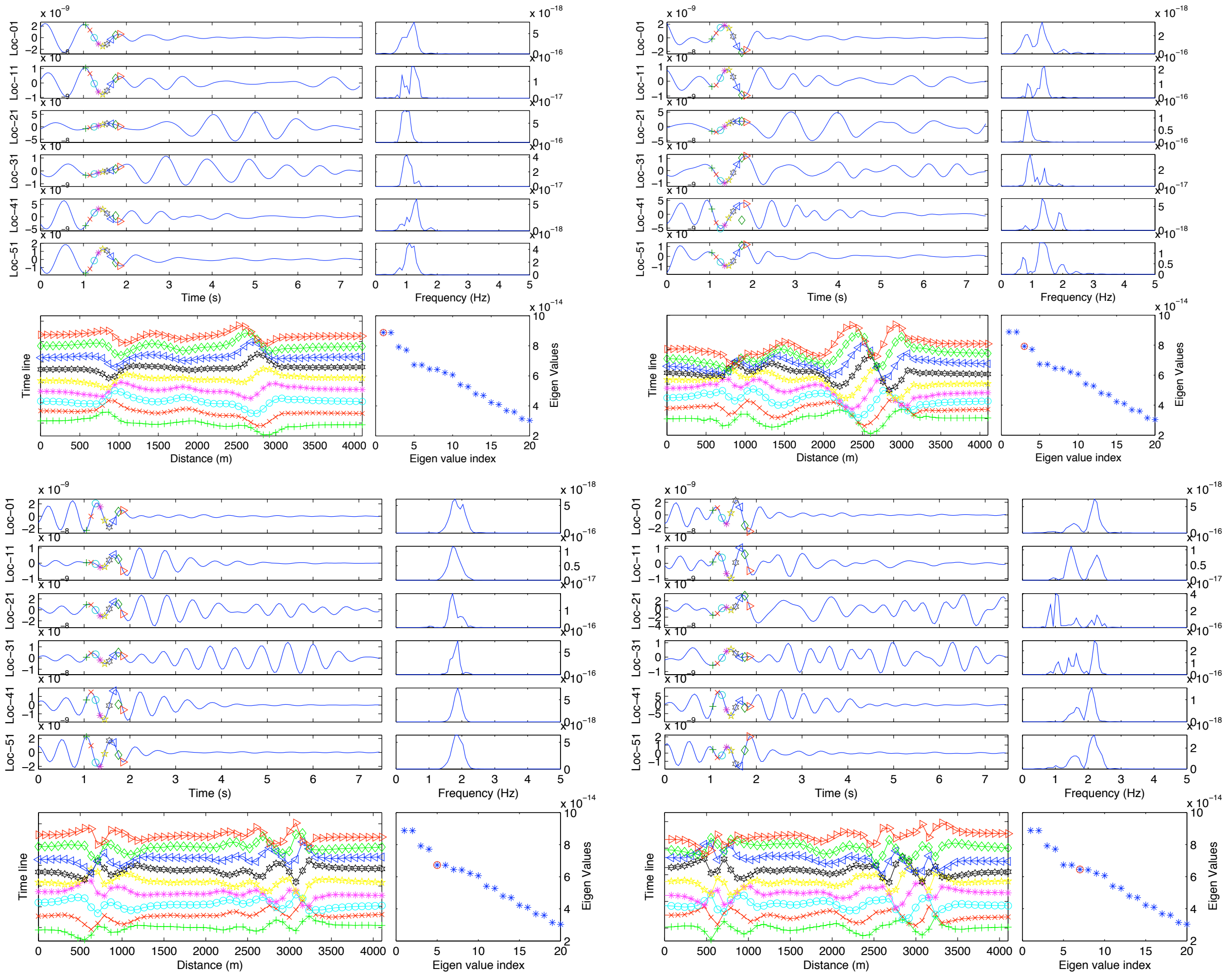

Figure 7. First four odd principal components obtained by MSSA of the simulated time series at Volvi site by the spectral element method.

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