Constant-damage Yield Strength Spectra

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SUMMARY:
The constant-damage yield strength spectrum (CDYSS) is defined as the yield strength demand of a single-degree-of-freedom (SDOF) system of varying natural vibration period with the target constant damage index and specified ductility capacity. In this study CDYSS is derived by averaging the nonlinear time-history analysis results of a SDOF system under different earthquake ground motions. The damage index is calculated by the modified Park-Ang damage model proposed by the authors. In total 641 sets of earthquake ground motion records are adopted and classified into 12 groups in line with Chinese seismic design code. The yield strength demands of SDOF systems with given ductility capacity are obtained through trial and error method. The main characteristics of CDYSS are analyzed. The empirical formulas of CDYSS are then developed by multivariable nonlinear regression analysis. CDYSS derived in this study can be applied to damage-control seismic design of RC structures.

Keywords: inelastic response spectra, strength demand, nonlinear time-history analysis, seismic damage index, nonlinear regression analysis

1. INTRODUCTION

Currently the required design strength is derived from constant-ductility strength demand spectra (CDSDS) computed by non-linear response analysis based on many earthquake ground motions. Although CDSDS have been widely accepted among code writers and design engineers, recent studies have shown that constant displacement ductility is not a reliable criterion for determining the strength demand. Actually, the cumulative damage resulting from inelastic cycles has significant effect on the damage state of a structure. A more reliable criterion should be based on the simultaneous consideration of the ductility capacity and other response parameters such as hysteretic energy dissipated by the structure and cumulative ductility ratio. Hence, a structure designed on the basis of CDSDS may not possess a sufficient margin of safety against desired damage level.

To include the effects of cumulative damage in determining the strength demand, some new concept and approaches have been proposed. These approaches can be generally classified into two categories: methods based on equivalent ductility factor and methods based on using of damage models directly. The first approach was introduced in terms of equivalent ductility factor (Fajfar, 1992), which is always lower than the ductility ratio, where the reduction is dependent on the amount of absorbed hysteretic energy of the system. Similar approaches were proposed in terms of a weighted ductility factor and an allowable ductility factor (Fajfar *et al*., 1992; Jean *et al.*, 1998). The second approach which directly uses damage indices of damage models for finding the strength demands was proposed by Cosenza *et al*. (Cosenza *et al*., 1993). Warnitchai and Panyakapo proposed the concept of constant-damage design spectra (Warnitchai and Panyakapo, 1999). Panyakapo investigated the validity of traditional CDSDS and developed constant-damage design strength spectra for RC structures (Panyakapo, 2004). The constant-damage strength spectrum is a plot of the yield strength of a SDOF system required to limit the seismic damage to a target value. Lu and Wei constructed constant-damage strength spectra incorporating performance consideration from the viewpoint of performance-based seismic design (Lu and Wei, 2008).

In this study, using the modified Park-Ang damage model proposed by the authors (Jiang *et al*., 2011), different from the damage model adopted in the literature, constant-damage yield strength spectra (CDYSS) for RC structures are constructed by a large number of nonlinear earthquake response analysis. To investigate the effect of local soil condition, the earthquake ground motions used in this study are classified into 12 groups in line with Chinese seismic design code. The simplified empirical expressions for CDYSS, which could be applied conveniently in engineering practice, are derived by nonlinear regression analysis.

2. EARTHQUAKE GROUND MOTIONS

Covering a great variety of characteristics, 641 sets of earthquake ground motion records collected by State Key Laboratory of Disaster Reduction in Civil Engineering of China are adopted here to conduct nonlinear time history analysis. These records are classified into 12 groups with four categories of site soil condition ranging from stiff to soft, i.e., Class I, II, III, and IV, each of which is further sorted into three design groups, Group 1, 2, and 3, according to the characteristic period of ground motions taking the earthquake characteristics such as magnitude and epicentral distance into account, which is conformed with current Chinese seismic design code. The characteristic period of earthquake ground motions is defined as

$$T_g = 2\pi \cdot \frac{EPV}{EPA} \tag{2.1}$$

where EPV and EPA are the effective peak velocity and effective peak acceleration respectively. The range of characteristic period for each group is listed in Table 1.1.

Table 2.1. Range of characteristic period for each group (s)

Category	I	II	III	IV
Group1	≤0.30	≤0.40	≤0.50	≤0.70
Group2	0.30~0.35	0.40~0.45	0.50~0.65	0.70~0.90
Group3	≥0.35	≥0.45	≥0.65	≥0.90

According to Eqn. 2.1 and Table 2.1, 641 sets of earthquake ground motion records are classified into 12 groups. The number of sets for each group is listed in Table 2.2.

Table 2.2. Number of sets for each group

Category	I	II	III	IV
Group1	20	34	82	83
Group2	19	15	44	22
Group3	88	91	112	31

3. SEISMIC DAMAGE MODEL AND HYSTERETIC MODEL

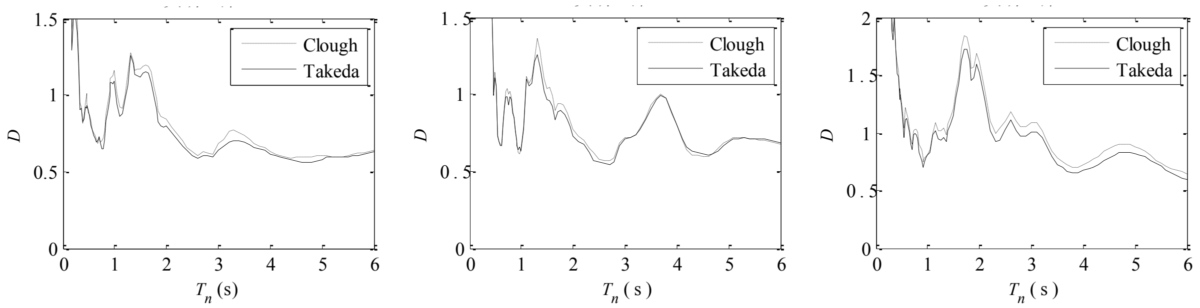
To eliminate the non-convergence problem at upper and lower limits existing in the original Park-Ang damage model (Park and Ang, 1985) which has been widely used for RC structures, a modification proposed by the authors (Jiang *et al*., 2011) as follows is applied here:

$$D = (1-\beta)\cdot\frac{\mu_m}{\mu_u} + \beta\cdot\frac{\int dE}{F_y\delta_y(\mu_u - 1)} \tag{3.1}$$

where μ_u and μ_m are the ultimate displacement ductility factor under monotonic loading and the maximum deformation ductility factor experienced in the time history under the earthquake ground motions respectively, F_y is the yield strength of the system, δ_y is the yield displacement, β is the combination coefficient, which is taken as 0.1 here for flexure-dominant RC structures with good deformation capacity, and dE is the incremental hysteretic energy.

The influence of hysteretic model on the prediction of seismic damage by using Eqn. 3.1 is first evaluated. The modified bilinear Clough model and trilinear Takeda model (Takeda, 1970), which are appropriate for analyzing flexure-dominant RC structures, are applied in the nonlinear earthquake response analysis and prediction of seismic damage index of the SDOF system. The yield strength and post-yield stiffness in this study are set as identical for the comparability of the two models. The degradation coefficient of stiffness is taken as 0.4. The ratio of the post-yield stiffness to the secant stiffness at the yield point is taken as 0.1. In Takeda model the cracking load is taken as one third of the yield load, the secant stiffness at the yield point is taken as one half of the elastic stiffness before cracking. The ultimate displacement ductility ratio of the SDOF system under monotonic loading is taken as 4.0. The ratio of the yield strength to the maximum elastic earthquake force acting on the SDF system keeping elastic is taken as one third. The damping ratio is taken as 0.05, which is appropriate for RC structures.

Nonlinear time history analysis are conducted on the SDOF system with the above two hysteretic models under the earthquake ground motions in Section 2. The mean damage spectra for 5 sets of ground motions of the same group are obtained, part of which are shown in Fig. 3.1. In general, the differences between the results obtained by modified Clough model and those by Takeda model are not significant. The damage index by modified Clough model is a little bit larger than that by Takeda model. Takeda model is employed in the latter analysis because it has been considered more adequate and representative for RC structures.



(a) Group 2 of Class I (b) Group 3 of Class II (b) Group 1 of Class IV

Figure 3.1. Effect of hysteretic model on mean damage spectra

4. CONSTANT-DAMAGE YIELD STRENGTH SPECTRA

4.1. Calculation Procedure

The yield strength coefficient, which is defined as follows, is used to reflect the yield strength demand of the SDOF system

$$C_y = \frac{F_y}{mS_a} \tag{4.1}$$

where F_y is the yield strength of the SDOF system, m is the mass, and S_a is the elastic acceleration spectrum. For each set of earthquake ground motions, the yield strength coefficient C_y to achieve the target damage index of the SDOF system with given ductility capacity can be obtained by iterative computation. The initial yield strength coefficient is taken as 1. It is decreased gradually in each running of iteration until the damage index is very close to the target value within the specified allowable error range. For each set of ground motions, CDYSS can be acquired by following the flowchart as illustrated in Fig. 4.1. To be consistent with Chinese seismic design code, CDYSS in the range of 0.1 to 6s is derived. To improve the computation efficiency, the incremental period is taken as 0.025s in the range of 0.1 to1.0s, from 1.0 to 2.0s, it is taken as 0.05s, and from 2.0 to 6.0s, 0.1s is adopted.

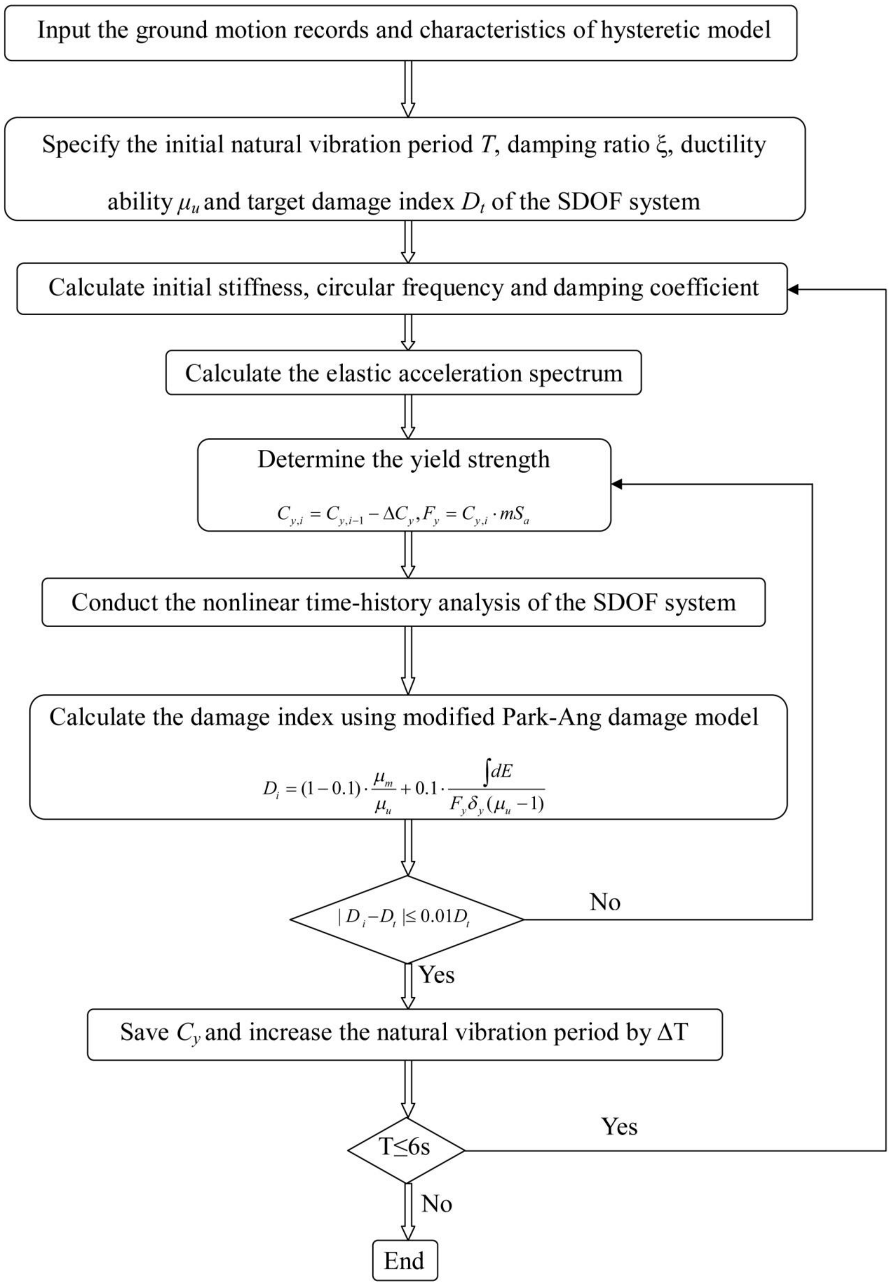


Figure 4.1. Flowchart for calculation of CDYSS

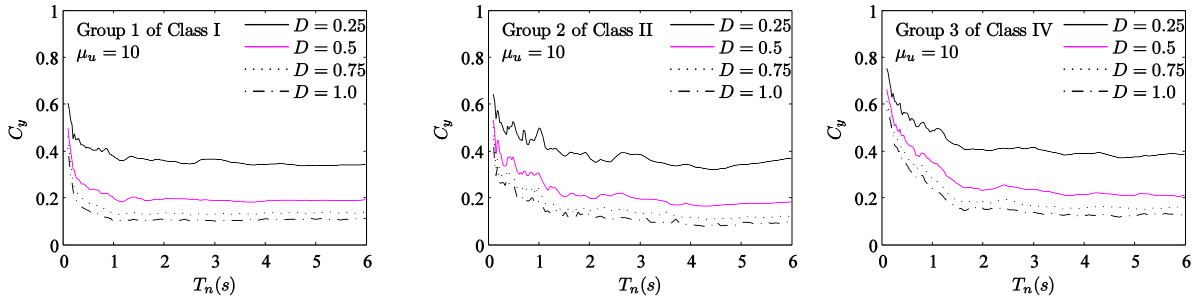
4.2. Mean Yield Strength Spectra

The mean yield strength spectra and the corresponding coefficient of variation (COV) for different category of site soil condition are obtained. Due to the limited space, only part of results with μ_u=10 are illustrated in Figs. 4.2 and 4.3. From Fig. 4.2, it can be found that:

(1) Although there exists fluctuation in the CDYSS curve, generally the yield strength coefficient decreases with the increase of period. It decreases very rapidly in the range of short period and tends to be constant in the range of long period.

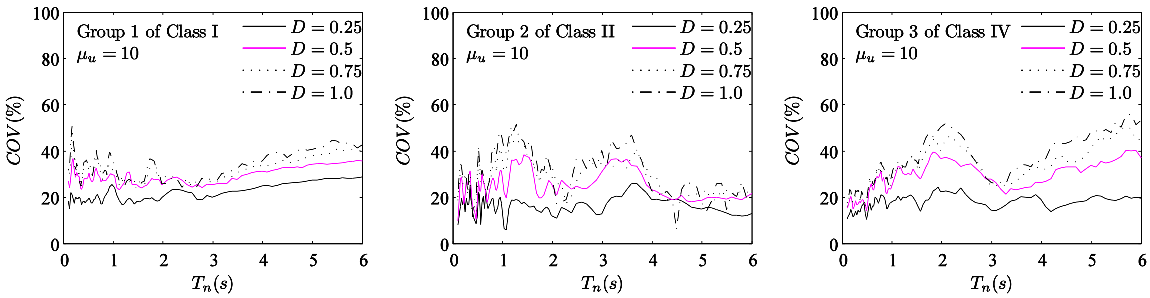
(2) The yield strength coefficient decreases nonlinearly with the increase of target damage index, that is, the coefficient decrease quickly at low damage level, and the decreasing becomes more slowly at higher damage level.

(3) With the increase of damage level, the COV of CDYSS becomes higher and fluctuates with higher amplitude, indicating that the influence of characteristics of earthquake ground motions on damage index become more significant when the structure reaches stronger nonlinear phase.



(a) Group 1 of Class I (b) Group 2 of Class II (c) Group 3 of Class IV

Figure 4.2. Mean CDYSS



(a) Group 1 of Class I (b) Group 2 of Class II (c) Group 3 of Class IV

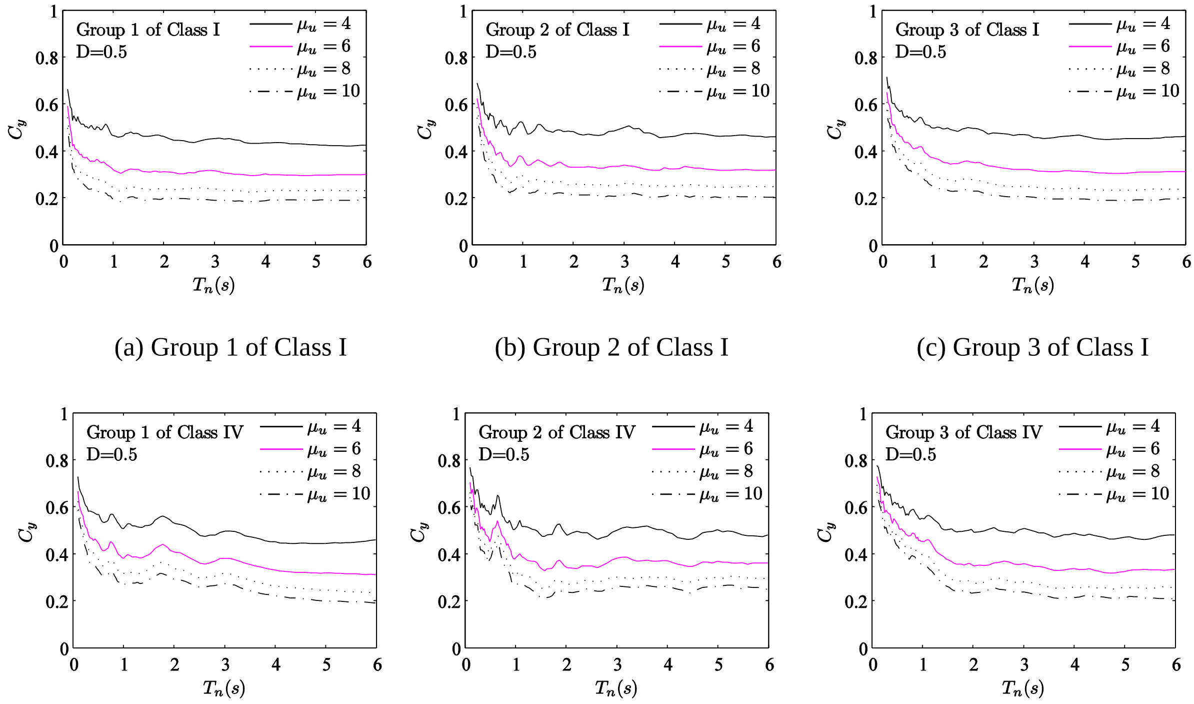
Figure 4.3. Coefficient of variation

4.2.1. Effect of ductility capacity

The CDYSS for the SDOF system with the same target damage index but different ultimate ductility factor are compared to evaluate the effect of ductility capacity. The comparison for site soil Class I and IV is illustrated in Fig. 4.4. It can be found from Fig. 4.4:

(1) The yield strength coefficient decreases with the enhancement of ductility capacity and this tendency is much more obvious if the ultimate ductility factor is relatively low.

(2) The effect of ductility capacity is enhanced with the increase of natural vibration period of the structure at the beginning and maintains at a relatively stable level in the range of long period. The same conclusions can be drawn for other site soil classes.



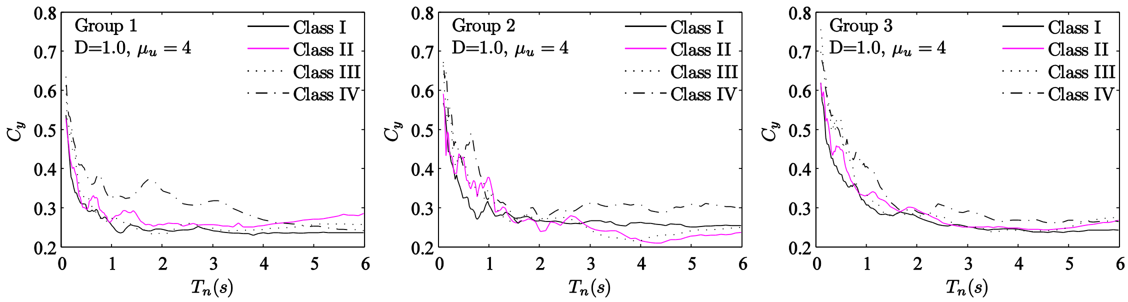
(a) Group 1 of Class I (b) Group 2 of Class I (c) Group 3 of Class I

(d) Group 1 of Class IV (e) Group 2 of Class IV (f) Group 3 of Class IV

Figure 4.4. Effect of ductility capacity on CDYSS

4.2.2. Effect of site soil condition

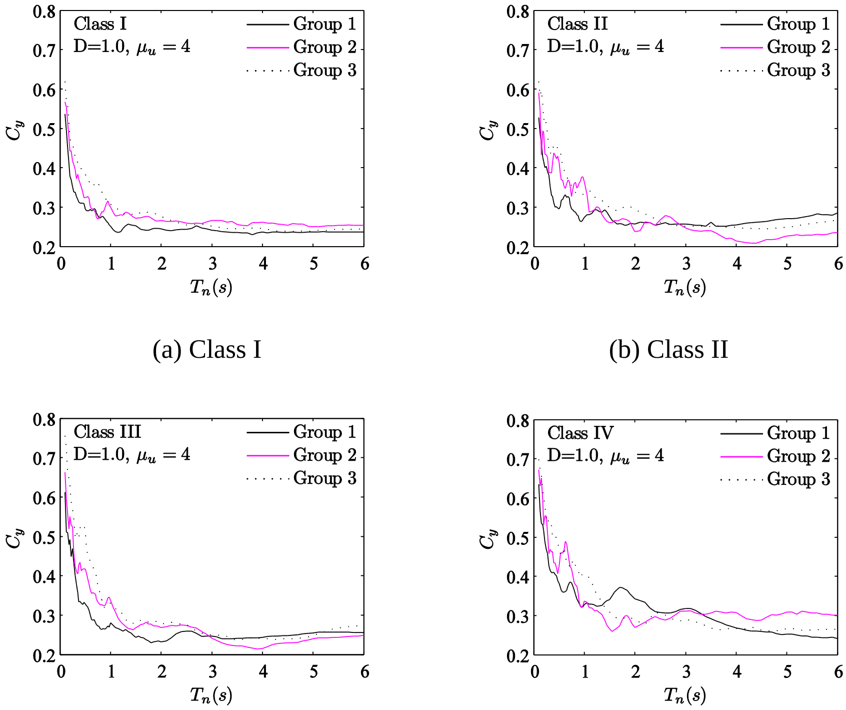
Fig. 4.5 illustrates the effect of site soil class on CDYSS for the SDOF system with the target damage index of 1.0 and ultimate ductility factor of 4, which indicates that in the range of short period, roughly less than 0.9s, C_y decreases when the site soil becomes stiffer, and this tendency is not clear in the other range of period.



(a) Group 1 (b) Group 2 (c) Group 3

Figure 4.5. Effect of site soil class on CDYSS (D=1.0)

The effect of design group is shown in Fig. 4.6. Similarly, in the range of short period, roughly less than 0.9s, C_y rises up with the increase of design group number which indicates the increase of epicentral distance and characteristic period of earthquake ground motions. This tendency is not clear in the other range of period.



(a) Class I (b) Class II

(c) Class III (d) Class IV

Figure 4.6. Effect of design group on CDYSS

5. EXPRESSIONS FOR CDYSS

5.1. Simplified Expression Proposed for Regression Analysis

The general expression to predict yield strength coefficient, considering the influence of category of site soil condition, damage level, ductility capacity and natural period of vibration, is as follows:

$$C_y = f\,(D, \mu_u, T_n, C_s) \tag{5.1}$$

where D is the target damage index, μ_u is the ultimate displacement ductility ratio under monotonic loading, T_{n} is the natural vibration period, and C_S is the category of site soil condition. After repeated calculation and comparison, the following functions are adopted for regression analysis:

$$C_y = A + \frac{B}{T_n^C} \tag{5.2}$$

$$A = a_1 \cdot (\mu_u D)^{a_2} \tag{5.3}$$

$$B = b_1 \cdot (\mu_u D)^{b_2} \tag{5.4}$$

$$C = c_1 \cdot e^{-\frac{c_2}{\mu_u D}} \tag{5.5}$$

where a_1, a_2, b_1, b_2, c_1, and c_2 are the coefficients related with category of site soil condition.

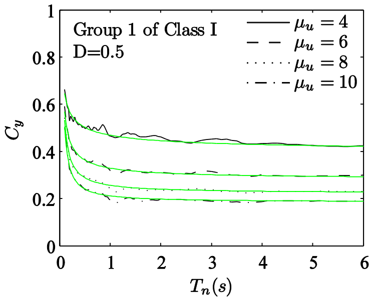
5.2. Results of Regression Analysis

Multiple nonlinear regression analysis are conducted to obtain the above six coefficients related with category of site soil condition with the aid of Matlab. The results of regression analysis are shown in Table 5.1. The comparison between the actual CDYSS and the simplified expressions using Eqns. 5.2

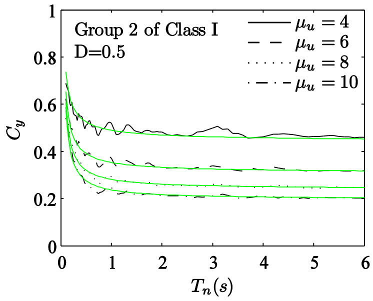
to 5.4 and Table 5.1 is shown in Figs. 5.1 and 5.2 for the combination of two damage indices, 0.5, and 1.0, two site soil classes, Class 1 and Class 2, and four ultimate displacement ductility factors, 4, 6, 8, and 10. The simplified expressions coincide with the actual CDYSS very well.

Table 5.1. Values of coefficients obtained by regression analysis

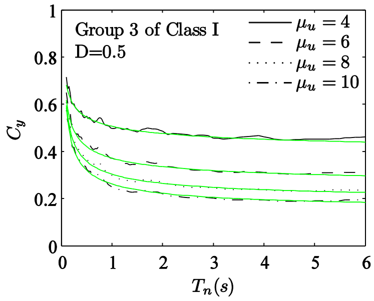
Category of site soil condition		a_1	a_2	b_1	b_2	c_1	c_2
I	Group1	0.6840	-0.8103	0.1824	-1.2145	1.8104	2.5311
	Group2	0.8125	-0.8822	0.0567	-0.2104	1.1314	0.7391
	Group3	0.7478	-1.0461	0.1626	-0.1736	0.7726	1.6100
II	Group1	0.8292	-0.8628	0.0419	-0.2763	1.3684	1.2589
	Group2	0.0045	-0.3812	0.8356	-0.7092	0.5540	3.2929
	Group3	0.7319	-1.2743	0.2393	-0.1196	0.5998	1.7793
III	Group1	0.8376	-0.9501	0.0560	-0.1689	1.2563	1.2068
	Group2	0.7589	-1.1692	0.1988	-0.1822	0.7638	1.7431
	Group3	0.6158	-1.1562	0.3015	-0.2265	0.7339	1.7729
IV	Group1	0.4448	-0.5865	0.3719	-0.6908	0.8747	2.6377
	Group2	0.7243	-0.8095	0.1292	-0.1080	0.8198	1.1491
	Group3	0.4511	-1.2311	0.4348	-0.2910	0.5884	2.1495



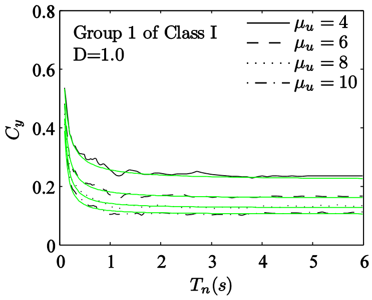
(a) Group 1 of Class I (D=0.5)



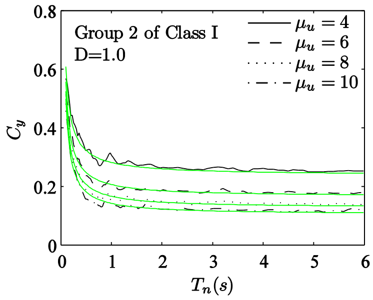
(b) Group 2 of Class I (D=0.5)



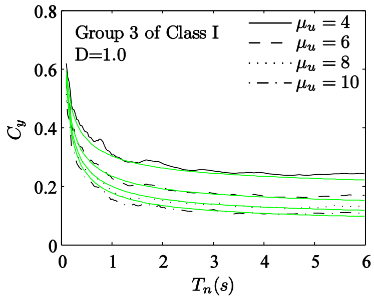
(c) Group 3 of Class I (D=0.5)



(d) Group 1 of Class I (D=1.0)

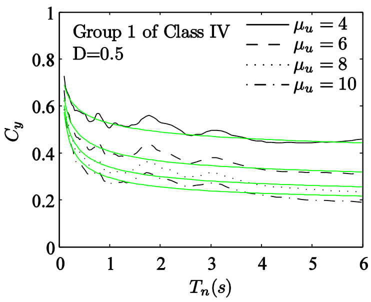


(e) Group 2 of Class I (D=1.0)

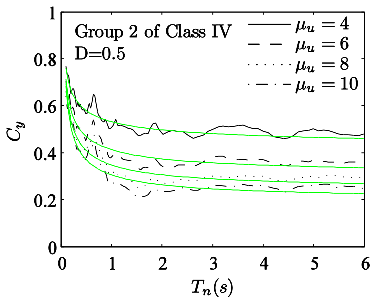


(f) Group 3 of Class I (D=1.0)

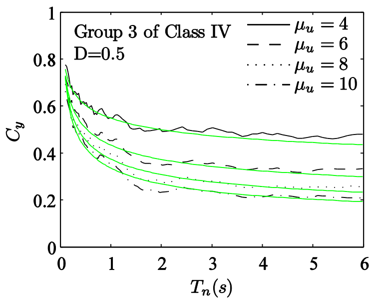
Figure 5.1. Comparison between actual CDYSS and the simplified expressions (Class I)



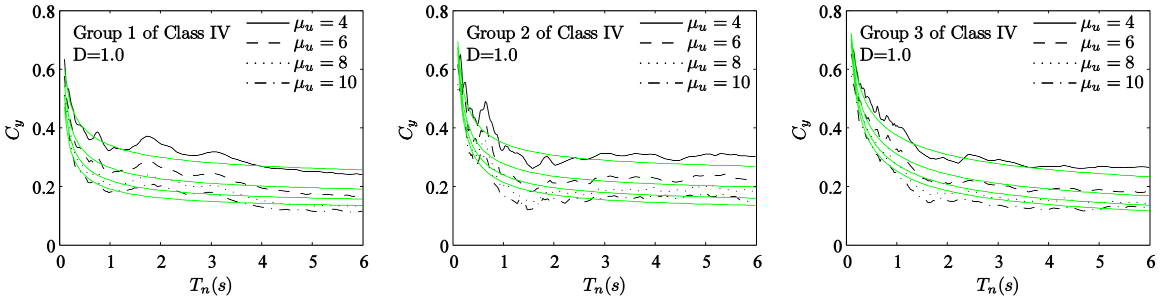
(a) Group 1 of Class IV (D=0.5)



(b) Group 2 of Class IV (D=0.5)



(c) Group 3 of Class IV (D=0.5)



(d) Group 1 of Class IV (D=1.0) (e) Group 2 of Class IV (D=1.0) (f) Group 3 of Class IV (D=1.0)

Figure 5.2. Comparison between actual CDYSS and the simplified expressions (Class IV)

6. CONCLUSIONS

In this study the constant-damage yield strength spectra considering the effect of target damage index, ductility capacity, and site soil condition are constructed by a great number of nonlinear time-history analysis of a SDOF system using the modified Park-Ang damage model and Takeda hysteretic model. The simplified expressions for CDYSS are developed by nonlinear regression analysis. CDYSS derived in this study can be applied for damage-control seismic design of RC structures with the uniform seismic hazard.

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