Analytical Formulations of Fragility Functions with Applications to Probabilistic Seismic Risk Analysis

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SUMMARY:
The role of seismic fragility functions in the PBEE probabilistic framework formula is revisited, and four categories of seismic fragility are identified. The analytical formulae of seismic demand and damage fragility functions are derived respectively, in which the analytical relationships of the median and the dispersion of the two kinds of fragility models with the parameters of probabilistic seismic demand model (PSDM) and probabilistic seismic capacity model (PSCM) are found. By applying the two analytical formulae of seismic fragility functions to the IM-based analytical formulation of seismic risk, it is discovered that the analytical formulae of seismic demand and damage hazards are two specific cases of the IM-based analytical formulation of seismic risk. A RC frame structure designed according to the current Chinese codes is taken as a case study. The seismic performance of the code-conforming reinforced concrete frame buildings is evaluated by the analytical seismic risk formulae.

Keywords: seismic fragility; seismic risk; demand fragility; damage fragility; analytical formulation

1. INTRODUCTION

The new-generation performance-based earthquake engineering (PBEE) developed in the PEER center involves explicit evaluation of system-level performance of civil infrastructures and rigorous treatment of randomness existing in earthquake strong motions, structural properties, physical damage, and economic and human losses, as well as uncertainties due to the lack of knowledge and statistical samples (Porter 2003; Moehle and Deierlein 2004). The role of seismic fragility functions in the PBEE probabilistic framework formula is revisited, and four categories of seismic fragility are identified, namely, seismic demand fragility, structural capacity fragility, seismic damage fragility, and seismic loss fragility. With the assumptions of lognormal distributions of seismic demand and structural capacity, as well as the assumption of power relation of conditional median EDP and IM, the analytical formulae of seismic demand fragility and seismic damage fragility functions are derived respectively from the viewpoint of product-format probability demand and capacity models, in which the analytical relationships of the median and the dispersion of the two kinds of fragility models with the parameters of probabilistic seismic demand model (PSDM) and probabilistic seismic capacity model (PSCM) are found. By applying the two analytical formulae of seismic fragility functions to the IM-based analytical formulation of seismic reliability under the assumption of Cauchy-Pareto approximation of seismic hazard function (Cornell *et al*. 2002), it is discovered that the analytical formula of seismic demand hazard and the analytical one of seismic damage hazard are two specific cases of the IM-based analytical formulation of seismic reliability. A RC frame structure designed according to the current Chinese codes is taken as a case study. The fragility curves of seismic demand, structural capacity, and seismic damage of the structure are obtained, and the seismic risk curves of seismic demand only considering the record-to-record variability inherent in earthquake strong motions, and seismic damage with also consideration of the building-to-building modeling uncertainty of structural capacity, are derived and compared. The seismic performance of the code-conforming reinforced concrete frame buildings in mainland of China is evaluated by the analytical seismic risk formulae, coupled with the rigorous utilization of nonlinear static and dynamic analysis and efficient random simulation techniques.

2. THE ROLE OF SEISMIC FRAGILITY FUNCTIONS IN PEER PBEE FRAMEWORK

The PEER PBEE methodology is based on a so-called framework formula that estimates the mean annual frequency (MAF) of a performance measure exceeding a specified threshold. Originally proposed by Cornell and Krawinkler (2000), the formula now has the commonly accepted form (Porter 2003; Moehle and Deierlein 2004):

$$\lambda(dv) = \int_{dm}\int_{edp}\int_{im} G(dv \mid dm) \mid dG(dm \mid edp) \mid\mid dG(edp \mid im) \mid\mid d\lambda(im) \mid \quad (2.1)$$

where, *im* = intensity measure (e.g. the peak ground acceleration PGA or the spectral acceleration *Sa* at the fundamental period T_1); *edp* = engineering demand parameter (e.g. the peak interstory drift angle); *dm* = damage measure corresponding to a damage state (e.g. the slight damage state); *dv* = decision variable (e.g. financial loss, casualty, downtime); $G(x|y) = G(X \geq x|Y = y)$ is the conditional complementary cumulative distribution function (CCDF) of random variable X given $Y = y$; $\lambda(z)$ denotes the mean rate of event $\{Z \geq z\}$ per year; $dG(x|y)$ and $d\lambda(z)$ are the differentials of $G(x|y)$ and $\lambda(z)$, respectively.
The triple-integral formula can be de-coupled into successive double or single integrals based on the Markovian assumption of one step memory for each of the intermediate measures. Specifically,

$$\lambda(dm) = \int_{edp}\int_{im} dG(dm \mid edp) \mid dG(edp \mid im) \mid\mid d\lambda(im) \mid \quad (2.2)$$

gives the output of probabilistic seismic safety analysis (PSSA), i.e., MAF of reaching or exceeding a damage state $\{DM \geq dm\}$, whereas

$$\lambda(edp) = \int_{im} G(edp \mid im) \mid d\lambda(im) \mid \quad (2.3)$$

gives the result of probabilistic seismic demand analysis (PSDA), i.e., MAF of reaching or exceeding a specified demand level $\{EDP \geq edp\}$.
The PEER's PBEE framework formula disaggregates the problem of probabilistic seismic risk assessment (PSRA) into four models: the hazard model that predicts the intensity measure IM, $\lambda(im)$; the demand model that predicts the structural response referred to as engineering demand parameter EDP, $G(edp|im)$; the capacity model that predicts the damage measure DM, $G(dm|edp)$; and finally the loss model that predicts the decision variables DV, $G(dv|dm)$. If we take IM (such as PGA or *Sa*) as the input variable of a seismic fragility function corresponding to EDP, then the conditional distribution $G(edp|im)$ can be called "demand fragility function". Similarly, the damage and loss fragility functions corresponding to DM and DV respectively can be derived from the inner integrals in Eqn. 2.1:

$$G(dv \mid im) = \int_{dm}\int_{edp} G(dv \mid dm) dG(dm \mid edp) dG(edp \mid im) \quad (2.4)$$

$$G(dm \mid im) = \int_{edp} G(dm \mid edp) dG(edp \mid im) \quad (2.5)$$

Substituting Eqn. 2.4 and Eqn. 2.5 into Eqn. 2.1 and Eqn. 2.2 respectively, the latter two formulae are reduced to

$$\lambda(dv) = \int_{im} G(dv \mid im) \mid d\lambda(im) \mid \quad (2.6)$$

$$\lambda(dm) = \int_{im} G(dm \mid im) \mid d\lambda(im) \mid \quad (2.7)$$

Note that Eqn. 2.6 and Eqn. 2.7 have the same "structure" as Eqn. 2.3, they all are specific forms of the general risk formula "risk=hazard×fragility". Actually, we can treat the events $\{DV \geq dv\}$, $\{DM \geq dm\}$ and $\{EDP \geq edp\}$ as reaching or exceeding a limit state through performance measures DV, DM and EDP, respectively, so conditional probabilities $G(dv|im)$, $G(dm|im)$ and $G(edp|im)$ can be denoted as a general fragility function

$$F_R(x) = P[LS \mid IM = x] \tag{2.8}$$

then Eqn. 2.3, Eqn. 2.6 and Eqn. 2.7 would take the same formulation as

$$\lambda_{\mathrm{LS}} = \int_x F_R(x) \mid d\lambda_{IM}(x) \mid \tag{2.9}$$

where, $\lambda_{IM}(x)=\lambda(im=x)$. Eqn. 2.9 is nothing but the well-known probability interference formula in structural reliability theory (Melchers 1999).
In Eqn. 2.9, the seismic fragility is customarily modeled by a lognormal cumulative distribution function (CDF) (Ellingwood 2001; Wen *et al*. 2003):

$$F_R(x) = \Phi\left[\frac{\ln\left(x / m_R\right)}{\beta_R}\right] \tag{2.10}$$

in which $\Phi[\cdot]$ is the standard normal probability integral, m_R is the median fragility, and $\beta_R=\sigma_{\ln R}$ is the logarithmic standard deviation (or dispersion) of the fragility.
In the following section, we will derive the analytical formulations as well as their parameter relationships of seismic demand fragility and seismic damage fragility functions. To simplify the problem, we will not consider the loss fragility any more, but the results of this paper can be easily extended to the case of seismic loss fragility and risk assessment.

3. ANALYTICAL FORMULATIONS OF SEISMIC FRAGILITY FUNCTIONS

3.1. Analytical formulation of seismic demand fragility

The seismic demand fragility is the conditional probability of reaching or exceeding a specific value of the demand limit state without considering capacity uncertainty:

$$F_{R_D}(x) = P[D \geq d^{\mathrm{LS}} \mid IM = x] \tag{3.1}$$

where d^{LS} is the specific value of the seismic demand D which defines the determinative threshold of one limit state. Due to the record-to-record variability in earthquake strong motions, the seismic demand D also is random over any suite of ground motion records applied to the structure. For a given value of the ground motion intensity measure, such as spectral acceleration, the conditional demand model is normally modeled by a product of its conditional median m_D with a random variable ε_D:

$$D = m_D \cdot \varepsilon_D \tag{3.2}$$

where ε_D is a lognormal random variable with median equal to unity and conditional logarithmic standard deviation $\sigma_{\ln\varepsilon} = \beta_{D|Sa}$, the functional relationship between conditional median m_d and spectral acceleration S_a generally follows a power law (Cornell *et al*. 2002):

$$m_D = a(IM)^b \tag{3.3}$$

where a and b are regression parameters. The parameters a, b, and $\beta_{D|IM}$ can be determined by linear regression of $\ln m_D$ vs. $\ln IM$ from the results of any probabilistic seismic demand analysis (PSDA) method, e.g. cloud analysis, strip analysis, or incremental dynamic analysis (IDA) (Jalayer 2003).
With the above assumptions, it follows that Eqn. 3.1 can be further derived as:

$$\begin{aligned} F_{R_D}(x) &= 1-\Phi\left[\frac{\ln\left(d/ax^b\right)}{\beta_{D|IM}}\right] = \Phi\left[\frac{\ln\left(ax^b\right)-\ln d}{\beta_{D|IM}}\right] = \Phi\left[\frac{\ln x - \ln\left(d/a\right)^{1/b}}{\frac{1}{b}\beta_{D|IM}}\right] \\ &= \Phi\left[\frac{\ln\left(x/m_{R_D}\right)}{\beta_{R_D}}\right] \end{aligned} \tag{3.4}$$

in which m_{R_D} and β_{R_D} are the median and dispersion of the demand fragility, respectively. Obviously, the parameters of the lognormal demand fragility model can be re-formulated as:

$$m_{R_D} = \left(\frac{d^{\mathrm{LS}}}{a}\right)^{1/b} = im^d \tag{3.5}$$

$$\beta_{R_D} = \frac{1}{b}\beta_{D|IM} \tag{3.6}$$

where $im^d = (d^{\mathrm{LS}}/a)^{1/b}$ is the IM value corresponding to the threshold d^{LS} of one limit state. From Eqn. 3.5 it can be seen that the median m_{R_D} of the demand fragility is equal to this specific spectral acceleration value im^d, which controls the location of the demand fragility curve; whereas the dispersion β_{R_D} controls the shape of the demand fragility curve.

3.2. Analytical formulation of seismic damage fragility

The damage fragility considers the randomness existed both in demand and capacity. The damage fragility function can be defined as the conditional failure probability of seismic demand reaching or exceeding random structural capacity given a specific value of strong motion intensity:

$$F_{R_C}(x) = P[D \geq C \mid IM = x] \tag{3.7}$$

Again we assume that the structural capacity C is a lognormal variable with median m_C and dispersion β_C, then the CDF of the structural capacity C is

$$F_C(d) = P[C \leq d] = \Phi\left[\frac{\ln\left(d/m_C\right)}{\beta_C}\right] \tag{3.8}$$

With the lognormal assumptions of both demand and capacity defined above, the damage fragility Eqn. 3.7 can be derived according to the basic principle in structural reliability theory (Melchers, 1999) as

$$F_{R_C}(x) = \Phi\left[-\frac{\ln m_C - \ln m_D}{\sqrt{\beta_C^2 + \beta_{D|IM}^2}}\right] \tag{3.9}$$

Substituting Eqn. 3.3 into the above Eqn. yields

$$F_{R_C}(x) = \Phi\left[\frac{\ln m_{D|IM} - \ln m_C}{\sqrt{\beta_{D|IM}^2 + \beta_C^2}}\right] = \Phi\left[\frac{\ln(ax^b) - \ln m_C}{\sqrt{\beta_{D|IM}^2 + \beta_C^2}}\right] = \Phi\left[\frac{\ln x - \ln\left(m_C / a\right)^{1/b}}{\frac{1}{b}\sqrt{\beta_{D|IM}^2 + \beta_C^2}}\right]$$
$$= \Phi\left[\frac{\ln\left(x / m_{R_C}\right)}{\beta_{R_C}}\right] \tag{3.10}$$

where m_{R_C} and β_{R_C} are the median and dispersion of the damage fragility, respectively. Obviously, the parameters of the lognormal damage fragility model can be re-formulated as:

$$m_{R_C} = \left(\frac{m_C}{a}\right)^{1/b} = im^{m_C} \tag{3.11}$$

$$\beta_{R_C} = \frac{1}{b}\sqrt{\beta_{D|IM}^2 + \beta_C^2} \tag{3.12}$$

where $im^{m_C} = (m_c / a)^{1/b}$ is the IM value corresponding to the demand level d which is equal to the median capacity m_C. From Eqn. 3.11 it can be seen that the median m_{R_C} of the damage fragility is equal to this specific intensity measure value im^{m_C}, which controls the location of the damage fragility curve; whereas the dispersion β_{R_C} controls the shape of the damage fragility curve. It is obvious that if the randomness in structural capacity is not considered, then Eqn. 3.12 will reduce to Eqn. 3.6.

4. APPLICATIONS OF ANALYTICAL FRAGILITY FUNCTIONS TO PROBABILISTIC SEISMIC RISK ANALYSIS

4.1 Analytical formulation of probabilistic seismic risk at the IM level

Generally assumed to be a Type II distribution of the largest values, the probability distribution of the annual extreme earthquake strong motion intensity measure (e.g., spectral acceleration) can be approximated over the range of significance by (Cornell *et al*. 2002):

$$\lambda_{IM}(x) = P[IM \geq x] = 1 - \exp\left[-\left(\frac{x}{u}\right)^{-k}\right] \approx \left(\frac{x}{u}\right)^{-k} \approx k_0 x^{-k} \tag{4.1}$$

where u = scale parameter, k = shape parameter, constant $k_0 = u^k$.
Eqn. 4.1 represents a linear relationship on a log-log plot in the region of interest, i.e., where the contribution to the total probability integral is greatest (Cornell *et al*. 2002).
Substituting Eqn. 2.10 and Eqn. 4.1 into Eqn. 2.9 and carrying out the integral, one can obtain the analytical formulation of IM-based seismic risk problem (Cornell 1994; Cornell 1996a, 1996b):

$$\lambda_{\text{LS}} = \lambda_{IM}(m_R)\exp\left(\frac{1}{2}k^2\beta_R^2\right) \tag{4.2}$$

4.2. Analytical formulation of seismic demand hazard

With the analytical formulation of the demand fragility, the analytical formulation of the demand hazard can be directly obtained by substituting Eqns. 3.5 and 3.6 into the analytical formulation of the seismic risk Eqn. 4.2:

$$\lambda_{EDP}(d)=\lambda(D\geq d)=\int F_{R_D}(x)\left|d\lambda_{IM}(x)\right|=\lambda_{IM}(m_{R_D})\exp\left[\frac{1}{2}k^2\beta_{R_D}^2\right]$$
$$=\lambda_{IM}(im^d)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\beta_{D|IM}^2\right] \quad (4.3)$$

The above displacement-based explicit format for probabilistic seismic demand analysis (PSDA) is derived firstly by Bazzurro *et al.* (1998), and widely applied in PSDA, e.g., Shome (1999), Luco (2002), etc.

4.3. Analytical formulation of seismic damage hazard

Similarly, with the analytical formulation of the damage fragility, we can obtain the analytical formulation of the damage hazard, or the limit state probability, directly by substituting Eqns. 3.11 and 3.12 into the analytical formulation of the seismic risk Eqn. 4.2:

$$\lambda_{DM}(C)=\lambda(D\geq C)=\int F_{R_C}(x)\left|d\lambda_{IM}(x)\right|=\lambda_{IM}(m_{R_C})\exp\left[\frac{1}{2}k^2\beta_{R_C}^2\right]$$
$$=\lambda_{IM}(im^{m_C})\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{D|IM}^2+\beta_C^2\right)\right] \quad (4.4)$$

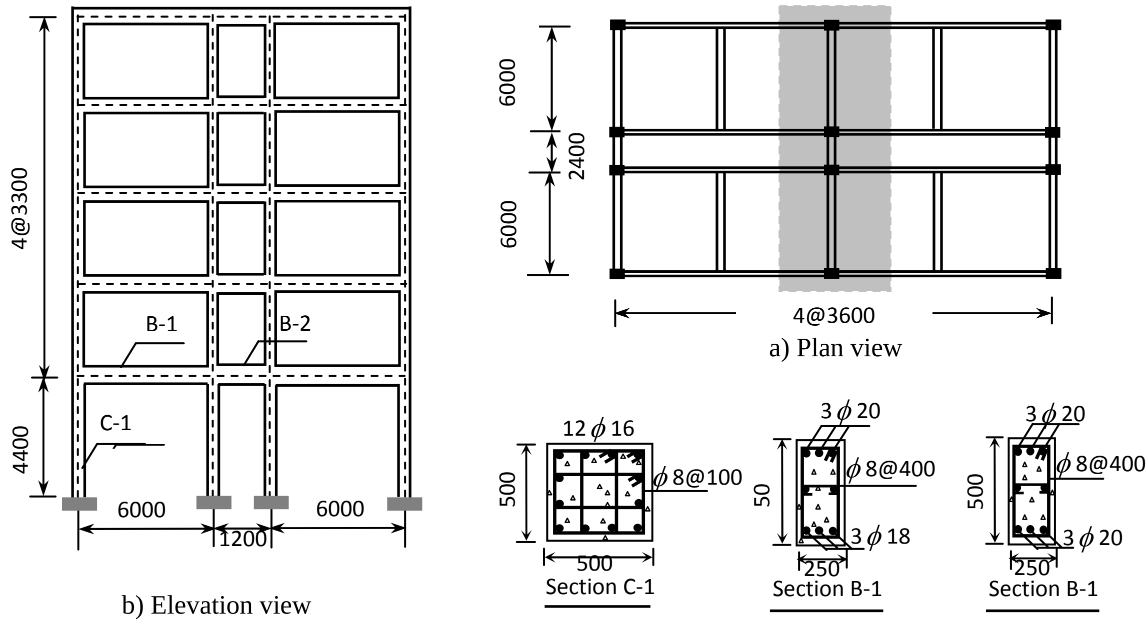
The above displacement-based explicit format for annual limit state frequency is used by PEER as a basis for probabilistic performance-based assessment and design in the framework of demand and capacity factor design (DCFD) methodology adopted by FEMA350 (FEMA 2000a; Cornell *et al.* 2002; Jalayer and Cornell 2003).

5. CASE STUDY: A CODE-CONFORMING RC FRAME

A five-storey RC frame structure was designed according to Chinese codes GB50011-2010 and GB 50010-2002, which represented the typical mid-rise RC frame construction in China. The designed structure is located on firm rock (site Class II in GB50011-2010). The fortification intensity is degree VII, and the design characteristic period of the location site is 0.35s. The frame structure is in bi-axially plan configuration with negligible torsional eccentricities, as shown in Figure 1. The elevation of the case-study frame with its beam and column steel rebar layouts are shown in Figure 1. The design value of concrete compressive strength is 14.3MPa, whereas the design steel yield strength is 300MPa.

A 2-D model is selected for nonlinear static and dynamic analyses. The analytical modeling and the finite element analysis are performed using OpenSees (Mazzoni *et al*. 2007). Each column and beam consists of a single force-based nonlinear beam-column element with four Gauss-Legendre integration points. The nonlinear beam-column elements are characterized by fiber sections, which are discretized in fibers of confined and unconfined concrete, and steel reinforcement. Using available material models in OpenSees, the reinforcing steel is characterized by the Giuffre-Menegotto-Pinto model (Menegotto *et al*. 1973), while the concrete behavior is modeled by a uniaxial model with degrading linear unloading/loading stiffness, and no strength in tension. Accounting for the level of confinement offered by the actual layout, diameter and spacing of stirrups, formulas developed by Scott *et al.* (1982) are used to characterize the behavior of core concrete.

A total of fourteen material parameters are used to model the various structural materials, i.e., four parameters for the unconfined concrete ($f_{\mathrm{cp,cover}}$ = peak strength, $\varepsilon_{\mathrm{cp,cover}}$ = the strain at peak strength; $f_{\mathrm{cu,cover}}$= the residual strength; $\varepsilon_{\mathrm{cu,cover}}$=strain at which the residual strength is reached), four parameters for the confined concrete ($f_{\mathrm{cp,core}}$, $\varepsilon_{\mathrm{cp,core}}$, $f_{\mathrm{cu,core}}$, $\varepsilon_{\mathrm{cu,core}}$), and six parameters for the reinforcing steel (E_S=initial stiffness, f_y=yield strength, α=post yield to initial stiffness ratio, CR_1, CR_2, R_0=the parameters controlling the transition from elastic to plastic branches). Two concrete parameters and four steel parameters are treated as deterministic values ($\varepsilon_{\mathrm{cp,cover}}$=0.002, $f_{\mathrm{cu,cover}}$=0, CR_1=0.925, CR_2=0.15, R_0=20, α=0), while the other eight parameters are taken as random variables. From elastic analysis, the first mode period T_1 of the structure is 0.72s.



c) Steel rebar layout of beams and columns

Figure 1. Case-study structure

One hundred ordinary (defined here as closest distance to rupture greater than 10 km to avoid considering directivity pulse-type effects) ground motions are selected from the PEER-NGA database (http://peer.berkeley.edu/nga/) to represent the RTR variability.

The parameters k and k_0 are fitted through seismic hazard data at the design basis earthquake (DBE) and the maximum considered earthquake (MCE) intensity levels which have 10% and 2% probabilities of exceedance in 50 years, respectively. The power law models of PSHA for PGA and S_a are estimated by

$$\lambda_{PGA}(x) = 1.70\times10^{-5}x^{-2.09} \tag{5.1a}$$
$$\lambda_{S_a}(x) = 1.03\times10^{-5}x^{-2.38} \tag{5.1b}$$

The seismic demand is assessed using nonlinear time history analysis (NTHA) with the selected one hundred earthquake records. Regression of the NTHA results leads to the following power relationships:

$$\theta_{\max} = 0.024PGA^{0.84}, \quad a = 0.024, \quad b = 0.84, \quad \beta_{D|PGA} = 0.25 \tag{5.2a}$$
$$\theta_{\max} = 0.024Sa^{0.90}, \quad a = 0.024, \quad b = 0.90, \quad \beta_{D|Sa} = 0.14 \tag{5.2b}$$

The maximum interstory drift angle, $\theta_{\max}$, is selected as capacity parameter. In order to quantitatively define limit states, pushover analysis is used to evaluate drift capacity. Four damage states, namely, slight damage (LS_1), moderate damage (LS_2), extensive damage (LS_3) and complete damage (LS_4) are considered. The first limit state, LS_1, is defined as the linear limit of the global pushover curve, which represents the first significant change of global performance of the structure. The second limit state, LS_2, is identified by the equivalent yield point, which is obtained by the idealized equivalent elasto-plastic system with energy absorption equivalent to that of the original system. The third limit state, LS_3, is assumed as the peak shear resistance, while the last limit state, LS_4, is assumed to correspond to the 20% reduced post-peak capacity or the ultimate point of the pushover curve.

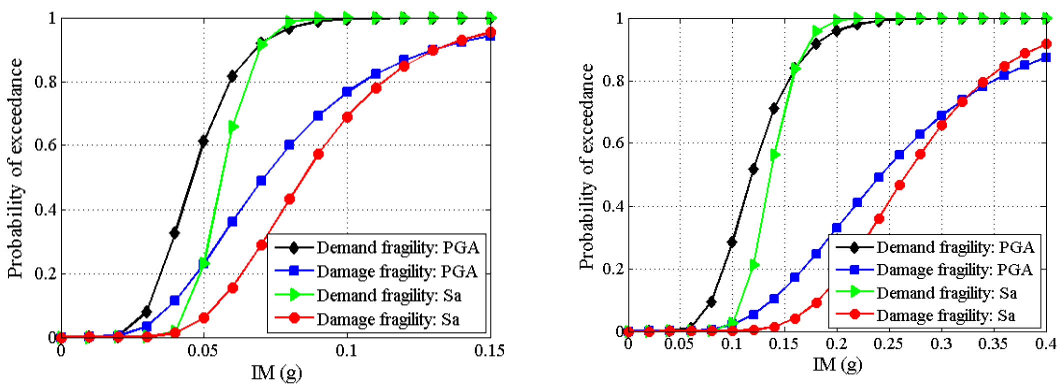
To consider the randomness in structural properties, a random pushover approach (RPA) via a transformed correlation Latin Hypercube Sampling (TCLHS) algorithm is applied. Ten random variables (n=10) are considered in random pushover analysis, and one hundred structural samples (N=100) are generated.
From the viewpoints of uncertainty modeling and propagation, β_{C} contains two components of uncertainty: the aleatory component, β_{CR}, and the epistemic component, β_{CU}:

$$\beta_C = \sqrt{\beta_{CR}^2 + \beta_{CU}^2} \tag{5.3}$$

where β_{CR} represents the results of structural uncertainty propagation, which is obtained using random pushover analysis procedure; while β_{CU} arises from the assumptions in structural modeling and RPA, which are generally assumed to be specified values using expert judgments. In this paper, the epistemic uncertainty, β_{CU} is assumed to be 0.2 for LS_i (i=1,2,3,4) according to Wen *et al*. 2003.
The distribution parameters for the threshold values for different limit states LS_i (i=1,2,3,4) are: m_C=0.26%, 0.73%, 1.44%, and 2.44% for LS_i (i=1,2,3,4), while the corresponding β_{CR} are 0.16, 0.05, 0.08, 0.18, respectively. Substituting the values of β_{CU} and β_{CR} in Eqn. 5.3, β_C are calculated as 0.26, 0.21, 0.22, and 0.27 for LS_i (i=1,2,3,4), respectively.
For seismic demand fragility analysis, the deterministic limit state thresholds are identified according to GB50011-2010, which are defined as 0.18%, 0.40%, 0.83% and 2.00% for LS_i (i=1,2,3,4), respectively. Using the analytical demand fragility functions (Eqns. 3.4-3.6), the seismic demand fragility curves are derived as shown in Figure 2 with their corresponding medians and dispersion listed in Table 1. For seismic damage fragility analysis incorporating the capacity uncertainty, the seismic damage fragility curves are generated using the analytical damage fragility functions (Eqns. 3.10-3.12), as shown in Figure 2 with their medians and dispersions listed in Table 1.

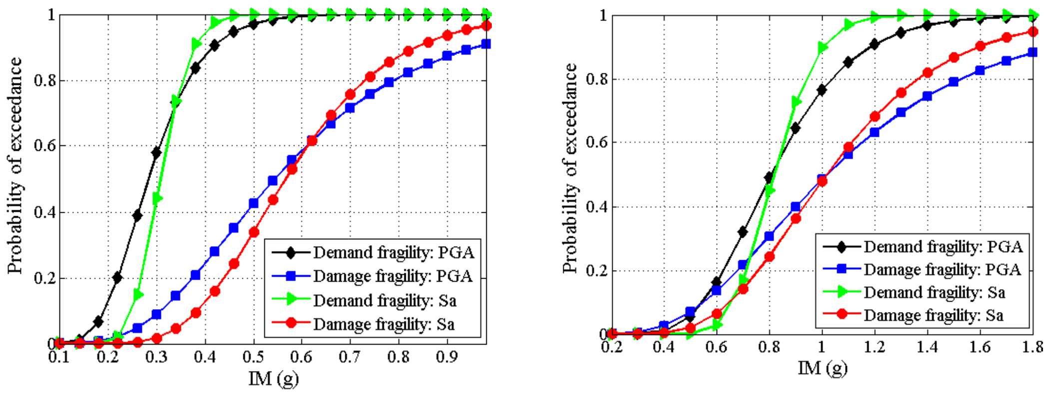
Table 1. Parameters of seismic hazard, demand fragility and damage fragility functions

Limit states	IM	PSHA		PSDA			d^{LS} (%)	PSCA		Demand fragility		Damage fragility	
		k_0 (×10^{-5})	k	a	b	$\beta_{D\|IM}$		m_C (%)	β_C	m_{R_D} (g)	β_{R_D}	m_{R_C} (g)	β_{R_C}
LS_1	PGA	1.70	2.09	0.024	0.84	0.25	0.18	0.26	0.26	0.05	0.30	0.07	0.43
	S_a	1.03	2.38	0.024	0.90	0.14				0.06	0.16	0.08	0.32
LS_2	PGA	1.70	2.09	0.024	0.84	0.25	0.40	0.73	0.21	0.12	0.30	0.24	0.39
	S_a	1.03	2.38	0.024	0.90	0.14				0.14	0.16	0.27	0.28
LS_3	PGA	1.70	2.09	0.024	0.84	0.25	0.83	1.44	0.22	0.28	0.30	0.54	0.39
	S_a	1.03	2.38	0.024	0.90	0.14				0.31	0.16	0.57	0.29
LS_4	PGA	1.70	2.09	0.024	0.84	0.25	2.00	2.44	0.27	0.80	0.30	1.02	0.43
	S_a	1.03	2.38	0.024	0.90	0.14				0.82	0.16	1.02	0.34



a) LS_1: slight damage

b) LS_2: moderate damage



c) LS_3: extensive damage

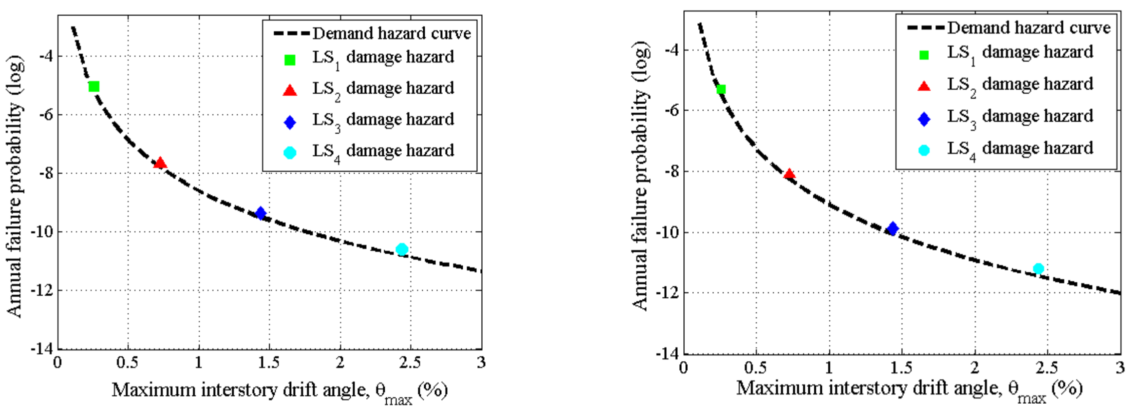
d) LS_4: complete damage

Figure 2. Seismic demand and damage fragility curves

Using Eqn. 4.3, the probabilistic seismic demand hazard curves for both PGA and S_a are derived, as shown in Figure 3, where seismic damage hazard points by using Eqn. 4.4 are also illustrated. In order to compare with each other, the limit state frequency values (seismic demand and damage hazards) are also listed in Table 2. It can be seen that the damage hazard values are relatively smaller than the corresponding demand hazard levels due to the existence of capacity variation and the higher median capacity identified by RPA. In addition, different choices of IMs (*PGA* or S_a) also have significant effects on seismic risk regardless of whether the limit states are deterministic or probabilistic.

Table 2. MAFs for seismic demand and damage hazards

Limit states	λ_{EDP} using PGA	λ_{EDP} using *Sa*	λ_{DM} using PGA	λ_{DM} using *Sa*
LS_1	1.30×10^{-2}	1.05×10^{-2}	6.43×10^{-3}	4.92×10^{-3}
LS_2	1.79×10^{-3}	1.26×10^{-3}	4.58×10^{-4}	2.99×10^{-4}
LS_3	2.91×10^{-4}	1.84×10^{-4}	8.45×10^{-5}	5.05×10^{-5}
LS_4	3.25×10^{-5}	1.79×10^{-5}	2.44×10^{-5}	1.37×10^{-5}



a) Seismic demand and damage hazard using PGA

b) Seismic demand and damage hazard using *Sa*

Figure 3. Seismic demand and damage hazard curves

6. CONCLUSION

In this study, through the derivation of the analytical functions and their parameter relationships of the demand fragility and the damage fragility, it is found that the demand hazard and the damage hazard are two specific formulations of the general IM-based seismic risk problem, depending on the chosen analytical fragility functions. Therefore, the force-based seismic risk formulation for intensity measure

(IM) and the displacement-based seismic risk formulations for engineering demand parameter (EDP) and damage measure (DM) are consistent in nature.

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