Dynamic behaviour of reinforced soils and deep foundations Theoretical analysis and experiments

J. Soubestre, C. Boutin & S. Hans
Université de Lyon, ENTPE/CNRS - France

M. Dietz, L. Dihoru, E. Ibraim & C. Taylor
University of Bristol, UK

SUMMARY:
The dynamic response of soil-pile-group systems are modelled both analytically, using homogenisation theory, and physically, using a shaking table to excite a soft elastic material periodically reinforced by vertical slender inclusions. A large soil/pile stiffness contrast is shown to lead to full coupling in the transverse direction of the bending behaviour from the piles and the shear behaviour from the soil. Analytically derived performance predictions capture important characteristics of the experimentally observed response. Thus the shear/bending analytical modelling should provide a simple manner to design and describe soil/piles system submitted dynamically to lateral ground motions.

Keywords: Reinforced soils, Deep Foundations, Dynamic, Homogenization, Experiments

1. INTRODUCTION

The performance of the global behaviour of pile-reinforced soil when subjected to lateral ground motions has been a topic of research interest in recent decades (e.g. Makris and Gazetas (1992), Mylonakis and Gazetas (1999), Koo et al (2003)). Numerical finite element studies of pile-reinforced soils are generally conducted. However, the resulting models are complex and require significant computing time due to the fine mesh needed to account for the heterogeneities in the medium. In fact, the problem is ill-conditioned because of the high number of piles and the high contrast between the mechanical properties of the soil and the piles. Another limitation lies in the purely numerical form of the result. An understanding of the interactions and the effective influence of the pile parameters can only be extracted by statistical back-analysis of numerous simulations.

An alternative approach is provided by the homogenisation of periodic media (Sanchez-Palencia's, 1980). Using asymptotic expansion techniques this method enables to derive the macroscopic equivalent behaviour of heterogeneous media. Herein, the key assumption lies in the separation of scale between the local size (i.e. here the distance between the piles) and the scale of evolution of the phenomena (i.e. the macroscopic deformation of the whole system). A direct application to piles/soil system leads to a classical behavior of composite, cf. Postel (1985). However, (Sudret and De Buhan, 1999) (De Buhan and Hassen (2008), argued that the slenderness of the embedded reinforcement should ensure that the response will involve bending, the classical assumption in earthquake engineering practice. Assuming a sparse reinforcement concentration and a large reinforcement/matrix stiffness ratio, they developed a phenomenological 'multiphase model' that accounts for the bending effect. An analytical model linking the previous approaches and based on homogenisation was proposed by Boutin and Soubestre (2011) to characterize the dynamic behaviour of pile-reinforced soils. In accordance to the configuration of the problem, the model can evaluate the contributions made to the global behavior by both shear (in the soil) and bending (in the pile). Both homogenised and multiphase models belong to the framework of generalized elastic continua in which the integration of the bending is related to a scale effect.

This paper presents the physical arguments at the basis of homogenised model proposed by Boutin and Soubestre (2011), and to provide an experimental validation of this latter. In section 2, the main aspect of the theoretical modelling are summarised. In section 3, the findings of an experimental campaign conducted at the University of Bristol (UK) and under the auspices of the European Commission's SERIES project in order to validate the analytical modelling approach are described.

2. MODELING OF SOIL/PILES SYSTEM

The study is concerned by a soil (index m) in which parallel, identical, homogeneous, straight beams, (index p) are periodically embedded with perfect contact (Figure 1.a). The dimension H along the beam axis is significantly larger than the lateral dimension l of the order of period (Figure 1.b). The typical size of the beam section h is considered of the same order than l so that the reinforcements are in finite concentration (however weaker concentration can also be addressed). In the sequel, S_p denotes the beam section, S_m the soil section, Γ the soil/beam interface ; the section of the period is $S = S_p \cup S_m$, and its boundary is δS.

The soil and the piles are assumed to present an isotropic linear elastic behavior. The two constituents are characterized by their Lame coefficients λ_q and μ_q ($q = m$, p) or, equivalently, by their Young's modulus E_q and Poisson' ratio ν_q. For soils, the (quasi-) linear assumption is reasonable for sufficiently small strains (say less than 10^{-4} m/m), which is the operational case for deep foundation systems. This small strain level corresponds also to small amplitude earthquakes, industrial induced vibrations, ambient noise vibrations, and geophysical testing conditions.

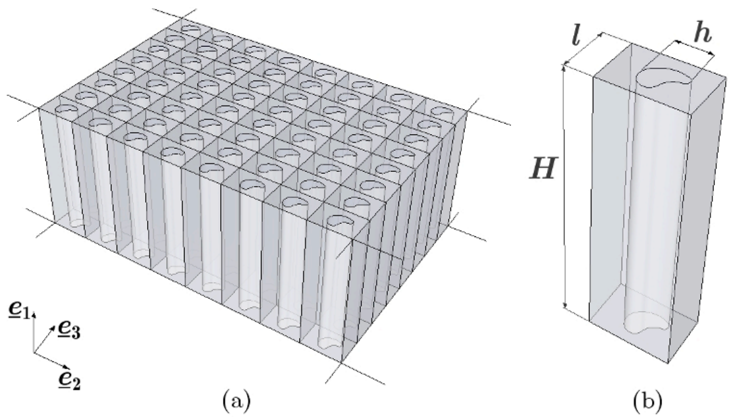

Figure 1. Deep foundation : soil reinforced by piles. (a) Periodic distribution of parallel identical homogeneous reinforcing piles embedded in the soil matrix (b) Period geometry and dimensions.

The geometry of the reinforced media naturally introduces (i) a distinction between the axial direction (unit vector $\underline{e}_1$) and the in-plane directions of the section ($\underline{e}_2$, $\underline{e}_3$) and (ii) a scale parameter $\varepsilon = l/L \ll 1$, where the macroscopic length L (generally of the order of H) is much larger than l.

2.1. Condition for shear/bending coupling

The contrast between the elastic properties of the matrix and reinforcement plays a crucial role in the global behaviour of the reinforced soil. Without the matrix, the pile lattice is governed by bending ; if the matrix and the reinforcement materials present identical properties, we would have a homogeneous medium governed by shear. This extreme cases lead to think that, for a given contrast of properties, both shear and coupling effects would be of the same order of magnitude.

The shear/bending model is based on the following idea : the (quasi)-static equilibrium and the elastic shear behaviour of the matrix (e.g. in the direction $\underline{e}_2$) reads :

$$\frac{d\,{}^m\tau}{dx_1}=0 \quad ; \quad {}^m\tau=\mu_m\frac{d\,{}^m u}{dx_1}$$

while for a beam governed by bending, we have :

$$\frac{d\,{}^pT_2}{dx_1}=0 \quad ; \quad {}^pT_2=\frac{d\,{}^pM}{dx_1} \qquad \text{and} \qquad {}^pM=-E_pI_p\frac{d^2\,{}^pu}{dx_1^2}$$

Hence, the coupling between the beam behavior in bending and the shear behavior of the matrix occurs when the transverse forces in both constituents are of the same order of magnitude. This implies that ${}^pT = S_m\,{}^m\sigma_{12}$, and therefore :

$$E_pI_p\frac{{}^pu}{L^3}=O\left(\mu_mS_m\frac{{}^mu}{L}\right)$$

As $I_p = O(l^4)$, $S_m = O(l^2)$, and since the displacement in the soil and the reinforcement are of the same order of magnitude (${}^pu = O({}^mu)$), the contrast of the shear modulus μ_m/μ_p (or equivalently of the Young modulus E_m/E_p) has to be of the order of magnitude of the squared scale ratio ε^2:

$$\frac{\mu_m}{\mu_p}=O\left(\frac{\mu_m}{E_p}\right)=O\left(\frac{l^2}{L^2}\right)=O\left(\varepsilon^2\right)$$

2.2. Global behavior derived by homogenization

Such a strong contrasted situation can be handle through homogenization method (Sanchez-Palencia, 1980). The physical variables are expressed with the relevant dimensionless space variables (x_1/L, x_2/l, x_3/l) or more conveniently with the appropriate physical space variables (x_1, y_2, y_3), where $y_2 = \varepsilon^{-1}x_2$; $y_3 = \varepsilon^{-1}x_3$. The dynamic equilibrium of both constituents is rewritten in a two-scale formulation (variables x and y) and the asymptotic behavior reached when ε tends to zero is determined. In this purpose, the displacement, strain and stress fields of the reinforcement and the matrix are expanded asymptotically according to the powers of ε and the ε^2-stiffness contrast is integrated. These expansions introduced in the balance equations leads to a series of problems in powers of ε. Their successive resolutions are performed up to derive the governing equations at the macroscopic scale.

For a stiffness contrast $\mu_m = O(\varepsilon^2\mu_p)$ and for a bi-symetric period geometry (as is usual in practice when piles are arranged in a square or a hexagonal grid) the resolution, detailed in Boutin and Soubestre (2011), shows that for harmonic motions at circular frequency ω :

– Along the axial direction, the macroscopic vertical motion $U_1(x_1)$ of the soil/pile system is driven by the piles that suffers a classic kinematic of compression :

$$\frac{d\langle\sigma_n\rangle}{dx_1}=0 \quad ; \quad \langle\sigma_n\rangle=E_pS_p\frac{dU_1}{dx_1}$$

– As for the transverse direction ($\underline{e}_2$ for example), the macroscopic behavior of the soil/pile system undergoing a macroscopic horizontal motion $U_2(x_1)$ reads:

$$\frac{d\langle\tau\rangle}{dx_1} = -\frac{\langle\rho\rangle}{|S|}\omega^2 U_2 \qquad ; \qquad \langle\tau\rangle = G\frac{dU_2}{dx_1} - \frac{E_p I_{P2}}{|S|}\frac{d^3U_2}{dx_1^3}$$

where $<\rho> = (\rho_p\ |S_p| + \rho_m\ |S_m|)/|S|$ is the mean density of the reinforced. The shear coefficient G is equal to the shear modulus of soil μ_m corrected by a form parameter κ that accounts for the presence of the pile. Parameter κ can either be calculated by numerical FE simulations, or be approximated a by self-consistent estimate, Hashin et al. (1964), with an excellent accuracy for weak reinforcement concentration (rather usual in engineering practice) :

$$G = \mu_m(1+\kappa) \quad ; \qquad \kappa = \frac{2c}{\dfrac{2}{\mu_p/\mu_m - 1} + (1+c)} \qquad \text{when } S_p/S < 0.1$$

Note that the behavior in transverse direction contains, (i) the classic shear term related to the distortion $U_{2,x1}$ and, (ii) the unusual contribution due to bending involving the gradient of the curvature $U_{2,x1x1x1}$. The bending inertia parameter is exactly that of the reinforcement (divided by the period section).

This result shows that the reinforced media behaves as an inner bending media (Boutin et Soubestre, 2011), that differs from the classic description of composites (Léné, 1978 – Sanchez-Palencia, 1980 – Postel, 1985). It gives a confirmation of the phenomenological « bi-phasic » approach developped by (de Buhan et al., 2008 – Sudret et al., 1999) and matches the mathematical aproaches of (Bellieud et al., 2002 - Pideri et al. 1997).

2.3. Energy and boundary conditions

The higher order of differentiation in the constitutive law requires enriched boundary conditions. These latter can be identify through the energetic formulation of the medium at the macrsoscopic scale. For an infinite layer of reinforced soil of height H along $\underline{e}_1$, taking the product of the equilibrium equation by the motion U_2 and integrating over the height, one obtains after twice integration by parts (and dropping index $_2$):

$$\int_0^H \left(G\left(\frac{dU}{dx_1}\right)^2 + \frac{E_p I_p}{|S|}\left(\frac{d^2U}{dx_1^2}\right)^2 \right) dx_1 - \int_0^H \langle\rho\rangle\omega^2 U^2 dx_1 = \left[\langle\tau\rangle U\right]_0^H - \left[\frac{^pM}{|S|}\frac{dU}{dx_1}\right]_0^H$$

where $^pM = -\,E_p I_p U_{,x1x1}$ is the momentum developed in beams. The energy accounts for kinetic energy and elastic energy related to both shear and bending deformations. It balances the work produced at the boundary ($x_1 = 0$ and $x_1 = H$) by (i) the mean stress vector $\langle\sigma_{12}\rangle$ submitted to the motion U_2 on the one hand, and (ii) by the momentum $^pM_2$ submitted to the pile section rotation $U_{2,x1}$ on the other. Hence, two boundary conditions must be specified at each extremity: one in terms of displacement or stress as for continuous media, and one in terms of rotation or momentum as for beams. By construction of the macroscopic modelling, the interpretation of these latter conditions is directly linked to the actual conditions imposed on the reinforcement.

3. EXPERIMENTAL VALIDATION

An experimental campaign was carried under the auspices of the European Commission's SERIES project to validate the theoretical model. Tests were conducted using the shaking table at the University of Bristol Earthquake and Large Structures (EQUALS) Laboratory. The objective was to identify and quantify the actual bending effect due to the reinforcement under transverse motions by analysing the spectral response of a reinforced matrix subjected to transverse excitation.

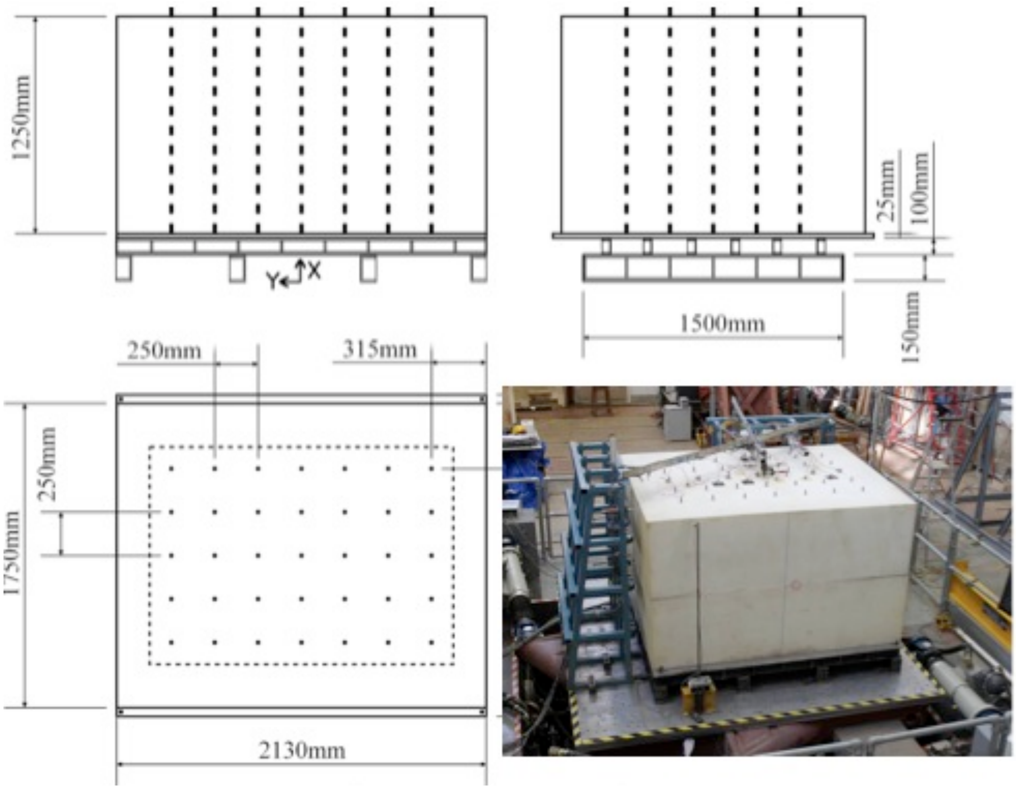

Figure 2. Scheme of the tested system and overview of the experimental set up.

3.1. Physical model

The physical model (Figure 2) is constructed from analogue materials that match the basic assumptions of the theoretical modelling, namely: linear-elastic matrix and reinforcement with large stiffness contrast and perfect adherence at their interface.

The matrix was a polyurethane foam block, 2.13 by 1.75 by 1.25m tall, and of density ρ_m = 48kg/m^3. From preliminary characterisation, tests the foam presents a linear elastic behaviour up to strain of 4% with a Young's modulus E_m = 54kPa and a Poisson's ratio ν_m = 0.11 (hence shear modulus μ_m = 24.3kPa). The experiments on the reinforced foam block were conducted with a global distortion level of about 0.1% to ensure that the foam remained within its linear elastic range.

The reinforcement was round, seamless, mild steel tube with 12.7mm outside diameter and 3.25mm wall thickness. The mechanical properties are Young's modulus E_p = 210GPa, Poisson's ratioν_p = 0.3 and density ρ_p = 7800kg/m^3. To reflect the pile-group attributes commonly seen in practice, 35 reinforcements (1.3m of lengths) were used on a seven by five grid at 250mm centers (Figure 2). An array of seven by five holes at 250mm centers was bored through the 1.25m deep block of foam. To ensure that the inclusions maintained good contact with the foam during testing, the bore diameter was 1mm less than the diameter of the inclusions.

The reinforcements were bolted to a base plate secured to the shaking table and the block of foam was adhered to the base plate. Hence, the model is clamped on bottom and free on top.

Acceleration and strain were measured. Accelerometers were mounted on the shaking table (to record the excitation), on the uppermost surface of the foam, and on the 50mm free length of reinforcement protruding from the top of the foam. The longitudinal strains generated in six of the 35 lengths of reinforcement were monitored using strain gauges. All strain gauge cabling was fed into the interior of the reinforcement through small holes (≈1 mm diameter) drilled through the wall. The instrumented lengths of reinforcement were each fitted with six strain gauges. The strain gauges were deployed in three pairs : one at the reinforcement bottom (38.5mm from the base), one at the middle (625mm from the base) and one at the top (1211.5 mm from the base). Each gauges pair faced in opposite directions, aligned with the axis of the reinforcement to allow the measurement of bending strains.

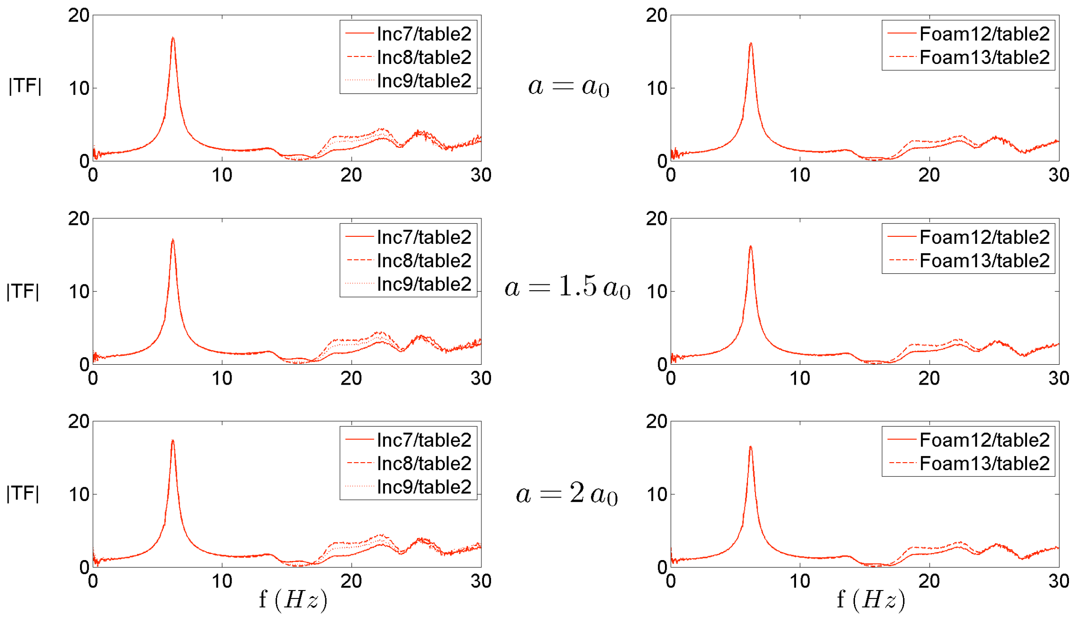

Figure 3. Transfer function modulus between accelerometers located on inclusions/table (left) and foam/table (right) for white noise excitations of different mean amplitudes: a0 (top), 1.5 a0 (middle) and 2 a0 (bottom).

Different magnitudes of random (white noise) excitation with frequency content between 1Hz and 30Hz were used to drive the shaking table. Harmonic sinusoidal waveforms were also used to excite the model at its eigen frequency for accurate mode shape determination.

3.2. Theoretical model applied to a reinforced layer

Assuming the experimental sample sufficiently large to neglect the boarder effects, the experiment can be interpreted by studying the transverse mode of an infinite lateral extension of reinforced matrix. The governing equation provided by the theoretical model reads :

$$G|S|\frac{d^2U}{dx^2} - E_p I_p \frac{d^4U}{dx^4} + <\rho> S\omega^2 U = 0$$

The general form of the solution is :

$$U(x) = a\cosh\left(\delta_2 \frac{x}{H}\right) + b\sinh\left(\delta_2 \frac{x}{H}\right) + c\cos\left(\delta_1 \frac{x}{H}\right) + d\sin\left(\delta_1 \frac{x}{H}\right)$$

where δ_1 and δ_2 are the roots of the characteristic equation :

$$G|S|\delta^2 - E_p I_p \delta^4 + <\rho> S\omega^2 = 0$$

The clamped free boundary conditions imposes that:

$$\begin{cases} U(0)=0 \\ U'(0)=0 \\ {}^{p}M_2(H)=0 \\ \langle\tau\rangle(H)=0 \end{cases} \Rightarrow \begin{cases} U(0)=0 \\ U'(0)=0 \\ U''(0)=0 \\ GU'(H)-E_pI_p/|S|U'''(H)=0 \end{cases}$$

Introducing these conditions in the general form of the solution provides the dispersion equation :

$$\frac{C\cos(\delta_1)}{\delta_1^2\delta_2^2}+\frac{\tanh(\delta_2)\sin(\delta_1)}{\delta_1\delta_2}+\frac{2}{C}\left(\cos(\delta_1)+\frac{1}{\cosh(\delta_2)}\right)=0 \quad ; \quad C=\frac{G|S|H^2}{E_pI_p}$$

The numerical resolution gives the values of δ_1 and δ_2, then the eigenfrequency (and eigenmodes)

$$f_i=\frac{\delta_{1i}\delta_{2i}}{2\pi H^2}\sqrt{\frac{E_pI_pS}{\langle\rho\rangle}}$$

3.3. Theory versus experiments

Comparisons between experiment and theory are based on the first mode response of the system.

Figure 3 displays the acceleration response of the system to three 0-30 Hz horizontal white noise excitation tests of different mean amplitudes: a_0, $1.5a_0$ and $2a_0$. The linear response of the system is confirmed by the independence of the transfer function modulus with the excitation level. Moreover the coincidence of TFs derived using accelerometers placed on foam and steel bars means that the whole system has an in-plane homogeneous kinematic for its first mode, as predicted by the theory. The measured eigen frequency $f_{1,exp}$ = 5.95Hz is close to the prediction of the shear/bending theoretical homogenised model ($f_{1,th}$ = 5.88Hz with the mechanical parameters of the reinforcement and foam) and clearly different from the pure shear (unreinforced foam: $f_{1,th}$ = 4.51Hz) or pure bending (negligible foam effect: $f_{1,th}$ = 6.56Hz) response.

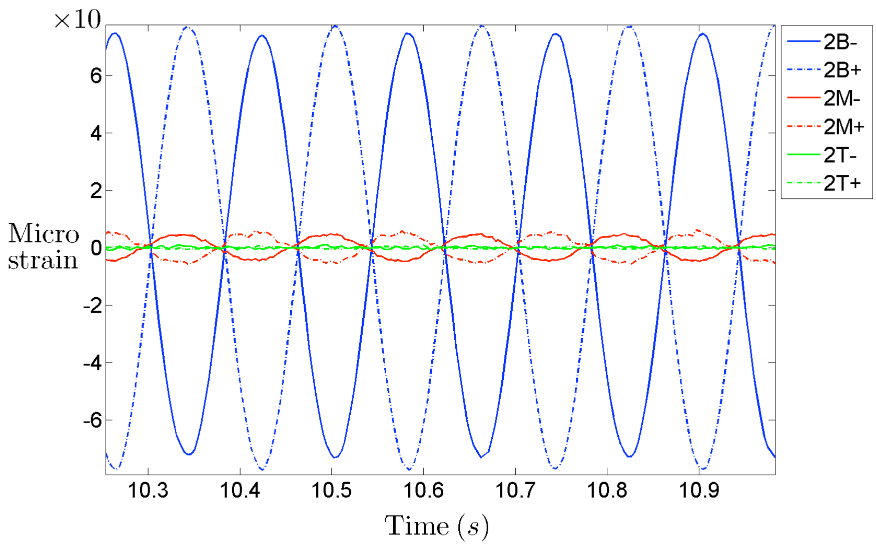

Figure 4. Strain response of a reinforcement bar under harmonic loading at the 1st mode frequency.

The system response to harmonic forcing at its first mode has also been recorded. The gauges indicate that there is no strain at the reinforcement top (T). At the bottom (B) and at the middle (M) we observe equal amplitude but opposite sign for opposite strain gauges, see Figure 4. Extension on one side of the inclusion and compression (of the same amplitude) on the other is clear evidence of bending. Moreover, the gauges 2B+ and 2M+ (or 2B- and 2M-) are in phase opposition.

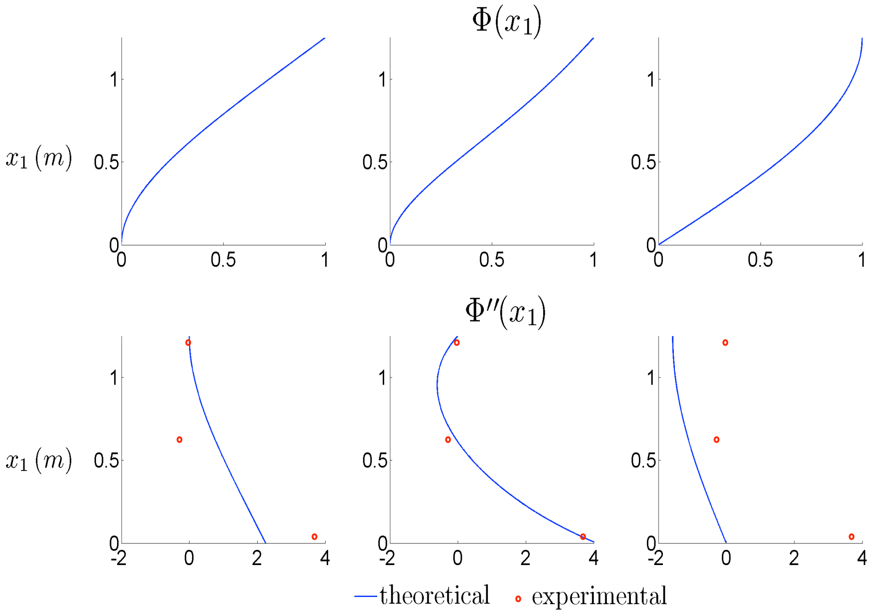

Figure 5. Mode shape (top) and curvature of the mode shape (bottom) of layer of media governed by bending (left), by shear (right) and by shear/bending coupling (centre, with the steeel and foam parameters).

These experimental observations are compared with the theoretical modeling in the following way. On the one hand, the axial strain recorded by the gauges $|^p e_n(x_1)|$ are related to the momentum in the reinforcement by the equality $^pM(x_1) = |^p e_n(x_1)|\; E_pI_p$. On the other hand, the momentum in the reinforcement is linked to the curvature of the mode shape $\Phi(x_1)$ by $^pM(x_1) = -\Phi''(x_1)\; E_pI_p$. Thus, the experimental strain data should fit the calculated curvature of the first mode shape (derived with the theoritical model, with the foam and steel bars parameters, under clamped / free boundary condition).

Such a comparison is presented on Figure 5. On the top are presented the first mode shape of (i) pure bending model (no matrix, on the left), (ii) pure shear model (no reinforcement, on the right) and (iii) of the theoretical shear/bending model (foam and reinforcement, on the center). On the bottom the curvature are given for the three models and compared to the strain data. It clearly appears that the shear/bending model (center) well captures the observed strain distribution, while both others (left and right) fails qualitatively and quantitatively to reproduce the experimental data (in particular the sign inversion of momentum).

These experimental observations provide a first validation of the theoretical modeling of inner momentum media. Furthermore, complementary experiments performed either with other type of boundary conditions, or with weaker number of reinforcement, are all in agreement with the theory.

4. CONCLUSIONS

An experimental programme has been conducted to validate the analytical modelling of pile-reinforced soils developed through homogenisation theory. The validated model provides new insights in the dynamic behaviour of pile-reinforced soils in the elastic range. In particular it evidences the coexistence of bending and shear, yielding to an atypical transverse behaviour.

The formulation of the inner bending (or shear bending) model is straight-forward and contains a few parameters easily related to the soil and reinforcement characteristics. Hence, the model can be used to perform parametric studies of pile-reinforced soil systems that cannot be achieved using the finite element method due to the problem being numerically ill-conditioned.

In earthquake engineering these results can directly applications either in the design of piles, or in structural health monitoring (through ambient noise measurement) of existing deep foundations.

It is worth mentioning that the shear/bending behaviour differs from the usual formulation based on the Wrinkler approach, as proposed by European norms. The key difference lies in the description of the soil effect on the pile. With the Wrinkler springs, the soil is assumed compressed by the piles (more precisely, the horizontal motion of the beam is taken as the driving variable for the soil response), conversely to the present analysis where the soil is sheared together with the piles. The experiments tend to prove that the design rules suggested by the norm do not account for the actual mechanism governing the soil/pile system (at least in the elastic domain).

Future work will expand the scope of the analytical model by considering soil/pile interface laws and poro-elastic soil behaviour of the soil.

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