Stochastic Simulation of Ground Motion Using Analytical Phase Spectrum

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SUMMARY:
In conventional stochastic simulation of ground motion, the acceleration time history at a site is obtained from the inverse Fast Fourier Transformation of a complex spectrum which combines a determined amplitude spectrum and a total random phase spectrum. In this article, an analytical phase spectrum considering the property of propagation medium and the source-site geometry was used instead of random phase spectrum to form the complex spectrum. To get an analytical phase spectrum, the analytical solution of displacement response to a shear dislocation source in infinite homogeneous space was calculated. The obtained analytical displacement time history was then differentiated into acceleration. The resulted acceleration time history was transformed by FFT technique into frequency domain where the phase spectrum of the acceleration time history was extracted. The extracted phase spectrum was then combined with an amplitude spectrum founded based on ω^2 source spectrum to form a complex spectrum. The stochastic approach based on analytical phase spectrum was tested by simulating ground motion from a fault scenario with varying fault mechanisms and locations of rupture starting point. The test results show that the response spectrum of the acceleration time history founded using analytical phase spectrum well matches the mean response spectrum of 100 realizations founded using different random phase spectra. The results also show that the disturbance of using random phase spectrum on the simulated ground motion is reduced by using analytical phase spectrum. Ground motions at certain stations recorded in the Northridge earthquake were also simulated by the new and conventional stochastic simulation approach. The comparison between the simulated and observed ground motions indicates that the new approach improves the matching between the simulated and observed waveforms.

Keywords: Stochastic Simulation, random phase spectrum, mean response spectrum, spatial pattern

1. INTRODUCTION

The stochastic synthesis technique is widely used in the high-frequency ground motion simulation (Boore, 1983, 2003). In this technique, the ground motion time history at a site is obtained from the inverse FFT transform of a complex spectrum. The amplitude of the complex spectrum is a convolution of a series of functions accounting for the effects of the source, propagation path and local site condition. The phase of the complex spectrum is usually interpreted as a series of random numbers uniformly distributed in the range (0, 2π). One reason of this interpretation is that ground motion in high frequency range shows a strong randomness due to the significant effect of local site condition. Another reason is that, the seismograms used in the statistical research of phase spectrum usually come from different earthquakes and thus include different propagation path effects and local site condition effects, this directly affect the regularity of the statistical phase spectrum.

Using random phase, however, will affect the simulated ground motion, especially the waveform of the simulated motion. Fig.1.1 shows the response spectra of 30 realizations at a site based on different random phase spectra. An unneglectable scatter of the spectra from their mean can be observed. At certain period, the largest response amplitude is almost two times of the smallest. In addition, using random phase spectrum will lead to a disturbance on the spatial pattern of the simulated ground motion field. For illustration, ground motions within a square area around a point source are computed by using the subroutine of point-source stochastic simulation in the stochastic finite-fault modelling program EXSIM (Motazedian and Atkinson, 2005). The distance interval of the sites in both the two directions parallel to the boundaries of the square is 200m. We first use the same phase spectrum for each site, and then let the program generate the random phase spectrum automatically. The resulted PGA contour is shown in Fig.1.2 and Fig.1.3. Fig.1.2 shows a clear attenuation trend and the clear demarcation lines between zones. Fig.1.3, by contrast, shows a severe interpenetration between different zones which makes it difficult to draw a demarcation line. This interpenetration would be more severe for a finite fault case than a single point source case. In engineering practice, the common method to weaken the disturbance of random phase spectrum is to take the average response spectrum of many random realizations. This, however, has two major flaws: i) the simulation will cost a huge time when applying to the ground motion field estimation for large earthquakes; ii) Taking average will result in the loss of the spatial coherency between ground motions at adjacent sites.

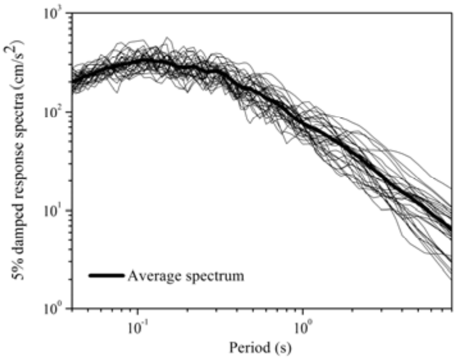

Figure 1.1 Acceleration response spectra and the mean spectrum from 30 realizations

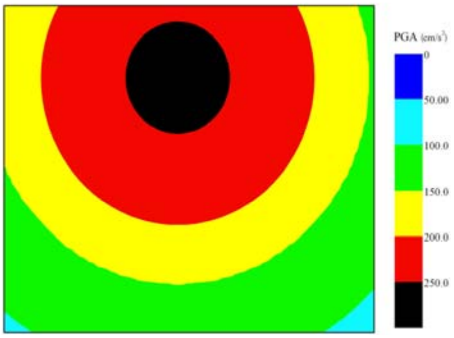

Figure 1.2 PGA distribution based on the same phase spectra

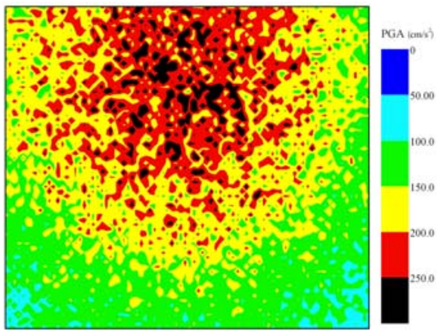

Figure 1.3 PGA distribution based on different random phase spectra

Most efforts to improve phase spectrum concern at the statistical analysis of the distribution parameter of phase difference angles (Nigam,1984; Kubo, 1987; Yokoyama *et al*, 1988; Zhu and Feng, 1993; Yang *et al*, 2001; Thráinsson, 2002), since the consensus is reached that the distribution of phase difference angles follows the lognormal distribution. Once the mean and deviation of the lognormal distribution are determined, the phase difference angles can be generated, and hence the phase angles will be easily obtained by given an initial phase angle. Some research address the statistical analysis on the lognormal distribution parameter of envelop delay time—the frequency difference of the phase difference spectrum (Lu, 2000; Sawada *et al*, 2000; Chai and Loh, 2002; Sato *et al*, 2002; Boore, 2003), while others suggest to use the phase spectra of real accelerograms to replace random phase spectra. These real accelerograms are strickly picked according to the magnitude, epicentral distance, local site condition or diveation of envelop delay time (Silva, 1990; Yamane and Nagahashi, 2008). In this paper, a new method for improving the phase spectrum of stochastic synthesis procedure is proposed. The main idea of the method is replacing random phase spectrum by the phase spectrum of the ground motion obtained from analytical computation.

2. METHODOLOGY

Analytical solution of the elastic displacement response to the shear dislocation in infinite homogeneous space is computed for the phase spectrum extraction. According to elastic dynamic theory, the elastic displacement in infinite homogeneous space caused by a displacement discontinuous can be calculated by the following time-spatial convolution (Aki and Richards, 2002):

$$u_n(x,t)=\iint_{\Sigma}\left[u_i(\xi,t)\right]c_{ijpq}\upsilon_k * G_{np,q}(x,t;\xi,\tau)\,d\sum(\xi) \tag{2.1}$$

Where, $G_{np,q}(x,\ t;\ \xi,\tau)$ denotes the displacement in n direction at location x and moment t, caused by the (p,q) couple at moment τ, location ξ on the rupture plane; $u_i(\xi,\ t)$ is the source time function; υ_k represents the direction cosine of the normal vector of rupture plane; c_{ijpq} is the elastic modulus of the medium; $d\Sigma(\xi)$ is the elemental area of rupture plane. By introducing the moment density tensor, $m_{p,q}=[u_i(\xi,\ t)]\ c_{ijpq}\upsilon_k$, Eqn. 2.1 can be rewritten into:

$$u_n(x,t)=\iint_{\Sigma} m_{p,q} * G_{np,q}(x,t;\xi,\tau)\,d\sum(\xi) \tag{2.2}$$

According to Eqn.2.2, the displacement caused by the entire rupture plane is actually the sum of the displacements from all point shear dislocations with area of $d\Sigma(\xi)$. The displacement from a single point shear dislocation can be simply represented by the product of the equivalent moment tensor of the 9 (p,q) couples, $M_{p,q}$, and the third-order Green's function, $G_{np,q}$ (Aki and Richards, 2002):

$$\begin{aligned}
du_n(x,t) &= M_{pq} * G_{np,q} \\
&= \left(\frac{30\gamma_n\gamma_p\gamma_q\upsilon_q-6\upsilon_n\gamma_p-6\delta_{np}\gamma_q\upsilon_q}{4\pi\rho r^4}\right)\mu d\sum(\xi)\int_{r/\alpha}^{r/\beta} t'\Delta u_p(t-t')\,dt' \\
&\quad+\left(\frac{12\gamma_n\gamma_p\gamma_q\upsilon_q-2\upsilon_n\gamma_p-2\delta_{np}\gamma_q\upsilon_q}{4\pi\alpha^2 r^2}\right)\mu d\sum(\xi)\Delta u_p\left(t-\frac{r}{\alpha}\right) \\
&\quad-\left(\frac{12\gamma_n\gamma_p\gamma_q\upsilon_q-3\upsilon_n\gamma_p-3\delta_{np}\gamma_q\upsilon_q}{4\pi\beta^2 r^2}\right)\mu d\sum(\xi)\Delta u_p\left(t-\frac{r}{\beta}\right) \\
&\quad+\left(\frac{2\gamma_n\gamma_p\gamma_q\upsilon_q}{4\pi\alpha^3 r}\right)\mu d\sum(\xi)\Delta\dot{u}_p\left(t-\frac{r}{\alpha}\right) \\
&\quad-\left(\frac{12\gamma_n\gamma_p\gamma_q\upsilon_q-\upsilon_n\gamma_p-\delta_{np}\gamma_q\upsilon_q}{4\pi\beta^3 r}\right)\mu d\sum(\xi)\Delta\dot{u}_p\left(t-\frac{r}{\beta}\right)
\end{aligned} \tag{2.3}$$

Where, γ_i denotes the cosine of the i-direction projection of the vector from the source to the observation site; r is the source-site distance; α and β are the wave velocity for P and S wave, respectively; υ_i is the projection of the normal vector $\boldsymbol{\upsilon}$ in i direction (i=x, y, z); $\triangle u_p$ is the slip time function in p direction on element $d\Sigma(\xi)$; δ_{np} is the Kronecker δ function.

To apply Eqn. 2.3, a source slip function is needed. Many published source slip functions are available, such as the Haskell function, Bell function, exponential function and triangle function. We chose the Bell slip function (Israel and Kovach, 1977), where τ in the function denotes rise time.:

$$s(t)=\begin{cases} 0 & t<0 \\ \dfrac{t}{\tau}-\dfrac{1}{2\tau}\sin\left(\dfrac{2\pi}{\tau}t\right) & 0\le t\le\tau \\ 1 & t>\tau \end{cases} \tag{2.4}$$

Using the above equations, the analytical displacement response is easy to get. Then, the acceleration motion can be obtained following the procedure: i) differentiate the analytical displacement motion into acceleration; ii) extract the phase spectrum of the resulted acceleration motion; iii) combine the extracted phase spectrum with the specific amplitude spectrum to form the complex spectrum, and then use the inverse FFT technique to get the acceleration time history. The ground motion obtained following this procedure is named analytical phase based ground motion below.

3. CASE STUDY

To test the effects of the new method, a case shown in Fig. 3.1 is used. The fault is assumed to be a normal fault (the rake λ=270°), and the orientation of the fault is φ=122° in strike, δ=40° in dip. The depth of the up edge of the fault is 6km.The dimension (15km by 6km) and the slip distribution on the rupture plane are derived by using a hybrid slip model (Sun, 2010). In the simulation, the rupture plane is discredized into 10 subfaults with the subfault size of 3km by 3km. The magnitude is assumed to be M_W 6.0, and the red star on the fault plane in Fig. 3.1 marks the rupture starting point. The property of the medium is taken from the Crust2.0 global velocity structure model (Bassin *et al*, 2000), as shown in Tab. 3.1.

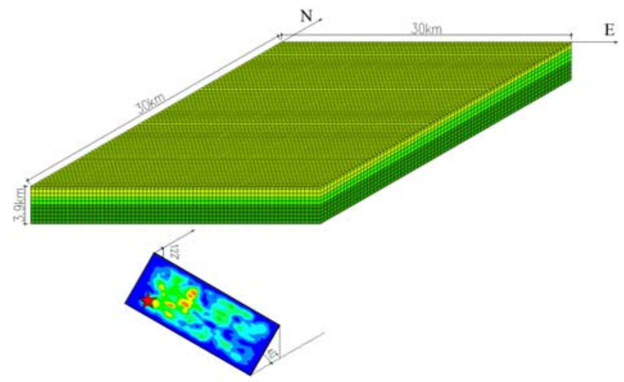

Figure 3.1 Computing model

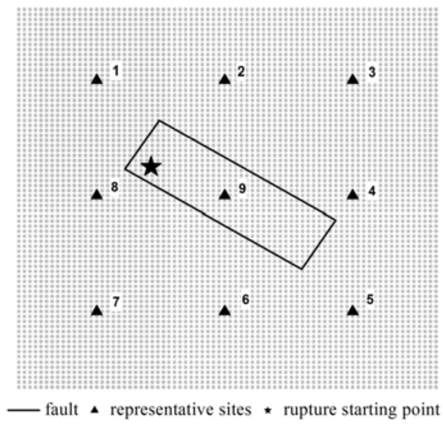

Figure 3.2 Locations of representative points

Table 3.1 Crustal velocity structure model

H（km）	α（km/s）	β（km/s）	ρ(g/cm^3)
1.0	2.5	1.2	2.1
2.0	4.4	2.5	2.5
8.0	6.1	3.5	2.75
10.0	6.3	3.6	2.8
10.0	6.6	3.6	2.9

The stochastic finite-fault modeling program EXSIM is used to simulate the surface ground motion. Nine sites located around the fault with different azimuths are picked out for comparison, as shown in Fig. 3.2. For each site, 100 realizations based on different random phase spectra are synthesized, and the average response spectrum of the 100 realizations is computed to compare with the response spectrum of the analytical phase based ground motion. The comparison is shown in Fig. 3.3, in which the solid curve represents the average response spectrum of the 100 realizations, while the dashed curve represents the response spectrum of the analytical phase based ground motion. Seen from Fig. 3.3, two types of response spectra match well.

We then test another three cases based on the model in Fig. 3.1: Case II) change the rupture starting point to be at the right bottom part of the rupture plane; Case III) change the rupture starting point to be at the right bottom part of the rupture plane, and meanwhile change the rupture mechanism to be rightward strike slip, i.e., the rake λ=180°; Case IV) change the rupture starting point to be at the middle of the rupture plane, i.e. the No.(3, 2) subfault, and meanwhile change the rupture mechanism to be inverse fault, i.e., the rake λ=90°. The response spectra for each case are shown in Fig. 3.4, 3.5 and 3.6, respectively. It can be found that, for all cases, the average spectrum of the 100 realizations match the analytical phase based spectrum well even though the location of the rupture starting point and the fault mechanism are different. The results may indicate that the ground motion based on analytical phase spectrum could represent the average level of large random realizations. This conclusion is important for simulating ground motions from future earthquakes. For future earthquake, it's still very difficult to tell the magnitude, fault mechanism, fault dimension and location of rupture starting point before the earthquake really happens. It's only reasonable to consider multiple fault conditions when estimating ground motions from future earthquakes. Certainly, taking the average of large random realizations could represent the average level of multiple cases. However, this will result in the loss of the spatial coherency between ground motions at adjacent points. By contrast, using analytical phase spectrum, only one realization is enough to represent the average level of multiple cases. This will largely reduce the computation time. In addition, the analytical phase spectrum is closely related to the source-site geometric relationship and the properties of the local medium since it is derived from the analytical solution of the elastic displacement response. This decides that the analytical phase spectrum itself contains the spatial coherency of the phase spectra at adjacent points, so the ground motions based on analytical phase spectrum will definitely contain the spatial coherency of the phase spectra at adjacent points.

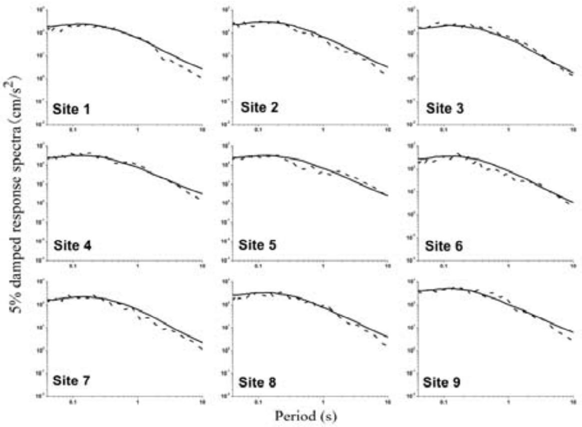

Figure 3.3 Comparison of response spectra for case I

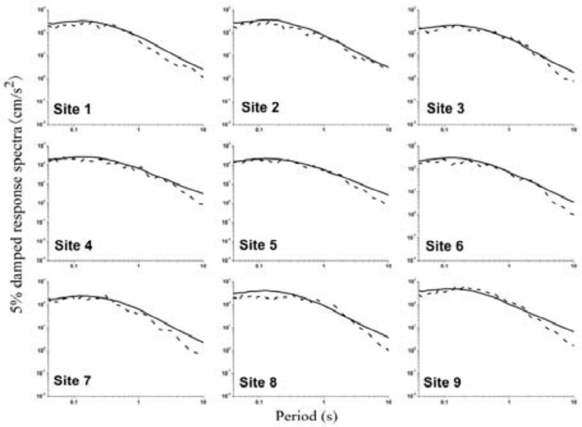

Figure 3.4 Comparison of response spectra for case II

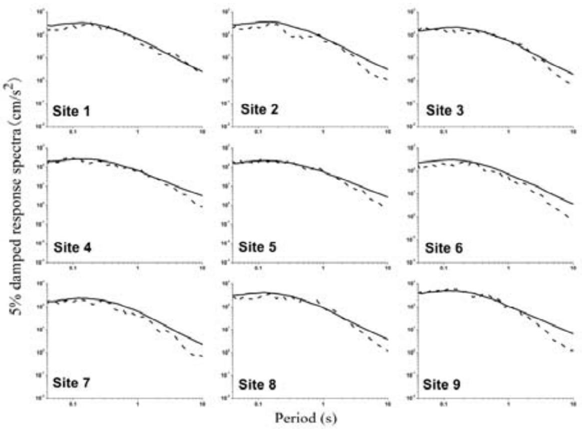

Figure 3.5 Comparison of response spectra for case III

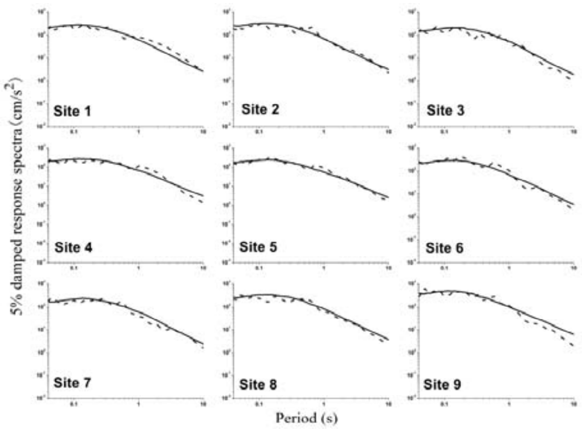

Figure 3.6 Comparison of response spectra for case IV

We also test the effect of the new method on reducing the disturbance of random number. For this test, 30 finite fault models with different dimensions and slip distribution patterns are generated by using the hybrid slip model. Ground motions at the 9 representative sites from each finite fault model are simulated by using the traditional stochastic approach and the analytical phase based stochastic approach separately. As illustration, the response spectra at site 8 from 30 finite fault models and the average response spectrum are shown in Fig. 3.7. In the figure, the left panel shows the response spectra obtained by using traditional stochastic approach while the right panel shows the results from the analytical phase based stochastic approach. Seen from Fig. 3.7, the scatter of the response spectra to the mean in the right panel is obviously less than that in the left panel. This indicates that the application of analytical phase spectrum effectively removes the large disturbance of random number.

Some may criticize on the process of computing the surface displacement motion by using an infinite homogeneous space Green's function. What we need to clarify is, this paper is just the first try to test the feasibility of the new idea of combining analytical phase spectrum into stochastic synthesis technique, so we only used the simplest analytical Green's function in infinite homogeneous space. The analytical Green's function in half space will be considered in the next step of the research. We will not use the ground motion from numerical simulation even though numerical simulation approach, like 3D finite element or finite difference technique, are the most effective methods to account for the response in near-surface inhomogeneous medium. Due to the limit of the discretion size, the numerical simulation result usually describe the high frequency contents poorly. Some phase information in high frequency range may lose, which directly leads to the error in the simulated ground motions in high frequencies. In addition, the phase spectrum replacement is preformed during the ground motion simulation for each subfault. Thus, combining the phase spectrum of the numerical simulation results means that, for every subfault, the numerical simulation over the entire computation model is needed. This will be an unbearable work.

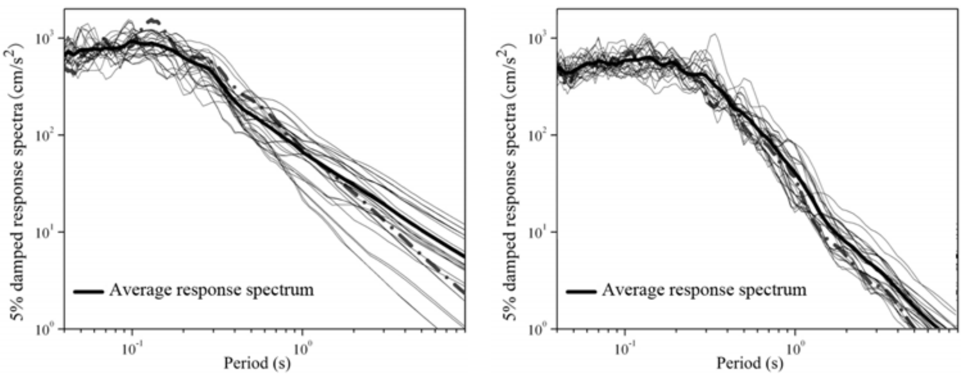

Figure 3.7 Comparison of the response spectra by the traditional stochastic approach (left panel) and the new approach (right panel)

4. NORTHRIDGE EARTHQUAKE APPLICATION

The analytical phase based stochastic approach is applied to simulate the ground motions at the near-field stations in Northridge earthquake, 1994. The finite fault model is adopted directly from the research of Wald *et al* (1996). The orientation of the fault is strike 122°, dip 40°, and the fault mechanism is inverse fault. The depth of the rupture starting point is 17.5km, and the moment magnitude is taken as 6.7. The simulation parameters used in the synthesis are listed in Tab. 4.1. The ground motion at 12 near-field stations are calculated by using the analytical phase based stochastic approach and compared to the records in terms of 5% damped response spectrum and time history, as shown in Fig. 4.2.

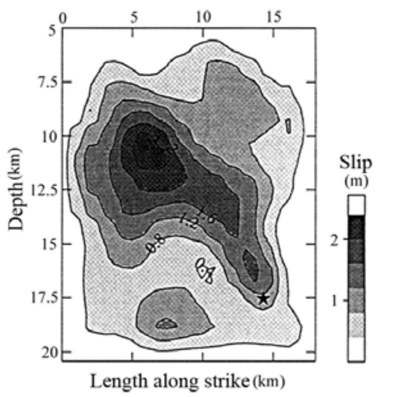

Figure 4.1. Source model of the Northridge earthquake from inversion

It can be found that, at most stations, the waveforms of the simulated time histories are much closed to the recorded waveforms and so do the response spectra. The best match is found at station LWE, whereas poor matches are found at station ORR and PKC. However, the poor precision at these two stations may results from the rough description of local site effects, topographic effect and basin effect in stochastic technique (Beresnev and Atkinson, 1998). In fact, the difference between the simulated ground motion and the records are not unexpected. Remind that, the result from the analytical phase based stochastic approach is comparable to the average level of large random realizations. In statistical point of view earthquake record is just one sample of the large realizations. No one can tell which sample of these realizations would become reality before earthquake really happens. From this point of view, estimating the average level of the large realizations seems more valuable.

Table 4.1 Simulation parameters

Q	Geometric model	k_0	Stress drop	Density	β	Amplification function
$150f^{0.5}$	$1/R \quad (R \le 70\text{km})$ $1/R^0 \quad (70\text{km} < R < 130\text{km})$ $1/R^{0.5} \quad (R \le 130\text{km})$	0.05	50 bars	2.8 g/cm^3	3.7 km/s	Boore & Joyner (1997)

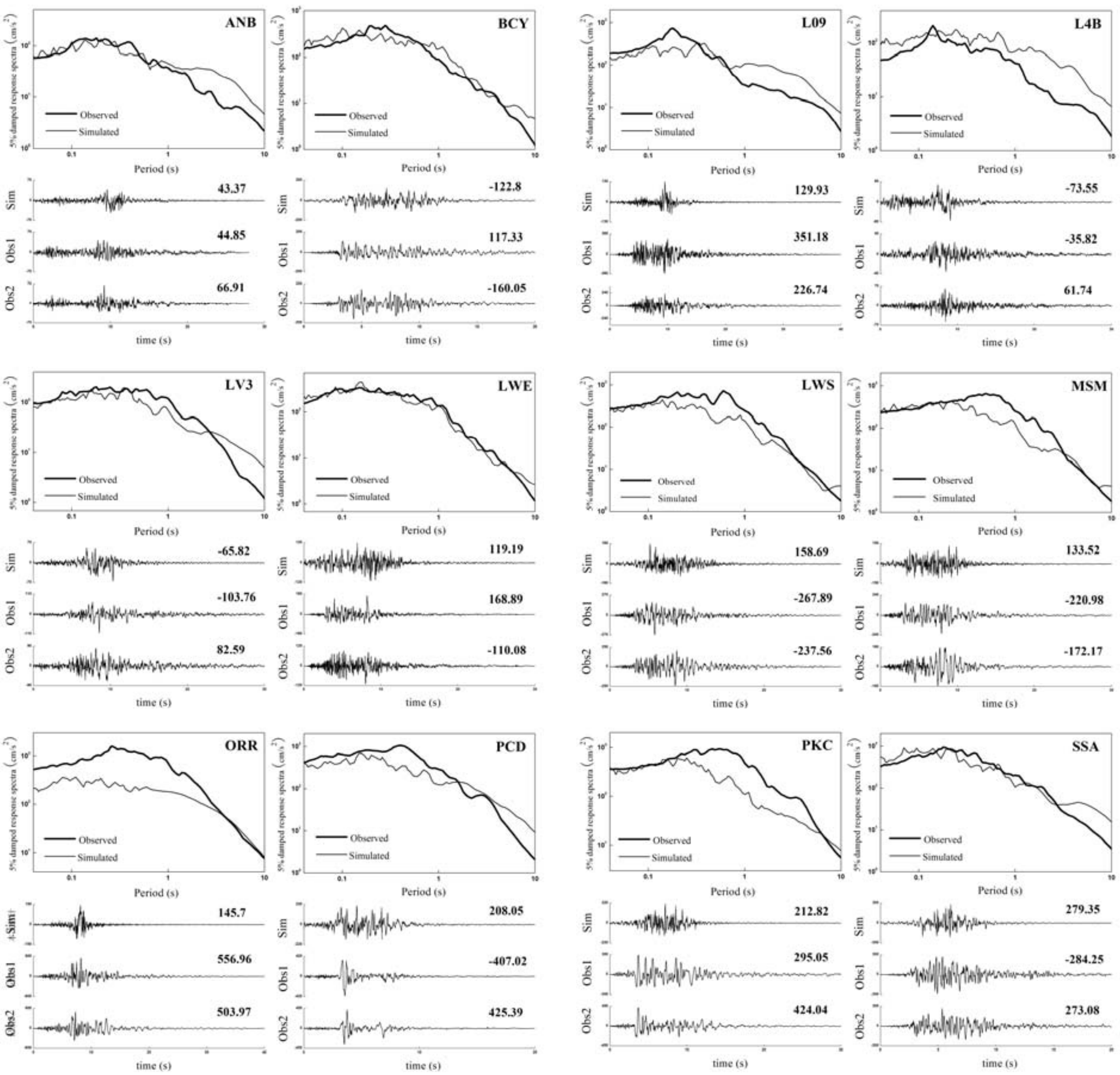

Figure 4.2. Comparison between the simulated and recorded motions at 12 stations

5. CONCLUSION AND DISCUSSION

An analytical phase spectrum considering the property of propagation medium and the source-site geometry was proposed to replace the random phase spectrum in stochastic synthesis technique. The analytical phase spectrum was extracted from the acceleration time history of the analytical Green's function from a shear dislocation source in infinite homogeneous space. The improved stochastic approach was tested for a fault scenario with varying fault mechanisms and locations of rupture starting point. The results show that the response spectrum of the acceleration time history founded using analytical phase spectrum is closed to the mean response spectrum of the 100 realizations founded using different random phase spectra. The results also show that the disturbance of random phase spectrum on the simulated ground motion could be reduced by using analytical phase spectrum. Ground motions at certain stations recorded in the Northridge earthquake are simulated by the improved and conventional stochastic simulation approach. The comparison between the simulated and observed ground motions indicates that the goodness of fit between the simulated and observed waveforms could be improved by using the analytical phase spectrum.

Certainly, adopting the analytical phase spectrum will cost more working time than the traditional stochastic approach, but the increase is comparable to the cost of computing multiple random realizations and taking their average. Considering the contribution of the improved approach on reducing the disturbance of random number, the increase of the working time is acceptable. Note that, we didn't expect the improved stochastic approach to work on both high- and low-frequency range. Our improvement only aim to reduce the disturbance of random number on the spatial pattern of the simulated ground motion in high frequency range. Thus, even with the analytical phase spectrum, the stochastic approach is still suggested to be applied only in high frequency range. The ground motion in low frequency is better simulated by the traditional methodologies based on 3D shear wave velocity structure.

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