Modeling of Phase Spectrum and Simulation of Nonstationary Earthquake Ground Motions

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SUMMARY:
A univariate model of phase spectrum, built up on a time argument associated with the concept of starting-time of phase evolution of frequency components, is proposed in the present paper whereby a family of simulation methods for nonstationary earthquake ground motions is developed. This phase model allows a feasible phase spectrum just using few variables of the starting-time in numerical implementation. In order to reduce the computational effort of the starting-time, a wave-group propagation formulation is also introduced. An observed ground motion at the type-II site, i.e. Northridge waves, is investigated for illustrative purposes. Numerical results prove the validity and applicability of the simulation scheme. This methodology provides a new perspective towards the representation of nonstationary stochastic ground motions.

Keywords: Phase spectrum; starting-time; wave-group propagation; earthquake ground motions; nonstationary

1. INTRODUCTION

The dimension reduction, both with the phenomenal models (site-based models) and physical models (source-based models), has been a challenging issue for the accurately modeling of nonstationary earthquake ground motions (Li and An, 2008). It results in the complication of numerical implementation regarding to highly-dimensional probabilistic space constructed by the parametric variables of stochastic ground motions. In a filtered Gaussian white-noise model, for example, there is a few of parameters need to be identified (Rezaeian and Der Kiureghian, 2008). While in the spectral representation model, it becomes difficult to rationally identify the phase spectrum and the associated physical quantities using only few variables. Traditionally, the phase spectrum or phase-difference spectrum is assumed to admit a probabilistic distribution, e.g. Uniform distribution(Shinozuka and Jan, 1972), Normal distribution (Ohsaki, 1979), Lognormal distribution (Zhu and Feng, 1992)and Beta distribution (Thrainsson and Kiremidjian, 2002). The phase spectrum is then built up using the method of randomly sampling. Recently, a polynomial fitting technique was employed to construct the accumulated phase spectrum whereby the phase spectrum was obtained (Shrikhande and Gupta, 2010). A numerical challenge, however, gives rise in the spectral representation model that tens even hundreds of variables are required towards achieving the desirable simulation results, which withdraws practical applications of the model.

Having this knowledge, we are in attempt to develop a family of simulation methods for ground motions employing a univariate phase model built up on a time argument associated with the concept of starting-time of phase evolution of frequency components. This treatment allows a feasible phase spectrum for the practical application just using few variables in numerical implementation. It bypasses the numerical challenge inherent in the classical spectral representation techniques. The sections arranged in this paper are distributed as follows. Section 2 is dedicated to illustrating the concept of starting-time of phase evolution. In order to reduce the computational effort of the starting-time for a complicated nonstationary ground motion process, Section 3 introduces the group propagation of earthquake waves of which the frequency components with significant contribution to ground motions are included. An observed ground motion is investigated in Section 4 for illustrative purposes. The concluding remarks are included in the final section.

2. STARTING-TIME OF PHASE EVOLUTION

2.1 Velocity of phase evolution

We consider a nonstationary earthquake ground motion $a(t)$ in a finite time domain [0 *T*], of which the Fourier transform is given by

$$A(\omega) = \int_0^T a(t)e^{-\mathrm{i}\omega t}dt = |A(\omega)|e^{-\mathrm{i}\varphi(\omega)} \tag{1}$$

where ω denotes the circular frequency; $A(\omega)$ denotes the Fourier spectrum in unilateral form, and $|\cdot|$ operates the magnitude of Fourier spectrum; $\varphi(\omega)$ denotes the principal value of Fourier phase spectrum, $\varphi(\omega) \in [0, 2\pi)$; i is the unit of imaginary number $\sqrt{-1}$.

In physics, the earthquake-induced vibration of a spatial point could be viewed as the integration of a series of harmonic waves with different scales in frequency components. The energy carried by the seismic ground acceleration can be expressed as its time-average variance $\bar{\sigma}^2$:

$$\bar{\sigma}^2 = \frac{1}{2\pi T}\sum_{n=0}^{\infty}|A(\omega)|\Delta\omega = \sum_{n=0}^{\infty}a_s^2(\omega) \tag{2}$$

where $a_s(\omega)$ denotes the acceleration of harmonic waves. The representative acceleration and velocity of harmonic waves are thus given as follows (Hinze, 1975):

$$a_s(\omega) = \sqrt{\frac{1}{2\pi T}|A(\omega)|^2\Delta\omega} \tag{3}$$

$$v_s(\omega) = \int_0^T a_s(\omega)dt = \sqrt{\frac{T}{2\pi}|A(\omega)|^2\Delta\omega} \tag{4}$$

The phase change of a harmonic wave in time interval Δt is typically a function of its velocity, which could be represented by

$$\Delta\varphi(\omega) = \frac{2\pi v_s(\omega)\Delta t}{l_s(\omega)} = 2\pi v_s(\omega)k_s(\omega)\Delta t \tag{5}$$

where $l_s(\omega)$ denotes the wave length; $k_s(\omega)$ denotes the wave number which is reciprocal to the wave length, and has the relationship with frequency as follows:

$$k_s(\omega) = \frac{\omega}{2\pi v_{se}} \tag{6}$$

where v_{se} is the equivalent shear-wave velocity of earthquake ground motions in an extensive site. It relies upon the type of site soil. As regards type-II site, for example, $v_{se} = 1500$ m/sec (Dziewonski and Anderson, 1981).

Equation (5) indicates the velocity of phase evolution with different frequency components:

$$\dot{\varphi}(\omega) = 2\pi v_s(\omega) k_s(\omega) \tag{7}$$

It is seen that the velocity of phase evolution is of time-independence.

2.2 Identification of starting-time of phase evolution

As indicated in equation (7), the phase velocity is the function in frequency components which varies with the frequency due to different frequency components endowed different values of Fourier amplitude. One might image that there is always a zero time where all the phases of frequency components are zero if the ground motion is evolved backward from the time of wave observation at the station towards the epicenter. A schematic diagram picturing the generator and observers of ground motions, as shown in Figure 1, is presented for illustrative purposes. The epicenter with depth h_0, labeled 'o', is the generator of ground motions (a hypothesis of point source on earthquakes is used in the investigated univariate phase model towards expediently simulating the observed earthquake ground motions though they might not admit this hypothesis). The station A with distance d from the epicenter, labeled 'o_1', is one of the observers of ground motions, and $a(t)$ denotes the observed ground motion. The ground motion $a(t)$ is artificially evolved backward to the zero-phase point labeled 'o_2' where, as mentioned previously, all the phases of frequency components of the ground motion are zero. For notation convenience, we define the time of wave observed at Station A as the current time, and define the time of wave reaching at the zero-phase point as the zero time. The time interval between the current time and the zero time is defined as the starting-time, which is labelled T_s as indicated in Figure 1.

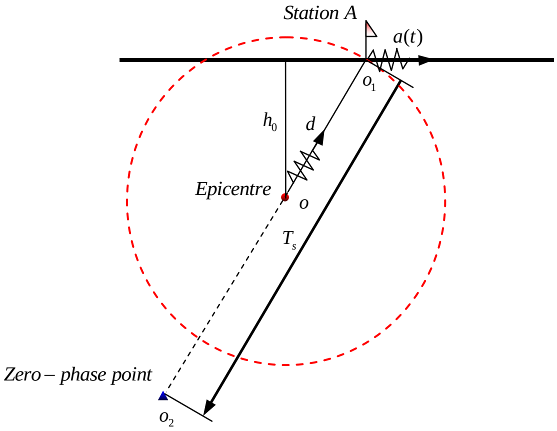


Figure 1. **S**chematic diagram for definition of starting-time of phase evolution.

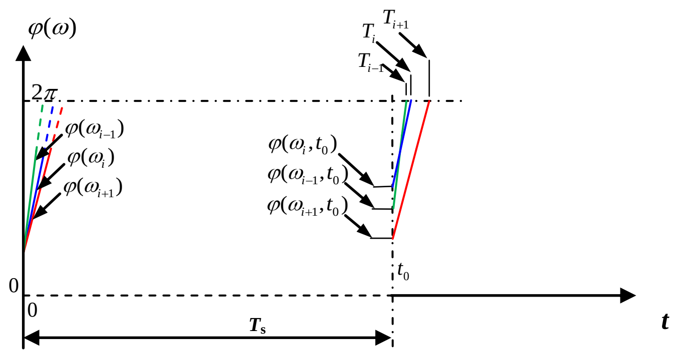


Figure 2. Schematic diagram for identification of starting-time of phase evolution.

The starting-time of phase evolution is actually an imaginary physical quantity with abstract meaning. It was indicated in the research of physical mechanism of earthquake ground motions that once the earthquake happens the phases of frequency components of ground motion generated from the earthquake are not all zero. The primary factors, however, that effect the phase of frequency at the current time are propagation distance and velocity of ground motions. It is understood that the propagation velocity of frequency components governs their phase evolutions, and the differences resulted from initial phases of frequency components can be ignored in case that the propagation distance of ground motions is long adequately. Without risk of confusion, we thus assume that the frequency components of earthquake waves exhibit the same initial phase with zero values at the instant of earthquake occurring, at least for the frequency components in a specific wave group that is to be mentioned in the next section. Having this knowledge, one comes to realize that if the initial zero-value phase hinges on wave groups, the epicenter 'o', as shown in Figure 1, and the zero-phase point 'o_2' are typically not identical in mathematical logic. Since the starting-time of phase evolution is an imaginary physical quantity, the zero-phase point denotes a spatial point with imaginary physical sense.

Figure 2 shows the phase evolutions of any three frequency components, labelled ω_{i-1}, ω_i, ω_{i+1}, respectively, from the zero-phase point at the zero time. In the figure, longitudinal coordinate indicates the principal value of phases. For illustrative conveniences, the velocities of phase evolution of these frequency components are assumed to have the following relative magnitudes: $\dot{\varphi}(\omega_{i+1}) < \dot{\varphi}(\omega_i) < \dot{\varphi}(\omega_{i-1})$; see the slopes at zero time in Figure 2. These frequency phases evolve forward from the initial zero phases at zero time, and would reach at 2π at in different time sequences due to their different velocities of phase evolution. It is seen from Figure 2 that the frequency component ω_{i-1} with relatively larger evolution velocity of the three frequencies will first arrive at 2π. The component, however, does not hold the largest primary phase in the following any instants since the frequency phase are of complementation with 2π when they reach at 2π for the consideration of primary values of phase. We might assume, at the current time labelled t_0, the primary phases of the these frequency components exhibiting the following relative magnitudes: $\varphi(\omega_{i+1}, t_0) < \varphi(\omega_{i-1}, t_0) < \varphi(\omega_i, t_0)$; see the slopes at current time in Figure 2. Continuing the evolution forward, one can see that the time cost of the three frequency components reaching to their nearest 2π, labelled T_{i-1}, T_i, T_{i+1}, respectively, has the following relative magnitudes: $T_{i+1} < T_i < T_{i-1}$. An equation set, including propagation parameters and starting-time of phase evolution of a collection of frequency components $\omega_i (i = 1, 2, \ldots, n)$, can be constructed of which the representative equation is shown as follows:

$$2\pi T_s v_s(\omega_i) k_s(\omega_i) + 2\pi T_i v_s(\omega_i) k_s(\omega_i) = N_i \tag{8}$$

where T_s denotes the starting-time of phase evolution; $N_i (i = 1, 2, \ldots, n)$ denotes the time crossing over 2π in the process of phase evolution of frequency components.

It is extreme difficult to derive the analytical solution of the starting-time. Numerical procedure is a practical choice that a simple scheme is proposed here: enumerate $N_1 (N_1 = 1, 2, \ldots, 1000000)$, and fix a N_1 according to the representative equation (8) by satisfying a loose condition, i.e. $\{N_i - [N_i]\} \le \varepsilon (i = 2, 3, \ldots, n)$, then N_i can be solved using the following equation:

$$N_i = \left[(T_1 - T_i) + \frac{N_1}{v_s(\omega_1) k_s(\omega_1)} \right] v_s(\omega_i) k_s(\omega_i) \tag{9}$$

With the solution of any $N_i (i = 1, 2, \ldots, n)$, the starting-time T_s can be deduced according to equation (8). Using N_1, for example, the expression of T_s is given by

$$T_s = \frac{N_1}{v_s(\omega_1) k_s(\omega_1)} - T_1 \tag{10}$$

3. GROUP PROPAGATION OF EARTHQUAKE WAVES

In the investigation of observed earthquake ground motions, we found that it is still difficult to fix a standard starting-time for all frequency components (there are hundreds of frequency components for a typical observed ground motion), even if the numerical procedure including loose condition is used.

A ground motion process, nevertheless, can be viewed as the composition of a series of narrowband waves with predominant frequency components. These narrowband waves derive from the superimposing of harmonic processes. Introducing the propagation theory of wave group, moreover, the amount of frequency components used in fixing the starting-time can be reduced significantly. There generally divide into three groups according to the Fourier amplitude spectrum of a typical ground motion, i.e. the group with low frequencies, the group with medium frequencies and the group with high frequencies which are labelled 'wavegroup A', 'wavegroup B', 'wavegroup C', respectively, as shown in Figure 3. In the figure, ω_i, ω_j, ω_k respectively represent a frequency component of the three groups. For each wave group, meanwhile, includes a series of narrowbands, one of which represented by ω_j is shadow area in Figure 3. $\Delta\omega_j$ denotes the width of the narrowband which is, in this paper, defined as times of interval of adjacent frequency components $\delta\omega$, i.e. $\Delta\omega_j = n\delta\omega$. Besides, the width of narrowbands in different wave group generally varies.

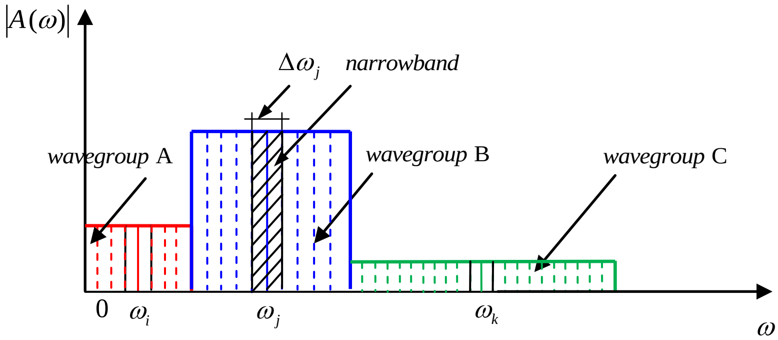


Figure 3. Schematic diagram of wave groups in accordance with Fourier amplitude spectrum.

It is also indicated in Figure 3 that for an observed earthquake ground motion, one could fix just one starting-time of phase evolution for all wave groups, or fix individual starting-time for different wave groups. The latter has more lively physical scenarios associated with earthquakes. The frequency components in one wave group thus have the same initial zero-value phase, while those in different wave groups have different initial zero-value phase, as indicated in the previous section. This hypothesis with loose conditions, in fact, is in accordance with the physical origin of earthquake ground motions.

As indicated previously, the integration of harmonic waves in narrowbands underlies the group propagation of earthquake waves. Taking the narrowband represented by ω_j in Figure 3 as an example, we further illustrate the principle of the superimposing which has the formulation as follows:

$$u = 2\,\mathrm{Re}\left\{ \int_{\omega_j - \frac{\Delta\omega_j}{2}}^{\omega_j + \frac{\Delta\omega_j}{2}} U(\omega_j) \exp[\mathrm{i}(\omega(t+t_c) - \varphi(\omega))] d\omega \right\} \tag{11}$$

where u denotes the narrowband wave; $\mathrm{Re}\{\cdot\}$ operates the real part of a complex; $U(\omega_j)$ denotes the Fourier amplitude value at frequency ω_j; t_c indicates a calibration time of group propagation, which will be addressed in the following sections. Fourier phase spectrum $\varphi(\omega)$ is expanded in its first-order Taylor series:

$$\varphi(\omega) = \varphi(\omega_j) + (\omega - \omega_j) \left.\frac{d\varphi}{d\omega}\right|_{\omega_j} \tag{12}$$

where $\varphi(\omega_j)$ denotes the Fourier phase value at frequency ω_j.

Substituting equation (12) into equation (11), we have

$$u = 2\,\mathrm{Re}\left\{ \int_{\omega_j - \frac{\Delta\omega_j}{2}}^{\omega_j + \frac{\Delta\omega_j}{2}} U(\omega_j) \exp\left[\mathrm{i}\left(\omega\left(t + t_c - \left.\frac{d\varphi}{d\omega}\right|_{\omega_j} \right) - \varphi(\omega_j) + \omega_j \left.\frac{d\varphi}{d\omega}\right|_{\omega_j} \right) \right] d\omega \right\} \tag{13}$$

Integral over the range of ω, we then have

$$u = 2\,\mathrm{Re}\left\{ \frac{-\mathrm{i}U(\omega_j)}{t + t_c - \left.\frac{d\varphi}{d\omega}\right|_{\omega_j}} \left[\exp\left[\mathrm{i}\left(\omega_j(t+t_c) - \varphi(\omega_j) + \frac{\Delta\omega_j}{2}\left(t + t_c - \left.\frac{d\varphi}{d\omega}\right|_{\omega_j} \right) \right) \right] - \exp\left[\mathrm{i}\left(\omega_j(t+t_c) - \varphi(\omega_j) - \frac{\Delta\omega_j}{2}\left(t + t_c - \left.\frac{d\varphi}{d\omega}\right|_{\omega_j} \right) \right) \right] \right] \right\} \tag{14}$$

Assigning *A* and *B* as the following expressions, respectively

$$A = \omega_j(t+t_c) - \varphi(\omega_j) \tag{15}$$

$$B = \frac{\Delta\omega_j}{2}\left(t + t_c - \left.\frac{d\varphi}{d\omega}\right|_{\omega_j} \right) \tag{16}$$

then the narrowband wave u is re-written into

$$u = \mathrm{Re}\left\{ \frac{-\mathrm{i}U(\omega_j)\Delta\omega_j}{B} [\cos(A+B) + \mathrm{i}\sin(A+B) - \cos(A-B) - \mathrm{i}\sin(A-B)] \right\} \tag{17}$$

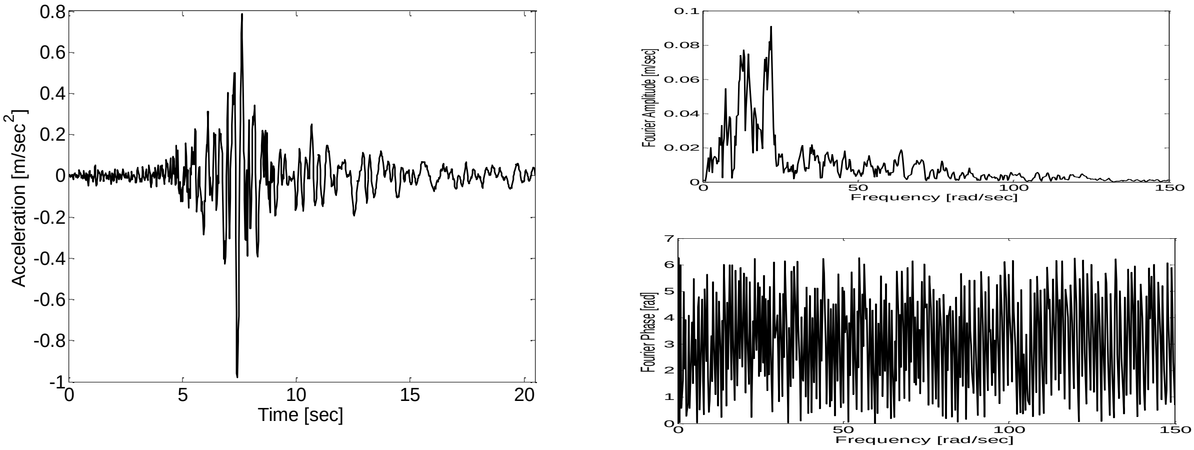
Introduction of rules in trigonometric functions, u is deduced as

$$u = 2\,\mathrm{Re}\left\{ \frac{U(\omega_j)\Delta\omega_j \sin B}{B} [\cos A + \mathrm{i}\sin A] \right\} \tag{18}$$

namely,

$$u = \frac{2U(\omega_j)\Delta\omega_j \cos A \sin B}{B} \tag{19}$$

4. CASE STUDIES



(a) (b)

Figure 4. Behaviors of Northridge earthquake ground motion: (a) representative time history and (b) Fourier spectrum.

An observed earthquake ground motions at type-II site is investigated for illustrative purposes: Northridge earthquake wave (scaled peak acceleration 100 gal, time interval 20.48 sec, sampling frequency 50 Hz, the distance between station and epicenter 36 km). The example ground motion and its Fourier spectra are shown in Figure 4. The numerical procedure on simulating earthquake ground motions is completely detailed in the following subsections.

4.1 Definition of parameters in loose conditions

As mentioned in Section 2, the parameter ε in loose condition is the coresidual of $N_i (i = 1, 2, \ldots, n)$ to 1, and it should be a value with magnitude far less than 1. The loose condition would result in a difference between original frequency phases and simulated frequency phases. In order to measure the effect acted by this difference on the simulated wave, an analysis step of phase sensitivity to the simulated wave needs to be carried out first. We investigate the L^2 error norm between original ground motion and simulated ground motion in case that the phase values of all frequency components are added $\pi/4$, $\pi/8$, $\pi/16$, $\pi/32$, respectively. It is indicated that the group with low frequencies significantly contributes to the simulated result than the group with high frequencies, and specifically, nearly all the frequency components have trivial effect on the simulated result in case that the phase increasement of frequency components is less than $\pi/16$. The parameter ε thus can be defined as follows:

$$\varepsilon = \underset{\max[\{N_i - [N_i]\}]}{\arg} \left\{ \max[|\{\hat{\varphi}(\omega_i) - \varphi(\omega_i)\}|] \le \frac{\pi}{16} \right\} \tag{20}$$

where $\hat{\varphi}(\omega_i)$ denotes the simulated phase spectrum.

It is also indicated from Figures 4(b) that the low frequency group exhibiting significant contribution to the simulated wave corresponds to larger magnitudes of Fourier amplitude spectrum. One might realize that the simulated result is sensitive to the frequency components carrying most energy. It is further understood that the medium and low frequency components including the most energy are related to the conventional engineering structures of which the fundamental period ranges from 0.1 to 10.0 sec, the high frequency components, therefore, occupying tiny energy can be dropped. Have this in mind, we just use, for the case of Northridge wave, the frequency components with number 1 to 200 in the following simulation corresponding to frequency range [0.0 61.0] rad/sec. This treatment further reduced the complex degree of the investigated case.

4.2 Starting-time of phase evolution of predominant frequencies

It is indicated from the above sections that using the principle of wave group, just the predominant frequency components and their adjacent components are critical in the numerical procedure. The deduced starting-times are used to simulate the phases of predominant components and their adjacent components in one wave group. The phase of adjacent components is required in estimating the gradient of phase spectrum at frequency ω_j; see $d\varphi/d\omega$ in equation (12).

According to the sensitive behavior of phases to frequency components, the frequency domain of interest is divided into three groups: low frequency group with range [0.0 8.9] rad/sec, medium frequency group with range [9.0 30.4] rad/sec, high frequency group with range [30.5 61.0] rad/sec. Further, 5 predominant frequency components are uniformly selected in each group, respectively, whereby a series of narrowbands in these predominant components are calculated, and then the ground motion is formed. The predominant frequency components and their crossing times N over 2π, starting-time of wave group T_s, parameter of loose condition ε are showed in Table 1.

It is seen from Table 1 that for the investigated ground motion, 3 wave groups respectively corresponding to low frequencies, medium frequencies and high frequencies, 15 narrowbands and 6 starting-times of phase evolution are sufficient to perform the simulation. In the 6 starting-times, 3 ones are used to represent the phase evolution of predominant components, and 3 ones are used to represent the phase evolution of adjacent components. Here, the latter 3 starting-times are independent to the former 3 starting-times though the corresponding frequency components are belonged to the same wave group, respectively. The predominant frequencies governing the wave-group propagation are those relevant to the former 3 starting-times. The latter 3 starting-times are just used to estimate the gradient of phase spectrum at frequency ω_j.

Table 1. Starting-time of phase evolution of wave groups for Northridge ground motion.

Group	Freq. (rad/s)	Predominant components				Adjacent components			
		No.	N	T_s (sec)	ε	No.	N	T_s (sec)	ε
1	0.0~8.9	4	63227	1.83634E11	0.00	5	12907	5.42631E9	0.00
		10	6822540			11	59235		
		16	5902030			17	220095		
		22	1952360			23	536106		
		28	26008200			29	1184610		
2	9.0~30.4	38	14810	4.99651E7	0.02	39	16160	3.57748E7	0.03
		52	21531			53	9788		
		66	47768			67	31185		
		80	8041			81	9657		
		94	8861			95	4049		
3	30.5~61.0	111	101036	5.56573E8	0.00	112	86929	4.90935E8	0.00
		131	247391			132	230361		
		151	233090			152	118969		
		171	226809			172	170529		
		191	242702			192	207004		

It is indicated that the phase values of simulated predominant and adjacent components match well to those of observed predominant and adjacent components since the difference between them is less than $\pi/16$ in case of the loose condition $\mathrm{mod}(N_i,1) \le \varepsilon (i=1,2,3,\ldots,n)$. The parameter of loose condition ε, in this case, nearly equals zero for the wave groups of interest, of which the maximum value is 0.03. The simulated and observed predominant and adjacent components are shown in a Fourier rose-diagram, as shown in Figure 5, where the polar radius denotes the Fourier amplitudes and the polar angle denotes the phases of investigated components. In the diagram, the circle 'o' indicates the simulated frequency components, while the dot '.' indicates the observed frequency components. It is revealed that the starting-time of phase evolution is effective to represent the phase spectrum in predominant frequencies whereby a univariate phase model can be achieved.

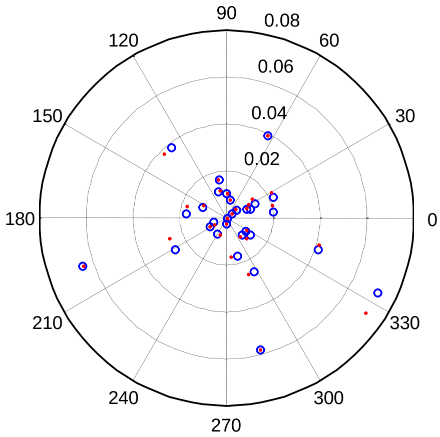


Figure 5. Rose diagram for Fourier spectrum of predominant frequencies.

It is also indicated in Table 1 that the crossing times N of predominant components over 2π are significantly different even if these frequencies are located in one wave group, which results in distinct starting-times of phase evolution for wave groups. It is explained that the above results are due to the differences between phase values and phase evolution velocities of predominant components at the current time t_0. As indicated previously, the starting-time of phase evolution is a nominate physical quantity introduced for simulating phase spectrum which lacks physical accounts.

4.3 Simulation of earthquake ground motions

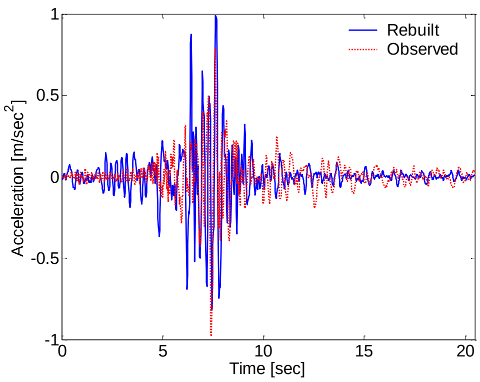


Figure 6. Comparison between simulated and observed Northridge ground motions.

Using the phase spectrum in predominant frequencies, the simulation of earthquake ground motion is performed where the amplitude spectrum is, for validating purpose of the univariate phase model, in accordance with the Fourier amplitudes of observed ground motion. The comparison between the time history of the simulated and observed ground motions is shown in Figure 6 (in simulated case: $t_c = 280$ st, where st is a time unit that equals to 0.02 sec). It is seen that the simulated ground motion arises to be sound. Therefore, the simulation of Northridge ground motion is reliable using 6 starting-times of phase evolution to construct 15 narrowbands with superimposing of harmonic processes.

In the previous numerical investigations, the amplitude values in equation (11) employs those of Fourier amplitudes of observed ground motions for validating purpose of the univariate phase model. The amplitude values actually can be rendered using a classical envelope function consisting of build-up, constant and decay portions (Ohsaki, 1979). For the simulation of nonstationary stochastic ground motions, the parameters in the envelope function and univariate phase model can be readily identified and synthesized through investigation of recorded ground motions at a certain site. The univariate phase model built on the proposed starting-time of phase evolution thus promotes a more feasible scheme for the representation of nonstationary stochastic ground motions.

5. CONCLUDING REMARKS

A new family of simulation methods for nonstationary earthquake ground motions is developed in the present paper which employs a univariate phase model built up on a time argument associated with the concept of starting-time of phase evolution of frequency components. Numerical investigations indicate that the proposed starting-time of phase evolution is a very useful concept which provides an efficient component to simulate ground motions just using few variables in numerical implementation. This treatment bypasses the numerical challenge inherent in the classical spectral representation techniques that tens even hundreds of variables are required in modeling the phase spectrum. It also promotes a more feasible scheme for the representation of nonstationary stochastic ground motions.

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