Comparison of the Total Uncertainty Associated With Alternative Approaches to Site Response Analysis

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ABSTRACT:
Within PSHA, the effects of site response can be treated in a variety of different manners. Typically, there is an assumed hierarchy of accuracy for the alternatives, but the validity of this hierarchy has not been formally assessed. Site response can be modelled directly using empirical Ground Motion Prediction Equations (GMPEs) and the precision of the analysis is related to the published variance components of the model. An alternative approach is to conduct site-specific site response analyses, and this approach is commonly assumed to be the most accurate. Using equivalent linear (EQL) and non-linear (NL) site response analysis for two magnitude-distance (M-R) scenarios this study focuses on the total uncertainty associated with this more computationally demanding route. The spectral ordinates and the variability of the site response methods are further compared to the predictions of the Campbell and Bozorgnia (2008) GMPE for these two M-R combinations.

Keywords: Site Response, Variability, PSHA, Uncertainty

1. INTRODUCTION

Site response analysis is widely used to calculate the surface ground motions for a suite of accelerograms that are deemed to be consistent with an earthquake scenario defined following a probabilistic seismic hazard analysis (PSHA) conducted for a reference rock site. In this analysis, the records are driven through a soil profile that has been defined in order to incorporate the effects of local site-specific soil conditions. There are various different record selection procedures (e.g. Bommer and Acevedo, 2004; Baker and Cornell, 2006; Hancock et al. 2006; Kottke and Rathje, 2008) which aim to select input motions that reflect the seismic hazard which is represented by some target spectrum. One common approach is to select input motions based on a scenario spectrum which is derived from the disaggregation of the hazard curves obtained from the PSHA. An alternative, less common approach, is to transform a GMPE using site specific amplification factors prior to conducting the PSHA. Using the transformed GMPE, the design ground-motions can be specific for the ground surface directly.

In the case that an approach involving post-PSHA site response analysis is chosen, then once the suite of records is identified they are driven through the soil profile within a numerical analysis framework. There are various options at this point; the site response can be performed using equivalent linear (EQL) or non-linear (NL) approaches and the equations of motion can be solved in the frequency or time domain (with the time domain being the only option for NL approaches).

Site response analysis is computational demanding; however it is commonly assumed that is also the most accurate approach. This paper aims to evaluate the total uncertainty associated with this computational demanding route. Through the application of two earthquake scenarios the challenges of using EQL and NL site response analysis will be further assessed. The results of both site response approaches will be also compared to the median predictions and published variances of the Campbell and Bozorgnia (2008) GMPE (abbreviated CB08) aiming to evaluate the benefits of using site-specific response analysis.

2. METHODOLOGY

2.1. Record selection process

Prior to performing any numerical simulation, as part of the site response analysis, a suite of records has to be selected. In the present study, this was achieved by using the semi-automated procedure of Kottke and Rathje (2008) which is incorporated in the software sigmaSpectra. Based on this procedure the median of a suite of records is matched to a target spectrum considering the root-mean-square-error (RMSE) at all defined periods. However the principal advantage of this approach is that the records are further linearly scaled to match the prescribed target standard deviation at each period considering the aleatory variability of the record set (Kottke and Rathje, 2008).

The target spectra used within sigmaSpectra were generated using the Campbell and Bozorgnia (2008) GMPE for the half space described in the subsequent section. Two scenarios were considered; one aiming to generate small-strain soil response and a high intensity one aiming to induce non-linear response. The selection of the appropriate earthquake magnitude (M) and Joyner-Boore distance (R) combinations that would induce this response was based on the predictions of CB08. The spectral accelerations of the latter were plotted for the natural period and average shear-wave velocity of the top 30m of the site for different distances and M combinations. From this plot the linear and non-linear response was inferred and two corresponding M-R combinations were selected. Particularly the low intensity scenario (abbreviated as Sc1) was characterised by an earthquake magnitude, M=7 and R=55km, whereas the second scenario (Sc2) was represented by M=7 and R=5km.

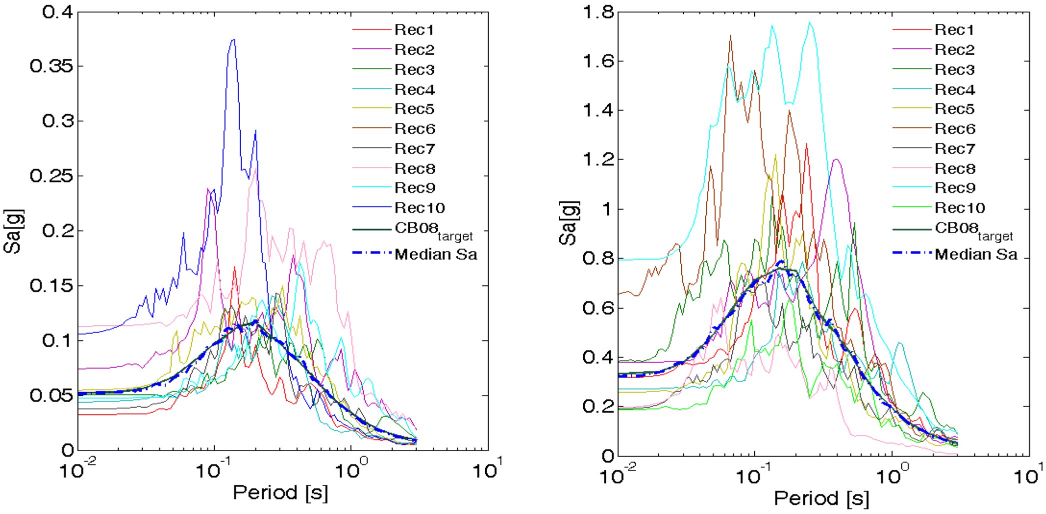
The next step in the record selection process was to generate the target spectra for these M-R combinations and the corresponding standard deviations at all periods. The latter was amended by removing the influence of the ergodic assumption (e.g. Lin et al., 2011; Morikawa et al., 2008; Walling, 2009; Montalva, 2010 etc). Since the published standard deviation of the GMPE was derived for records corresponding to a wide range of site conditions, distances from the source, type of sources and tectonic environments, etc, the variability of the predictions corresponding to a single site would be smaller (Lin et al., 2011). Therefore the published standard deviation of the model (σ_{CB08}) was reduced using the factors of Lin et al. (2011). Where factors were not provided for particular response periods the factors were obtained by linearly interpolating them in logarithmic period-space. The assumption here is that the reductions proposed by Lin et al. (2011) are globally applicable and thus can be used in conjunction to CB08. This assumption was loosely supported by comparing the reductions proposed by Lin et al. (2011) to those suggested by Rodriguez-Marek et al. (2011). This comparison showed that there are only marginal differences between the two studies despite Rodriguez-Marek et al. (2011) reductions being based on a different dataset.

It is appreciated that the removal of the effects of the ergodic assumption, described before, from the standard deviation should be coupled to the removal of the effects of the ergodic assumption from the mean and therefore a site-specific GMPE should be used to derive the median ground motions (Lin et al. 2011). It is however assumed that we do not need to modify the median of the GMPE for the purposes of our analysis. We must, however, consider more epistemic uncertainty on the mean.

Following the removal of the ergodic assumption, the σ_{CB08} was further modified by increasing the variability in order to account for the component-to-component variance. This was done as only 1D site response analysis is performed and hence only one arbitrary component is selected from each recording. As individual components are used, it was ensured that only one component from any recording was used so that the variability of the final suite of motions was not underestimated (Kottke and Rathje, 2008). The component-to-component variability that was added was as quantified by

Campbell & Bozorgnia (2008).

The developed target spectra with the reduced standard deviations for both M-R scenarios were then used as part of sigmaSpectra to select the final suite of motions using the NGA database (Chiou et al., 2008). For both scenarios, the records that did not fall into two preselected bins, of M=6.6-7.3 and R=30-70km for Sc1 and M=6.6-7.3 and R=1-30km for Sc2, were excluded. The M-R bins were the only two filtering criteria which were used to select the final catalogue of motions. Magnitude was valued as a parameter since it influences both the duration and the shape of the response spectrum. On the other hand the distance restriction ensured that the final records would not be drastically scaled in order to match the target spectrum. The final selected suite of records for both scenarios can be seen in Figure 1.



[a] **[b]**

Figure 1. Suite of selected records for low intensity [a] and high intensity scenario [b]

2.2. Site response analysis

The soil profile, that the selected records were driven through, was defined as a shallow stiff sand site with average shear wave velocity of the top 30m equal to V_{s30}=391m/s. The plasticity index was set equal to zero and the over consolidation ratio equal to one, while the coefficient of effective earth stress at rest was set equal to 0.5. The dynamic soil properties were determined by the empirical curves of Darendeli (2001).

The generated profile is illustrated in Figure 2a and was characterized by a natural frequency [f_o] equal to 2.65 Hz. This was calculated for the depth of the soil profile using the weighted-average shear wave velocity of the site. Site response analysis was then performed for the half space positioned at 65m below ground level with a bedrock shear wave velocity, of V_s=1270m/s.

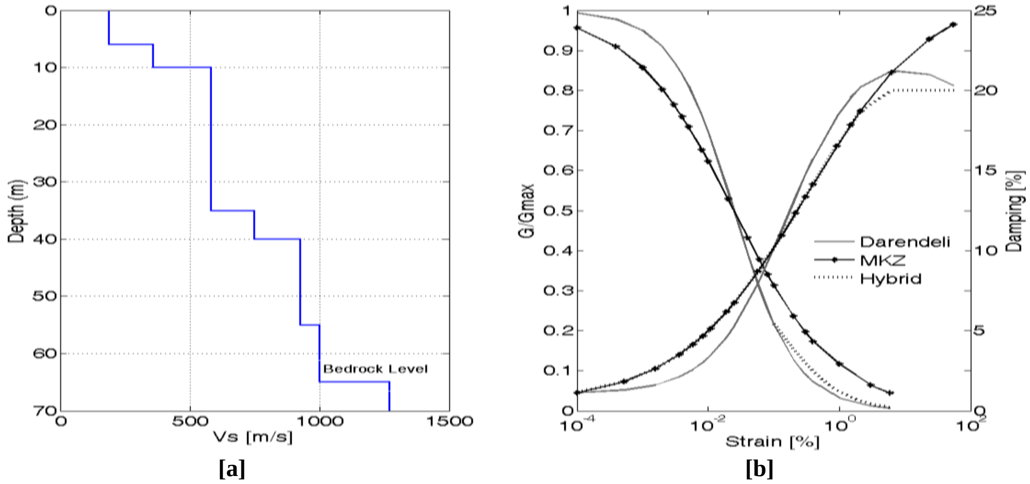


Figure 2. [a] Soil profile and [b] Darendeli (2001), MKZ and hybrid stiffness degradation and damping curves

The site response analysis was carried out using EQL analysis in the frequency domain through the software SHAKE (Schnabel et al., 1972). Non-linear analysis was also performed in the time domain using DMOD (Matasovic et al., 2011). The NL analysis within DMOD uses the hyperbolic modified model of Kodner and Zelasko (MKZ) which incorporates the extended Masing rules (Stewart et al., 2008). Due to the almost linear behaviour of the hyperbolic model at small-strains the predicted hysteretic damping is nearly zero. In order to overcome this deficiency additional viscous damping (in the form of Rayleigh damping) is considered which reflects the damping at small-strain level (Hashash and Park, 2002). The MKZ parameters were obtained by fitting the hyperbolic model to the empirical curves of Darendeli (2001). The resulting fitting of the MKZ model for layer 1 (surface) can be seen in Figure 2b.

3. RESULTS

3.1. Low strain scenario

The final suite of records selected for Sc1, as seen in Figure 1a, was driven through the soil profile described in the previous section. The resulting spectral ordinates and the variability in the predictions are summarised in Figure 3a and 3b respectively. These results are further compared to the predictions obtained using CB08 for V_{s30}=391m/s.

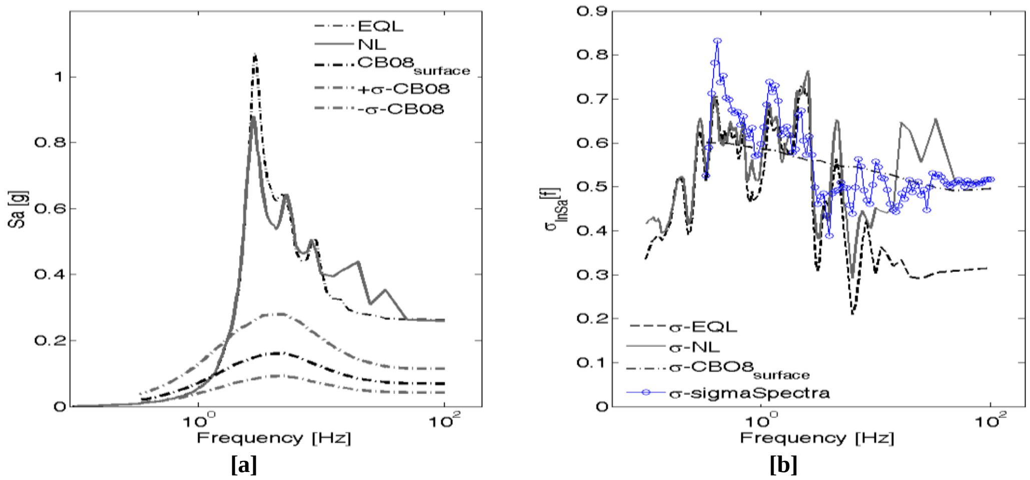


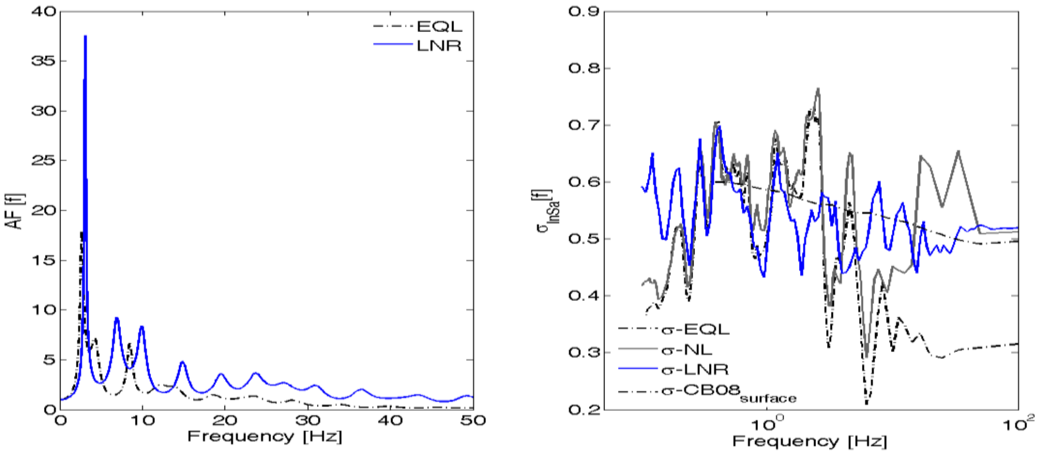
Figure 3. [a]Comparison of geomean Sa[g] and [b] standard deviations of EQL, NL and GMPE for Sc1

The largest difference, indicated in Figure 3a, between the EQL and NL predictions was at the soil principal frequency, whereas the majority of the variations in the spectral accelerations between the two analyses were clustered at frequencies over 10Hz. The larger peak motions predicted by EQL, in comparison to NL analysis, have also been noticed by several other researchers (e.g Yoshida et al., 2002 and Papaspiliou, 2010). The mechanism behind this variation was further investigated and as a first step the maximum stresses/strains were compared. From this it was confirmed that the EQL analysis predicted higher max stresses - and consequently higher maximum peak motions - for eight out of the ten records and for the majority of the soil profile layers. It also predicted higher strains for all input motions. It is worth emphasising that the two records for which EQL predicted smaller peak motions, in comparison to NL analysis, were also the two records that yielded the highest strains (0.377% and 0.38%). These two input motions had also the two highest spectral accelerations at the fundamental frequency of the site in the group. The strain levels of the remaining eight records were under 0.18%.

Correlating these strain levels to the stiffness degradation and damping curves, shown in Figure 2b, it was established that for strains below 0.3% the MKZ model corresponded to lower stiffness values and higher damping in comparison to the empirical curves of Darendeli (2001). This trend reversed for strains over 0.3%. This poor fit of the MKZ model to the Darendeli (2001) curves for the particular soil profile and thus the difference in stiffness and damping used by the two site response approaches is believed to be predominately responsible for the variation in the peak motions between the two (i.e. the lower predicted peak motions by NL analysis for strains below 0.3% and the higher peak motions for strains over 0.3%).

In light of the above observations, the EQL analysis was repeated using the MKZ stiffness and damping values replacing those of Darendeli (2001) for the same profile and suite of records. Both EQL and NL analysis then provided almost identical peak spectral ordinates with NL analysis still tending to under predict the maximum spectral acceleration but only marginally.

Further reviewing Figure 3a it can be observed that the EQL analysis predicts considerably lower spectral accelerations (the plot almost flattens out) for frequencies over 10Hz. This was attributed to the use of constant stiffness-smaller than the maximum stiffness - and constant damping throughout the analysis as opposed to an alternating stiffness which is used in the NL analysis. The constant reduced stiffness used in EQL corresponded to lower frequencies which resulted in the over damping of the higher frequency parts of the motions. In order to assess the last statement the amplification functions obtained by performing EQL analysis were compared to those obtained using linear analysis (LNR) employing the maximum stiffness G_{max}. The damping levels, used in the latter, were the converged damping values of the EQL analysis. Therefore the only difference between EQL and LNR analysis was the stiffness used. Figure 4a shows, that the use of the single reduced stiffness by EQL analysis, resulted in considerably low, almost flat AF[f] for frequencies over 10Hz, in contrary to the LNR analysis predictions.



[a] **[b]**

Figure 4. [a] Amplification functions of EQL and LNR and [b] standard deviations of different approaches

Further comparing the site response analyses to the predictions of CB08 in Figure 3a it can be observed that both the EQL and the NL analysis result in considerably higher ground-motions than the predictions of the GMPE, even higher than the $\sigma -$CB08$_{surface}$ values of the model for all frequencies over 1.9Hz. The GMPE predictions did not reflect the site response as accurately as the site response analysis in this case since they were based on a single site term, the $V_{s30.}$ Therefore the GMPE did not incorporate the contribution of parameters such as the impedance ratios of the soil layers, the layer thicknesses and resonances which were on the other hand captured by the site response analyses.

Figure 3b, shows that the variability of all the methods of analysis is comparable. Moreover both EQL and NL analysis reflected well the record-to-record aleatory variability represented by $\sigma -$ sigmaSpectra. The latter is the evaluated standard deviation of the suite of records which were selected and matched to the target median and standard deviation. It is also noticed that EQL variability significantly reduced in comparison to all the other approaches for higher frequencies. However this was due to the limitation of the EQL, established before, that tended to over damp the higher frequency parts of the motions. The fact that the reduction in the variability by EQL is not representative can be further supported by the σ-LNR curve plotted in Figure 4b for which no reduction occurs at the higher frequencies.

3.2 High strain scenario

Site response analysis using EQL and NL analysis was also performed for the high strain scenario Sc2 described before. The results are summarised in Figures 5a and 5b and are compared to the predictions of CB08 for the corresponding M-R combination.

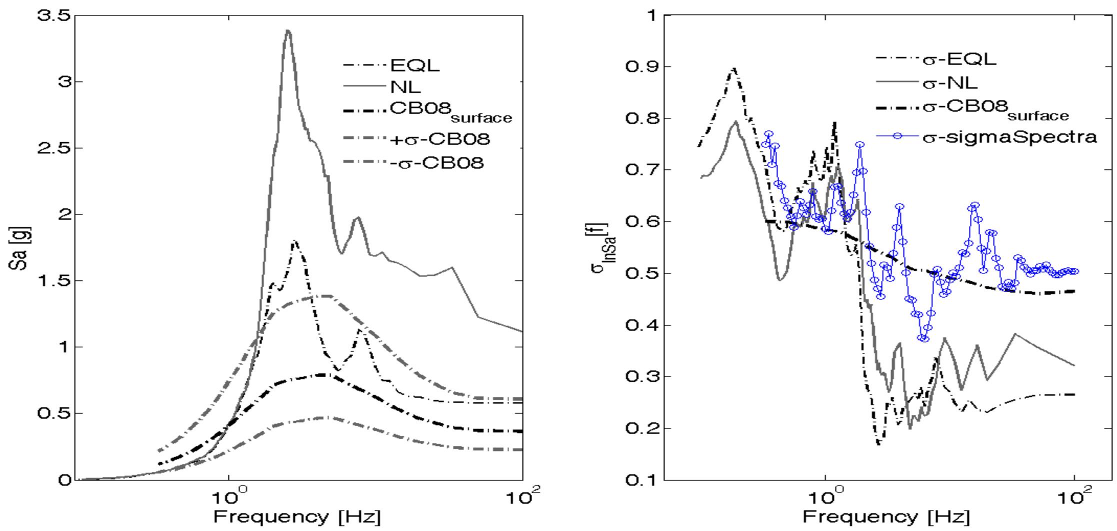


Figure 5. [a]Comparison of geomean of Sa[g] and [b] standard deviations of EQL, NL and GMPE for Sc2

As expected due to the high intensity of the input motions of Sc2 (see Figure 1b) stronger nonlinear behaviour was observed. This yielded considerable differences amongst the predictions of the EQL and the NL analysis. Specifically the former predicted almost twice the maximum strain levels which resulted in considerable over damping of the motions compared to the NL predictions (see Figure 5a). The NL analysis also predicted considerably higher spectral ordinates in comparison to the GMPE, whereas the EQL's predictions of ground motions were closer to those of the GMPE for the majority of the frequency range.

Further analysing the results it was observed that the EQL analysis converged at considerable lower stiffness in comparison to the stiffness used by DMOD for the same records. This phenomenon was attributed to the use of the empirical curves of Darendeli (2001) for strains over the recommended upper threshold of 1%. Focusing on the stiffness degradation curve of Darendeli (2001), an example of which can be seen in Figure 2b, it is noticed that the stiffness of the soil sharply decreases over that threshold. Hence, for the considered sandy deposit, the EQL analysis converged at very low values of stiffness which were considerably lower than the corresponding stiffness values of the MKZ model. Figure 6 represents an example of the sequence of iterations which shows that the stiffness very quickly drops to the low plateau and gets "trapped" there.

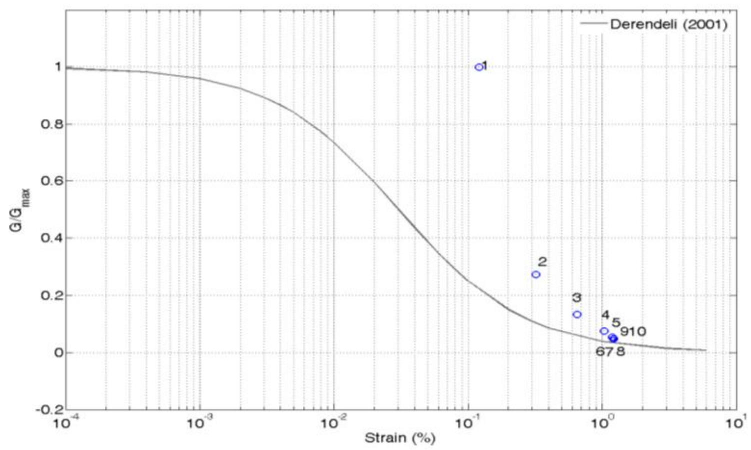


Figure 6. Converged stiffness of SHAKE for iteration 1-10 for layer 5.5m below ground level against the Darendeli (2001) curve for the corresponding depth

The shape of the stiffness degradation curve is of course determined by the soil type. Therefore the effect and thus the difference in predictions between the EQL and the NL analysis were further assessed by using a low plasticity clay (with plasticity index equal to 20) instead of sand. The comparison of the spectral ordinates between the EQL and the NL analysis for the clay case can be seen in Figure 7.

The spectral ordinates of the clay profile, for the EQL and the NL analysis, were further compared by using the root mean square error (RMSE). The RMSE of the spectral ordinates of the sand profile was also evaluated. It is recognised that RMSE will be biased towards lower frequencies due to the linear spacing of strain levels, but since the same bias will be true for both spectra (sand and clay) the RMSE can be used as a comparative mean. Conclusively, the RMSE for the sand profile was equal to 0.248 whereas the RMSE for the clay profile was equal to 0.193. The lower value of RMSE for the clay profile indicated a closer match between the predictions of EQL and NL analysis. This could be explained by the better match of the MKZ to Darendeli (2001) curves which was achieved since the stiffness degradation curve of the clay was less steep than the sand's.

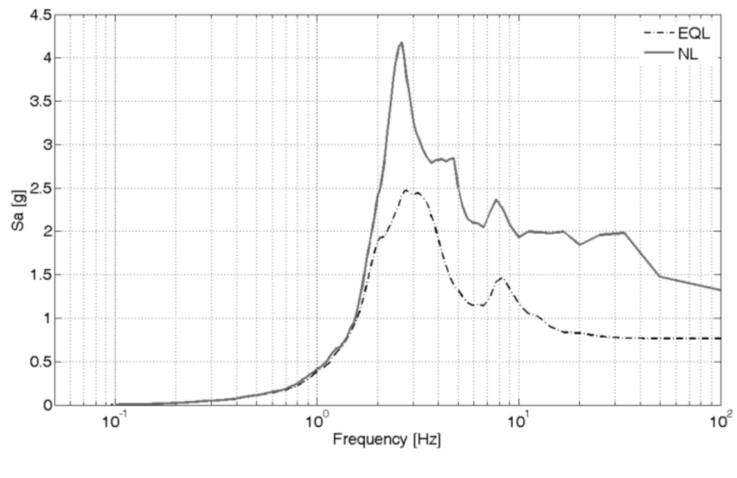
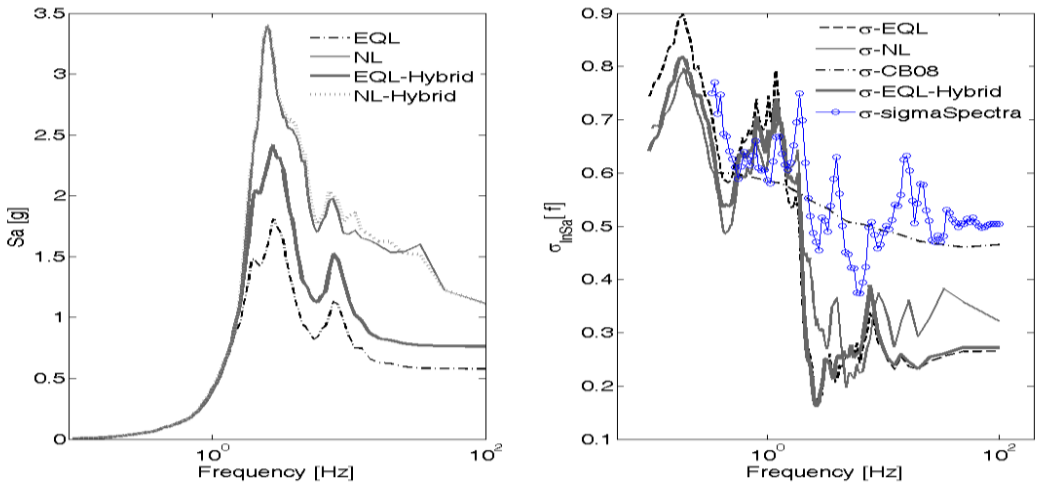


Figure 7. Comparison of geomean of Sa[g] of EQL and NL analysis for clay profile with PI=20% for Sc2

As an alternative to the Darendeli (2001) curves the hybrid method by Stewart et al. (2008) was also considered. Based on their recommendations and the example application of the method by Chiu et al. (2008), the pseudo-reference strain γ_r of Derendeli (2001) equal to the strain for which G/G_{max} =0.5 was replaced with the strength-based reference strain (τ_{ff}/G_{max}) for strains over 0.1% This approach attempts to account for the fact that at high strains the strength of the soil is a more representative parameter of the soil's behaviour.

Since no specific cyclic tests were available the static shear strength was increased by 15% for a constant angle of shearing resistance equal to 30° and the damping of the hybrid approach was limited to a maximum value of 20% (Chiu et al, 2008). The resulting hybrid curves can be seen in Figure 2b.The hybrid curves were used for both the EQL and the NL analysis and the results can be seen in Figure 8a and 8b.



[a] **[b]**

Figure 8. [a] Comparison of geomean Sa[g] and [b] standard deviations of EQL, NL and GMPE including the predictions using hybrid curves

Figure 8a shows that the use of the hybrid curves had a minimum effect to the predictions of the NL analysis for the examined case. This was due to the fact that the MKZ model was already using relatively high G/G_{max} values compared to the Darendeli (2001). Therefore no particular alteration to the stiffness was achieved by fitting the MKZ model to the new hybrid curves. However the use of the hybrid curves had a significant influence to the predictions of the EQL analysis. The spectral ordinates of the latter were considerably increased and the RMSE between the two new spectra (EQL and NL response spectra using hybrid curves) was significantly reduced to 0.198 from the former 0.248.

In addition to the spectral ordinates EQL and NL analysis were further compared in respect to their variability. Figure 5b shows that, similarly to the Sc1 case the standard deviations of both EQL and NL analysis reflected well the record-to-record variability, represented by σ-sigmaSpectra. The standard deviations of the site response methods increased in comparison to the σ-CB08$_{surface}$ for frequencies lower to the principal frequency of the site. This could be explained by the soil's non linear response which was related to the frequency content of the records.

A considerable reduction in the standard deviations for both EQL and NL analysis in comparison to the σ-CB08$_{surface}$ is also noticed in Figure 5b, for frequencies over 1.8Hz. This reduction complied with the general amplification factors degradation for higher modes (frequency dependence of AF[f]). It is also noticed that EQL analysis tended to predict lower σ values than the NL analysis for higher frequencies which was partially attributed to the EQL analysis's use of constant stiffness and damping (also identified and briefly explained in Sc1). Finally it can be seen in Figure 8b that although the effect of the hybrid method was considerable on the spectral ordinates of the EQL analysis their use did not reduce the variability of the predictions.

4. SUMMARY AND CONCLUSIONS

Equivalent linear and non-linear site response analysis were performed for two earthquake scenarios with M=7 and R=55km and M=7 and R=5km focusing on small and high strain soil response. The results of each method were compared in terms of the spectral ordinates and standard deviations for each case. The site response results were further compared to the predictions of the Campbell and Bozorgnia (2008) ground motion prediction equation for the corresponding M-R combinations.

From these analyses it was shown that the EQL approach predicted larger peak motions than the NL analysis for the case of the small strain-scenario (see Figure 3a), whereas it consistently predicted lower spectral ordinates for the majority of the frequency range for the case of the high intensity scenario (see Figure 5a). The difference between the peak motions, predicted by the two site response approaches, for the Sc1 case, was attributed to the poor fit of the MKZ model to the Darendeli (2001) curves for the particular soil profile. This was further assessed by using the fit MKZ modulus and damping values to the EQL analysis, replacing the corresponding stiffness and damping of the Darendeli (2001) curves. This then resulted in almost identical predictions of peak motion by the two site response approaches. What was also concluded was that the significant differences between the predictions of the EQL and the NL analysis, for the Sc2 case, were due to the inappropriate use of the Darendeli (2001) empirical curves for strain levels over 1%. As an alternative to the Darendeli (2001) curves the hybrid method by Stewart et al. (2008) was also considered. This had a minimum effect on the predictions of the NL analysis but considerably increased the spectral ordinates predicted by the EQL analysis. What was also shown was that for both scenarios the GMPE predictions were considerably lower to the predictions of the two site response approaches. The latter captured more accurately the response of the particular soil site with the GMPE only using a single site term, that of the $V_{s30,}$ to assess it.

The previous approaches were also compared based on their variability (see Figures 3b and 5b). As shown in Figure 3b the variability of all the different methods of analysis were comparable. However, the EQL analysis flattened the higher frequency motions due to the use of constant stiffness and damping. This created a fictitious reduction of the variability by the EQL analysis, noticed at both scenarios, at the upper frequency range which is not representative. For the Sc2 case the standard deviations of the site response approaches varied to the standard deviation of the GMPE reflecting both the non-linearity of the soil at small frequencies (increased variability) and the reduction of the spectral ordinates at higher frequencies (reduced variability). In both scenarios the site response methods reflected well the record-to record variability as represented by the standard deviation of the sigmaSpectra.

Performing the above deterministic analyses assisted in evaluating the limitations and the differences amongst the EQL analysis, the NL analysis and the GMPE as predictive methods of analysis. This could be particularly useful when a more generic probabilistic assessment is carried out. However it is believed that these methods should not be solely evaluated based on their comparative results but their accuracy should be further assessed, comparing for instance their predictions against vertical array data.

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