

# Frequency content indicators of strong ground motions

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## **SUMMARY:**

The frequency content of ground motions seems to be the most important parameter to explain the structural damage experienced during strong earthquakes. The frequency content of ground motions can be characterized using various stochastic and/or deterministic indicators. A comparative analysis of stochastic and deterministic frequency content indicators is applied to a set of 20 famous strong ground motion records having peak ground accelerations from 0.2g to 0.9g and recorded during the last 70 years. Since  $T_C$  is an important parameter for structural seismic design in use in many present day codes (Eurocode 8 and others), one main focus of the analyses is the comparison of the various definitions of the control period  $T_C$  of: Bommer et al. (2000), Lungu et al. (1997), Newmark & Hall (1969, 1982), ATC 3-06 (1978) as well as the definitions implicitly contained in ASCE/SEI 7-10 (2010).

Keywords: *earthquake records, stochastic modeling, response spectra, structural design*

## **1. INTRODUCTION**

The ground motions recorded worldwide in the last 70 years show various frequency contents, from wide and intermediate frequency bandwidth ground motions (recorded in hard and/or medium soil conditions) to narrow frequency band ground motions (recorded in soft soil conditions). The random frequency contents of ground motions generally depends on both source mechanism and magnitude as well as on epicentral distance and local soil conditions. The frequency contents of the strong ground motions is a key parameter for explaining and understanding structural damage experienced during strong seismic events.

Probabilistic-based assessment of the frequency contents of the ground motion records can be done using the Power Spectral Density concept (PSD) and its related dimensionless indicators  $\varepsilon$  (Cartwright&Longuet-Higgins) and  $q$  (Vanmarcke), or the fractile frequencies  $f_{10}$ ,  $f_{50}$  and  $f_{90}$  (Kennedy – Shinozuka indicators) below which 10%, 50% and 90% of the total cumulative power of the PSD occurs.

The deterministic assessment of the frequency contents of the ground motions records can be based on the concept of the control period of structural response spectra, historically introduced by Newmark & Hall (1969, 1982), as well as on the evolution of the concept during the last fifty years.

## **2. CHARACTERIZATION OF THE GROUND MOTION FREQUENCY CONTENT**

### **2.1. Stochastic indicators for the frequency content of seismic records**

The definition of the most reliable frequency content indicators of the ground motion records are based on modelling the strong phase of the recorded accelerogram as a stationary stochastic process.

The duration  $D$  of the stationary part of the motion may be selected as the time interval in which a significant fraction (say 70%, 80% or 90%) of the total cumulative power of the accelerogram  $a(t)$  is released i.e.  $D_{0.9} = t_{0.05} \div t_{0.95}$ ,  $D_{0.8} = t_{0.10} \div t_{0.90}$ , etc.

$$Cum.Power = \int_0^{\infty} [a(t)]^2 dt \quad (2.1)$$

Consequently, the power spectral density (PSD) of accelerograms considered in the present study was determined for the stationary part of the record modelled to be within the time interval  $t_{0.10} \div t_{0.90}$ .

The dimensionless indicators  $\varepsilon$  and  $q$  are defined as a function of the spectral moments of the PSD for the stationary process of the ground acceleration:

$$0 \leq \varepsilon = \sqrt{1 - \frac{\lambda_2^2}{\lambda_0 \lambda_4}} \leq 1 \quad (2.2)$$

$$0 \leq q = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}} \leq 1 \quad (2.3)$$

where  $\lambda_i$  is the  $i$ -th moment of the PSD.

$$\lambda_i = \int_{-\infty}^{+\infty} \omega^i S_x(\omega) d\omega \quad (2.4)$$

The guidance values for  $\varepsilon$  indicator in the case of actual ground motion accelerograms might be:

- $2/3 < \varepsilon < 0.85$  for a wide frequency band process;
- $0.85 < \varepsilon < 0.90$  for an intermediary band process;
- $\varepsilon > 0.90$  for narrow frequency band processes associated wide frequency band “noise”

## 2.2. Deterministic indicators for the frequency content of seismic records

The deterministic analysis of the frequency content of ground motions is related to the maximum response of a SDOF (single degree of freedom) system to the recorded ground motion. Two control periods of response spectra are  $T_C$  and  $T_D$ .  $T_C$  represents the border between the maximum acceleration branch and the maximum velocity branch of the response spectra and  $T_D$  is the border between the maximum velocity branch and the maximum displacement branch of the response spectra.

In Table 2.1 are presented various definitions for control periods  $T_C$  and  $T_D$  of the response spectra given in ATC 3-06 (1978), Newmark & Hall (1969, 1982), Lungu et al. (1997), Bommer et al. (2000) and resulting from the data in ASCE 7-10 (2010). There are two categories of definitions for  $T_C$  – the definitions based on the spectral values: acceleration, velocity and displacement given in: ATC 3-06 (1978), Lungu et al. (1997) and ASCE 7-10 (2010) and the definitions based on the peak values of the seismic ground motion: Newmark & Hall (1969, 1982) and Bommer et al. (2000). The definitions based on the spectral values use the effective peak acceleration, velocity and displacement: ATC 3-06 (1978),

Lungu et al. (1997), which represent averaged values, while the American Code ASCE 7-10 uses the spectral acceleration values at two periods corresponding to the short period range (0.2 s) and to the medium period range (1.0 s).

The relationships for the control period  $T_C$  from Newmark & Hall (1969, 1982) and Bommer et al. (2000) provide similar values. In the case of the control period  $T_D$ , the definition given in Bommer et al. (2000) provides values 1.5 times larger than the Newmark & Hall (1969, 1982) definition.

**Table 2.1.** Definitions of control periods  $T_C$  and  $T_D$  of the structural response spectra

ATC 3-06 (1978)	Lungu et al. (1997)	Newmark & Hall (1969, 1982)	Bommer et al. (2000)	ASCE 7-10 (2010)
$T_C = 2\pi \frac{EPV}{EPA}$	$T_C = 2\pi \frac{EPV}{EPA}$	$T_C = 2\pi \frac{\alpha_V PGV}{\alpha_A PGA} =$ $= 2\pi \frac{1.65 PGV}{2.12 PGA} = 4.89 \frac{PGV}{PGA}$	$T_C = 5 \frac{PGV}{PGA}$	$T_C = \frac{S_{D1}}{S_{DS}}$
-	$T_D = 2\pi \frac{EPD}{EPV}$	$T_D = 2\pi \frac{\alpha_V PGD}{\alpha_A PGV} =$ $= 2\pi \frac{1.39 PGV}{1.65 PGA} = 5.29 \frac{PGD}{PGV}$	$T_D = 8 \frac{PGD}{PGV}$	-
1) $EPA = \frac{\overline{SA}_{0.1+0.5s}}{2.5}$ $EPV = \frac{\overline{SV}_{0.8+1.2s}}{2.5}$	2) $EPA = \frac{\max \overline{SA}_{0.4}}{2.5}$ $EPV = \frac{\max \overline{SV}_{0.4}}{2.5}$ $EPD = \frac{\max \overline{SD}_{0.4}}{2.5}$	PGA – peak ground acceleration PGV – peak ground velocity PGD – peak ground displacement		$S_{DS}$ – design spectral response acceleration parameter at short periods (0.2 s) given by code; $S_{D1}$ – design spectral response acceleration parameter at 1.0 s given by code;
EPA – effective peak acceleration EPV – effective peak velocity EPD – effective peak displacement				

1) Definitions based on a fixed period window for computing EPA (0.1 ÷ 0.5 s) and EPV (0.8 ÷ 1.2 s);

2) Definitions based on a mobile period window (of 0.4 s width) for getting maximum effective values.

### 3. STRONG MOTION DATASET ANALYSIS

In this study a dataset of 20 seismic records are used from earthquakes in: Chile, Greece, Iran, Italy, Japan, Mexico, Montenegro, New Zealand, Romania, Taiwan, Turkey and USA. The peak ground acceleration (PGA) of the earthquake records varies between 0.2g (Mexico-City SCT 1985 EW) and 0.9g (Naghan 1977 Long).

The strong ground motion records are bordered by the narrowest frequency band record - Mexico-City SCT 1985 ( $\varepsilon=0.99$ ) and by the broadest frequency band record – Naghan 1977 ( $\varepsilon=0.67$ ).

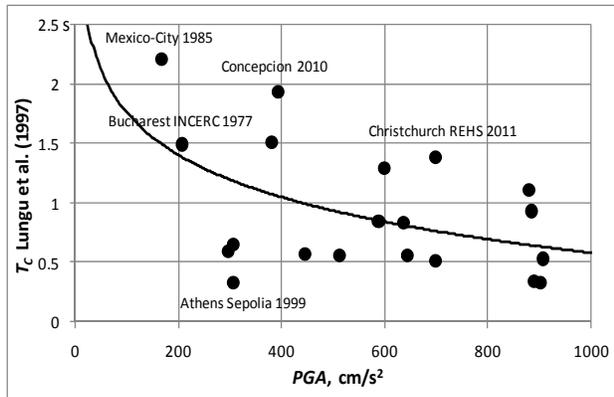


Figure 3.1. Control period,  $T_C$  versus PGA

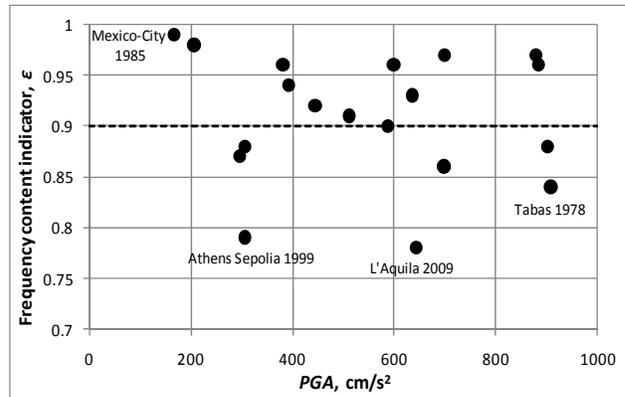


Figure 3.2. Dimensionless indicator,  $\varepsilon$  versus PGA

The distribution of the control period of response spectra  $T_C$ , stochastic dimensionless indicator  $\varepsilon$ , earthquake magnitude  $M_W$  and earthquake depth,  $h$  with PGA of the 20 analysed records is respectively given in Fig. 3.1, Fig. 3.2, Fig. 3.3 and Fig. 3.4.

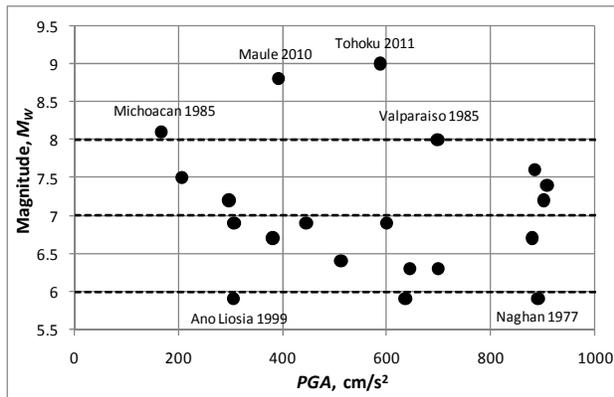


Figure 3.3. Event moment magnitude,  $M_W$  versus PGA

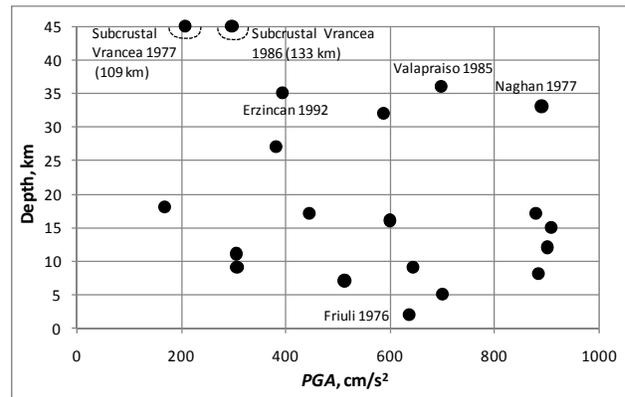


Figure 3.4. Earthquake depth,  $h$  versus PGA

The distribution of the number of analysed records with  $PGA$ ,  $M_W$ ,  $h$ , recording station epicentral distance and earthquake occurrence year are respectively given in Fig. 3.5, Fig. 3.6, Fig. 3.7, Fig. 3.8 and Fig. 3.9.

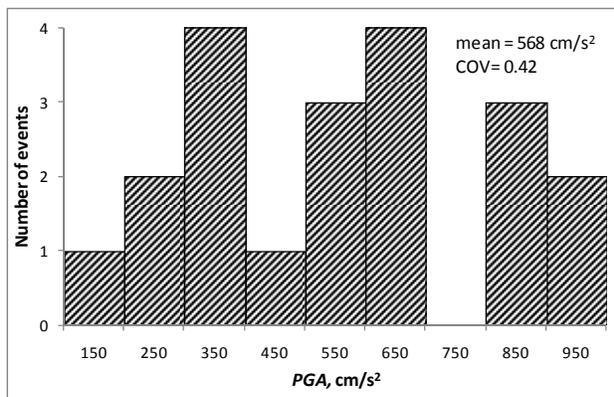


Figure 3.5. Frequency of recorded  $PGA$

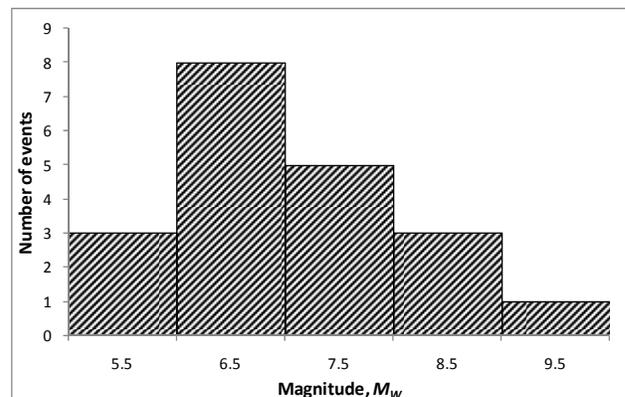
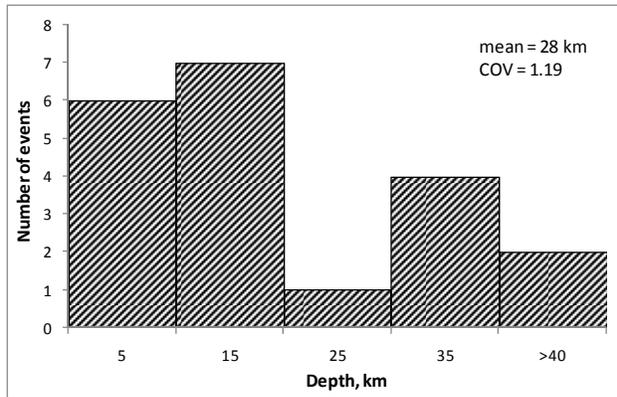
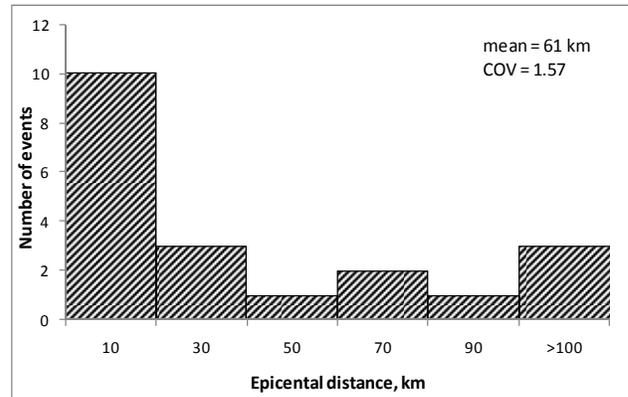


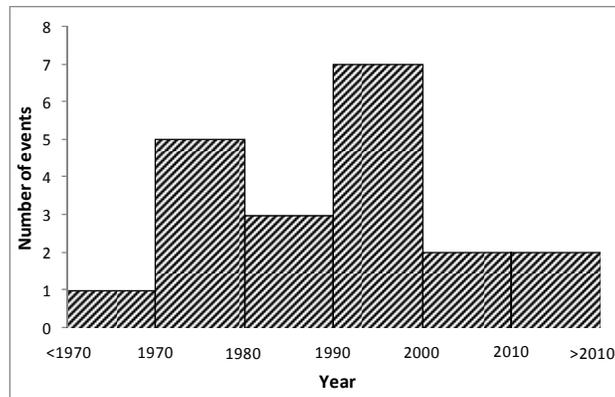
Figure 3.6. Frequency of event moment magnitude,  $M_W$



**Figure 3.7.** Frequency of earthquake depth, h



**Figure 3.8.** Frequency of recording station epicentral distance



**Figure 3.9.** Frequency of earthquake occurrence year

The values of the stochastic and of the deterministic frequency content indicators for the 20 strong ground motions selected in the dataset and computed using the definitions from Cap. 2 are shown in Table 3.1.

**Table 3.1.** Stochastic and deterministic indicators for analysis of the frequency content of selected ground motions

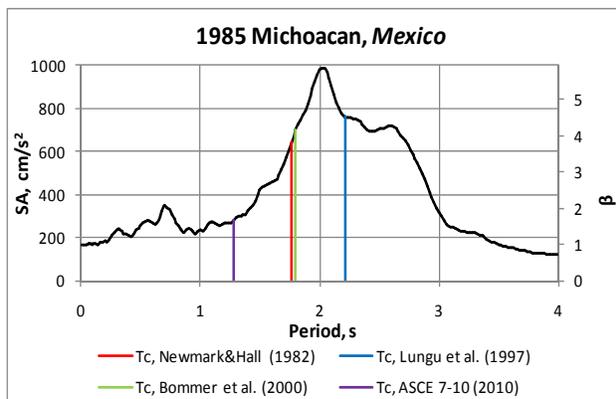
Earthquake record	Stochastic indicators			Deterministic indicators $T_C$ , s				
	Frequency content indicator, $\varepsilon$	$f_{10}$ , Hz	$f_{90}$ , Hz	Reference:				
				ATC 3-06 (1978)	Newmark & Hall (1982)	Lungu et al. (1997)	Boomer et al. (2000)	ASCE 7-10 (2010)
Naghan, <i>Naghan Long.</i>	0.67	1.42	12.52	0.32	0.31	0.34	0.32	0.23
L'Aquila, <i>Centro Valle EW</i>	0.78	1.60	9.73	0.49	0.31	0.56	0.31	0.40
Ano Liosia, <i>Athens Sepolia Transv.</i>	0.79	1.90	6.28	0.22	0.30	0.33	0.30	0.11
Tabas, <i>Tabas N74E</i>	0.84	0.91	7.94	0.49	0.45	0.53	0.46	0.28
Valparaiso, <i>Llolleo N10E</i>	0.86	1.38	7.27	0.51	0.28	0.51	0.29	0.36

**Table 3.1.**

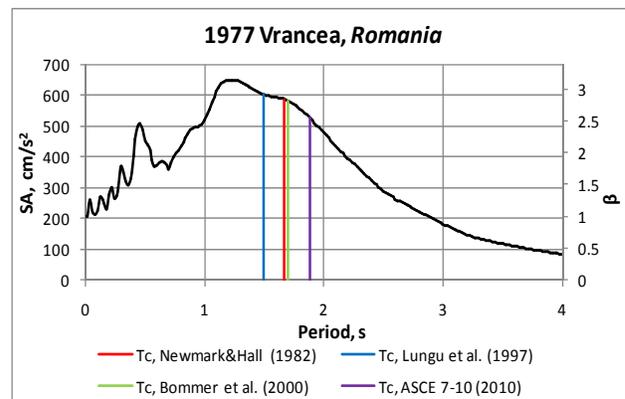
(continued)

Vrancea, <i>Petresti Focsani EW</i>	0.87	1.17	4.86	0.52	0.53	0.59	0.54	0.43
Duzce, <i>Lamont 375 NS</i>	0.88	2.41	8.76	0.17	0.21	0.33	0.21	0.09
Imperial Valley, <i>El Centro NS</i>	0.88	0.90	7.83	0.65	0.47	0.65	0.48	0.77
Tohoku, <i>Takahagi EW</i>	0.90	1.21	6.44	0.83	0.45	0.84	0.46	1.03
Aegion, <i>Aegion Transv.</i>	0.91	0.72	4.37	0.42	0.49	0.56	0.50	0.23
Montenegro, <i>Petrovac Hotel Oliva NS</i>	0.92	1.23	3.54	0.53	0.43	0.57	0.44	0.61
Friuli, <i>Gemona EW</i>	0.93	0.86	5.59	0.96	0.53	0.83	0.54	0.70
Maule, <i>Concepcion Long.</i>	0.94	0.47	4.32	0.53	0.84	1.93	0.86	0.39
Chi-Chi, <i>CHY080 NS</i>	0.96	0.81	2.09	1.28	0.57	0.93	0.58	1.37
Kobe, <i>Kobe Takatori NS</i>	0.96	0.42	2.44	1.02	1.04	1.29	1.06	0.77
Erzincan, <i>Erzincan N09E</i>	0.96	0.29	3.60	1.04	1.30	1.51	1.33	1.25
Northridge, <i>Sylmar Converter N142E</i>	0.97	0.52	2.99	1.19	0.57	1.11	0.58	1.02
Lytlington, <i>Christchurch REHS N92E</i>	0.97	0.41	2.14	1.67	0.60	1.38	0.62	2.10
Vrancea, <i>Bucharest INCERC NS</i>	0.98	0.34	2.10	1.21	1.67	1.49	1.70	1.88
Michoacan, <i>Mexico-City SCT EW</i>	0.99	0.35	0.58	0.81	1.76	2.21	1.80	1.28

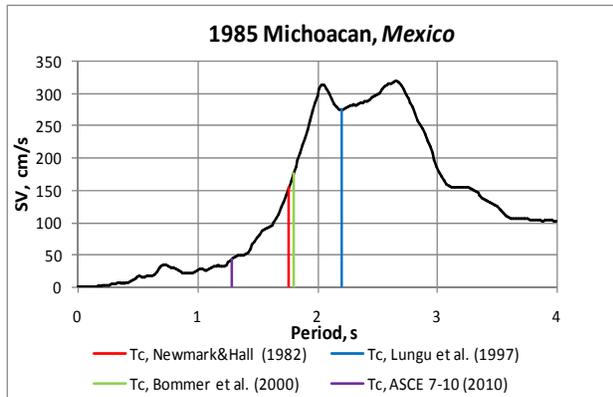
Absolute acceleration and dynamic amplification factor (DAF) response spectra for the two extreme narrow frequency bandwidth ground motions – Mexico-City, SCT, 1985, EW comp and Bucharest, INCERC, 1977, NS comp are plotted in Fig. 3.10 and Fig 3.11. The relative velocity response spectra are represented in Fig 3.12 and Fig 3.13.



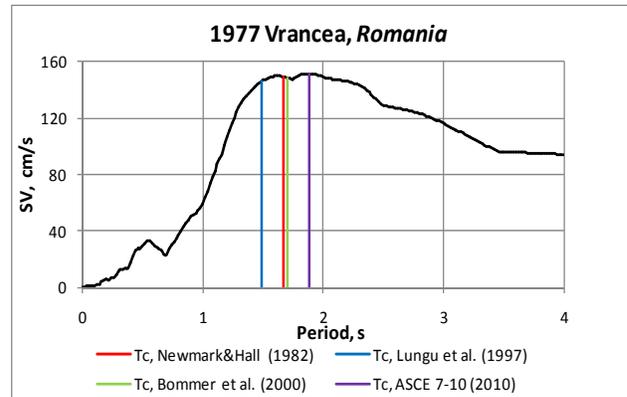
**Figure 3.10.** Acceleration response spectra and  $T_C$  definitions for Mexico-City, SCT, 1985, EW comp



**Figure 3.11.** Acceleration response spectra and  $T_C$  definitions for Bucharest, INCERC, 1977, NS comp

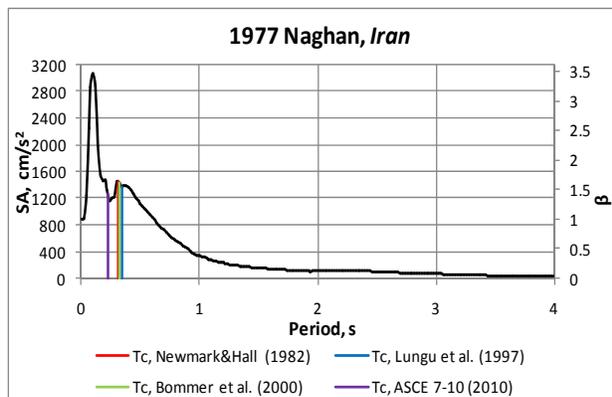


**Figure 3.12.** Velocity response spectrum and  $T_C$  definitions for Mexico-City, SCT, 1985, EW comp

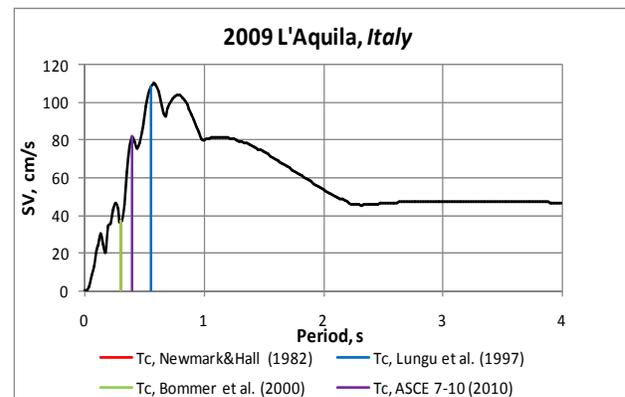


**Figure 3.13.** Velocity response spectrum and  $T_C$  definitions for Bucharest, INCERC, 1977, NS comp

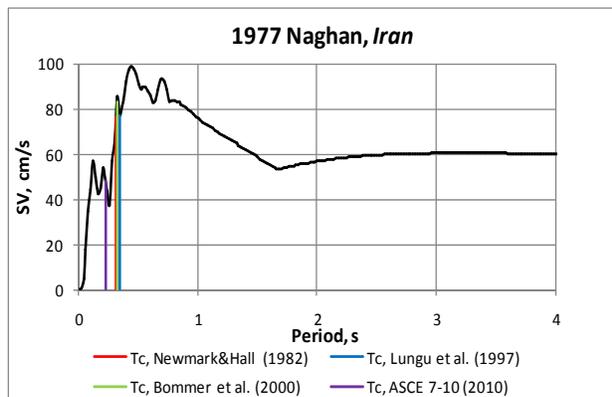
In Fig. 3.16 and Fig. 3.17 are shown the absolute acceleration and dynamic amplification factor (DAF) response spectra for two interesting wide frequency bandwidth ground motions – Naghan, 1977, Long comp. and L’Aquila, Centro Valle, 2009, EW comp. The relative velocity response spectra are represented in Fig. 3.18 and Fig. 3.19.



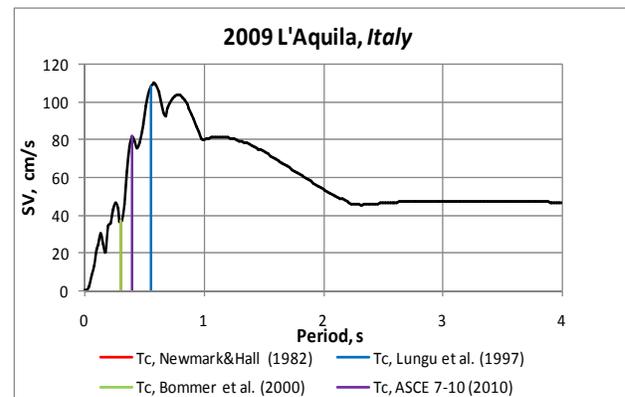
**Figure 3.14.** Acceleration response spectra and  $T_C$  definitions for Naghan, 1977, Long. comp



**Figure 3.15.** Acceleration response spectra and  $T_C$  definitions for L’Aquila Centro Valle 2009 EW comp



**Figure 3.16.** Velocity response spectrum and  $T_C$  definitions for Naghan, 1977, Long. comp



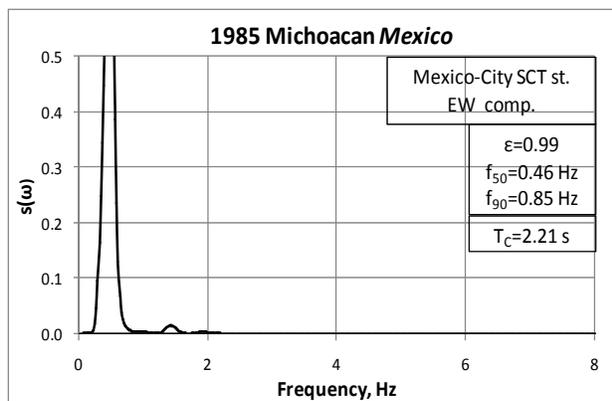
**Figure 3.17.** Velocity response spectrum and  $T_C$  definitions for L’Aquila Centro Valle 2009 EW comp

Table 3.1 and Fig. 3.10 ÷ 3.17 demonstrate the unexpected sensibility of the computed  $T_C$  values according to different authors' definitions, in spite of the fact that the  $T_C$  values are extremely important input parameters controlling structural design.

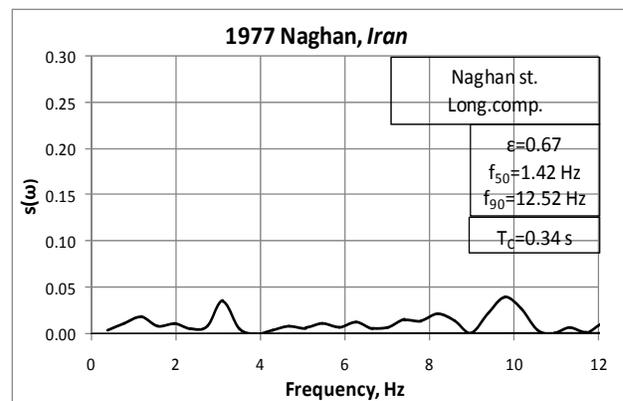
A surprisingly large variability of the values obtained for the control period  $T_C$  has been observed, especially for the narrow frequency bandwidth ground motions characterized by  $\varepsilon \geq 0.90$  and according to various definitions and authors.

The normalized power spectral density (PSD) functions of the narrowest frequency bandwidth ground motion and of the broadest frequency bandwidth ground motion from the dataset analyzed are displayed in Fig. 3.18 and Fig. 3.19.

One should note that earthquake engineering education might consider in parallel for structural analysis and design such opposite extremes of seismic input as well as its consequences on structural design.

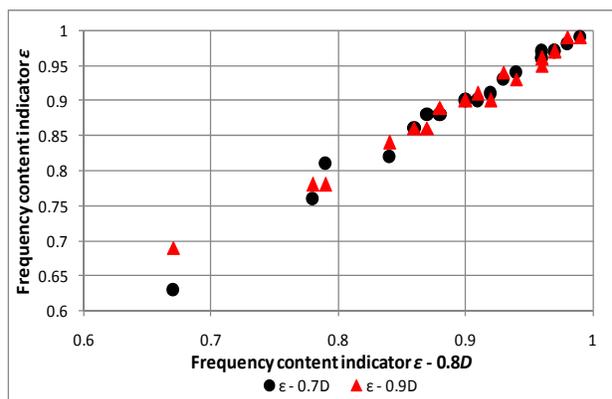


**Figure 3.18.** Normalized PSD for Mexico-City, SCT, 1985, EW comp

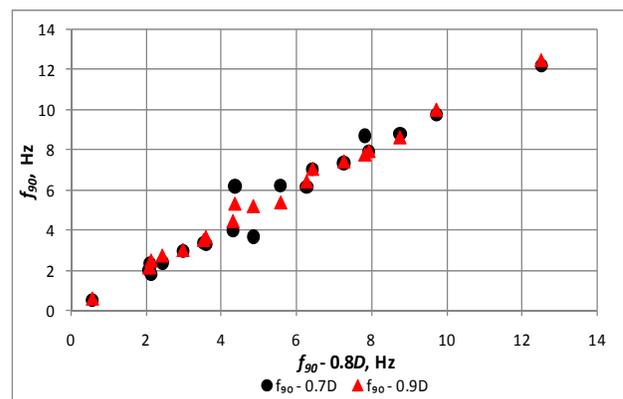


**Figure 3.19.** Normalized PSD for Naghan, 1977, Long comp

The influence of the strong motion duration on stochastic frequency content indicators is shown in Fig. 3.20 and Fig. 3.21. The two figures suggest that the influence is negligible.



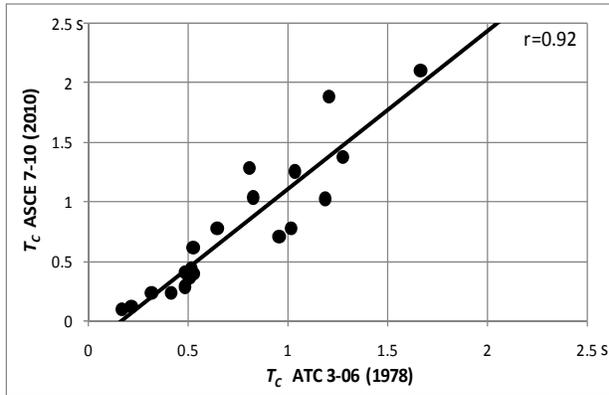
**Figure 3.20.** Influence of strong ground motion duration on  $\varepsilon$  dimensionless indicator



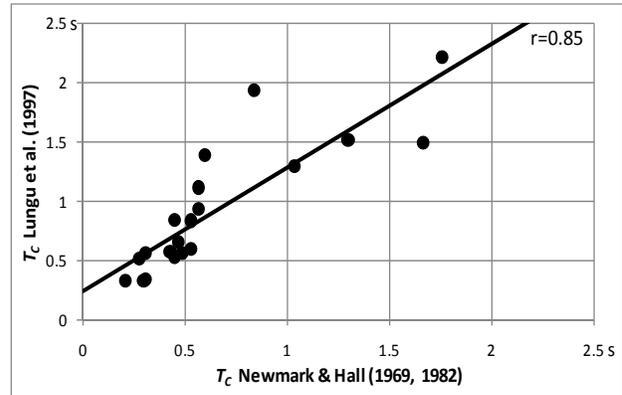
**Figure 3.21.** Influence of strong ground motion duration on  $f_{90}$  fractile frequency

#### 4. CORRELATIONS OF FREQUENCY CONTENT INDICATORS

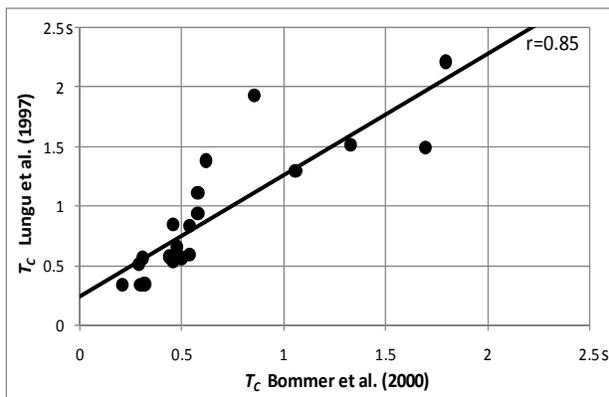
Several correlations between the control periods  $T_C$  determined according to the definitions given by various authors are plotted in Fig. 4.1, Fig. 4.2, Fig. 4.3 and Fig. 4.4.



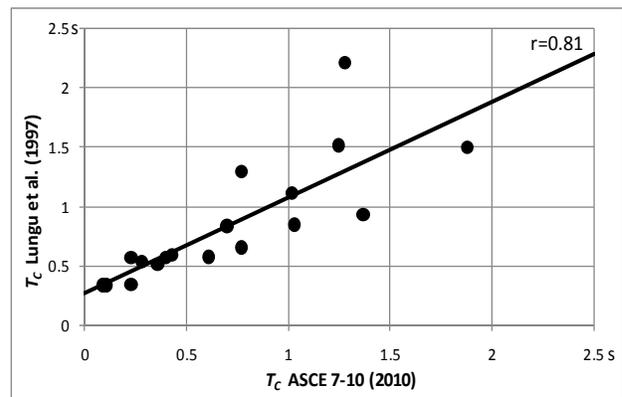
**Figure 4.1.**  $T_C$  ASCE 7-10 versus  $T_C$  ATC 3-06



**Figure 4.2.**  $T_C$  Lungu et al. versus  $T_C$  Newmark & Hall

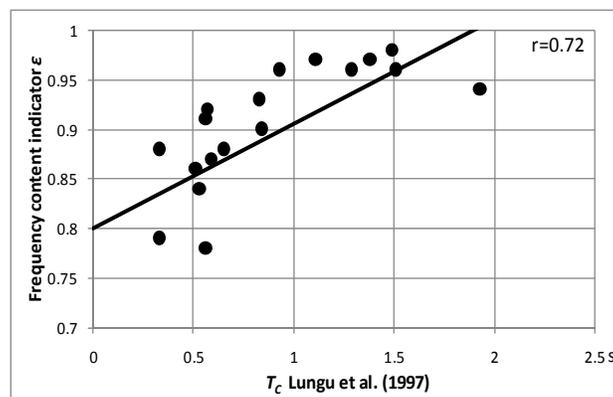


**Figure 4.3.**  $T_C$  Lungu et al. versus  $T_C$  Bommer et al.



**Figure 4.4.**  $T_C$  Lungu et al. versus  $T_C$  ASCE 7-10

The degree of reliability of the results in Fig. 4.1 ÷ 4.4 is controlled and explained by the value of the correlation coefficient  $r$ . Fig. 4.1 ÷ 4.4 may also suggest the definitions to be used in practical design.



**Figure 4.5.**  $\epsilon$  dimensionless indicator versus  $T_C$  Lungu et al.

Fig. 4.5 shows the fact that the increase in the value of the deterministic frequency content indicator  $T_C$  is accompanied by an increase of the value of the stochastic frequency content indicator  $\varepsilon$ .

## 5. CONCLUSIONS

- The values for the control period  $T_C$  given by Newmark & Hall (1982) or Bommer et al. (2000) and the values given by Lungu et al. (1997) are clearly well correlated;
- The mobile window procedure - Lungu et al. (1997) is considered the most adequate instrument for computing the values of the control period  $T_C$  of the response spectra;
- The values for the control period  $T_C$  based on two clearly different procedures for computing effective peak values (EPA, EPV, etc.) in Lungu et al. (1997) and ATC 3-06 (1978) are not well correlated;
- The values for the control period  $T_C$  given in ATC 3-06 (1978) and ASCE 7-10 (2010) produce similar results;
- The control period  $T_C$  of the response spectra and the frequency content indicator  $\varepsilon$  seem to be well correlated;
- The influence of the strong motion duration on the values of the probabilistic frequency indicators is negligible.

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