Accuracy of combination rules for MDOF and SDOF Systems

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SUMMARY:
The accuracy of the *30%* and the *SRSS* combination rules used to combine the effects of individual components of earthquakes is studied. Results indicate that for complex systems both rules may underestimate the axial load, but accurately estimate the base shear. The effect of individual components may be highly correlated for normal and principal components. The rules are not always inaccurate for large values of correlation coefficients of the individual effects, and small values of such coefficients are not always related to an accurate estimation of the response. Only for perfectly uncorrelated harmonic excitations and elastic analysis of SDOF systems, the individual effects are uncorrelated and the rules accurately estimate the combined response. The level of underestimation or overestimation varies with the degree of correlation of the components, the type of structural system, the response parameter, and the level of structural deformation.

Keywords: Combination rules, nonlinear analysis, MDOF system, principal components, correlation coefficients

1. INTRODUCTION AND OBJECTIVES

For seismic analysis purposes, energy released during an earthquake is represented in the form of two horizontal and one vertical translational acceleration time histories. For far-source ground motions, the effect of the vertical component is usually smaller than those of the horizontal components and is consequently neglected. Additional bases to neglect the vertical component effect are that building designs allow for gravity loads, which provides for a high factor of safety in the vertical direction (Newmark and Hall 1982). Thus, when a structure is analyzed, two horizontal recorded components are generally applied along their two major axes and then the individual effects are combined in many different ways. This concept has been implemented in many codes (IBC 2003, RCDF 2004). The commonly used procedures are the 30 percent (*30%*) and the Square Root of the Sum of the Squares (*SRSS*) combination rules. The codes, however, do not explicitly state the applicability of these rules. It is not specified the type of structures (simple or complex systems) to be considered nor if the rules can be applied to both, elastic and inelastic behavior. It is not specified either if the individual responses produced by each component should be collinear (axial load in columns) or non-collinear (base shear). The rules implicitly assume that the components and their corresponding effects are uncorrelated. The accuracy of these combination rules, essentially developed for linear modal analysis procedures is studied in this paper. Some of the abovementioned issues are explicitly considered. The effect of the correlation of the components is considered.

The ways of combining the individual effects of the earthquake components have been of interest to the civil engineering profession. Penzien and Watabe (1975) stated that the three components of an earthquake are uncorrelated along a set of axes generally denoted as principal axes. The major principal axis is horizontal and directed toward the epicenter, the intermediate axis is horizontal and perpendicular to the orientation of the major component, and the minor principal axis is vertical. The critical response could be obtained when these principal components are applied. Rosenblueth (1980) stated “lack of correlation of the principal accelerograms insures that responses are also uncorrelated”. Smeby and Der Kiureghian (1985) observed that, for response spectra analysis of linear structures, when the two horizontal principal components are not along the structural principal axes, the effect of correlation is small and that if the two horizontal components have identical or nearly identical intensities, then the effect of correlation disappears. Newmark (1975) and Rosenblueth and Contreras (1977) proposed the *Percentage Rule* to approximate the combined response as the sum of the 100% of the response resulting from one component and some percentage (λ) of the responses resulting from the other two components. To combine the two horizontal components, Newmark (1975) suggested λ to be 40% and Rosenblueth and Contreras (1977) suggested λ to be 30%.

Many other studies were reported to combine the seismic responses due to two or three components (Wilson et al 1995, Lopez et al 2006, Beyer and Bommer 2007, Rigato and Medina 2007 and Bisadi and Head 2010). In spite of the important contributions of these studies, most of them were limited to elastic analysis applied to SDOF systems or simplified plane concrete frames with a few stories connected by rigid diaphragms. They did not consider the inelastic behavior of the structural elements existing in actual structural systems and the appropriate energy dissipation mechanisms. Reyes-Salazar and Haldar (2001) found that strong-column weak-beam moment resisting steel frames are very efficient in dissipating earthquake-induced energy and that the dissipated energy has an important effect on the structural response. More recently, Reyes-Salazar et al (2004, 2008), by using nonlinear time history analysis of complex multi-degree of freedom (MDOF) systems, observed that both the *30%* and the *SRSS* rules could underestimate the combined response and that the energy dissipation mechanisms should be considered as accurately as possible. However, these studies did not consider realistic structural systems and did not estimate the effect of correlation of the earthquake components on the accuracy of the rules.

The above discussions clearly identify several issues that need our attention. The specific issues addressed in this study are: a) the accuracy of the commonly used combination rules for complex MDOF systems for elastic and inelastic behavior and for collinear en non-collinear response parameters and b) the accuracy of the rules for SDOF systems. To comprehensively study these issues, the seismic responses of some structural models are estimated as accurately as possible by using a sophisticated three-dimensional time history analysis. The degree of correlation of the seismic components and their effects for the normally recorded and uncorrelated principal components are considered. The responses of steel buildings with moment resisting steel frames (MRSFs) are specifically studied

2. MATHEMATICAL FORMULATION

To satisfy the objectives of the study, the seismic responses of some steel building models are evaluated as accurately as possible using an efficient assumed stress-based finite element algorithm developed by the authors and their associates (Gao and Haldar 1995, Reyes-Salazar 1997). The procedure estimates nonlinear seismic responses in time domain considering material and geometry nonlinearities. In this approach, an explicit form of the tangent stiffness matrix is derived without any numerical integration. Fewer elements can be used in describing a large deformation configuration without sacrificing any accuracy and the material and geometric nonlinearities can be incorporated without losing its basic simplicity. It gives very accurate results and is very efficient compared to the commonly used displacement-based approaches. The procedure and the algorithm, implemented in a computer program, have been extensively verified using available theoretical and experimental results (Reyes-Salazar and Haldar 2001). The development of the theory of this approach is out of the scope of this study.

3. STRUCTURAL MODELS

3.1. Complex MDOF systems

As part of the SAC steel project (FEMA, 2000) three consulting firms were commissioned to perform the design of several model buildings. They were 3-, 9- and 20- story buildings which were designed according to the code requirements for the following three cities: Los Angeles (UBC, 1994), Seattle

(UBC, 1994) and Boston (BOCA, 1993). The 3- and 9- story buildings, representing Los Angeles area and the Pre-Northridge Designs, are considered in this study to address all the issues raised earlier. They will be denoted hereafter as Models 1 and 2, respectively. The elevations, plans models showing the location of moment resisting frames (continuous lines), and the particular elements considered in the study, are showed in Fig. 1. Additional information for the models can be obtained from the SAC steel project reports (FEMA, 2000). In this study, the frames are modeled as MDOF systems. Each column is represented by one element and each girder of the perimeter MRFs is represented by two elements, having a node at the mid-span. Each node is considered to have six degrees of freedom. The total number of degrees of freedom is 846 and 3408, for Models 1 and 2, respectively. The models are excited by twenty recorded earthquake motion in time domain, recorded at the following stations:
Paraíso, Mammoth H.S., GymConvict Creek, Infiernillo N-120, La Unión, Relaciones Ext. 1, Relaciones Ext. 2, Long Valley Dam, K2-02, Redwood City, MT:Kalispell, Villita, Hall Valley 1, Hall Valley 2, K2-04, Dauville F.S. CA, Pleasant Hill F.S. 1, Pleasan Hill F.S. 2, Valdez City Hall and Hollister City Hall.

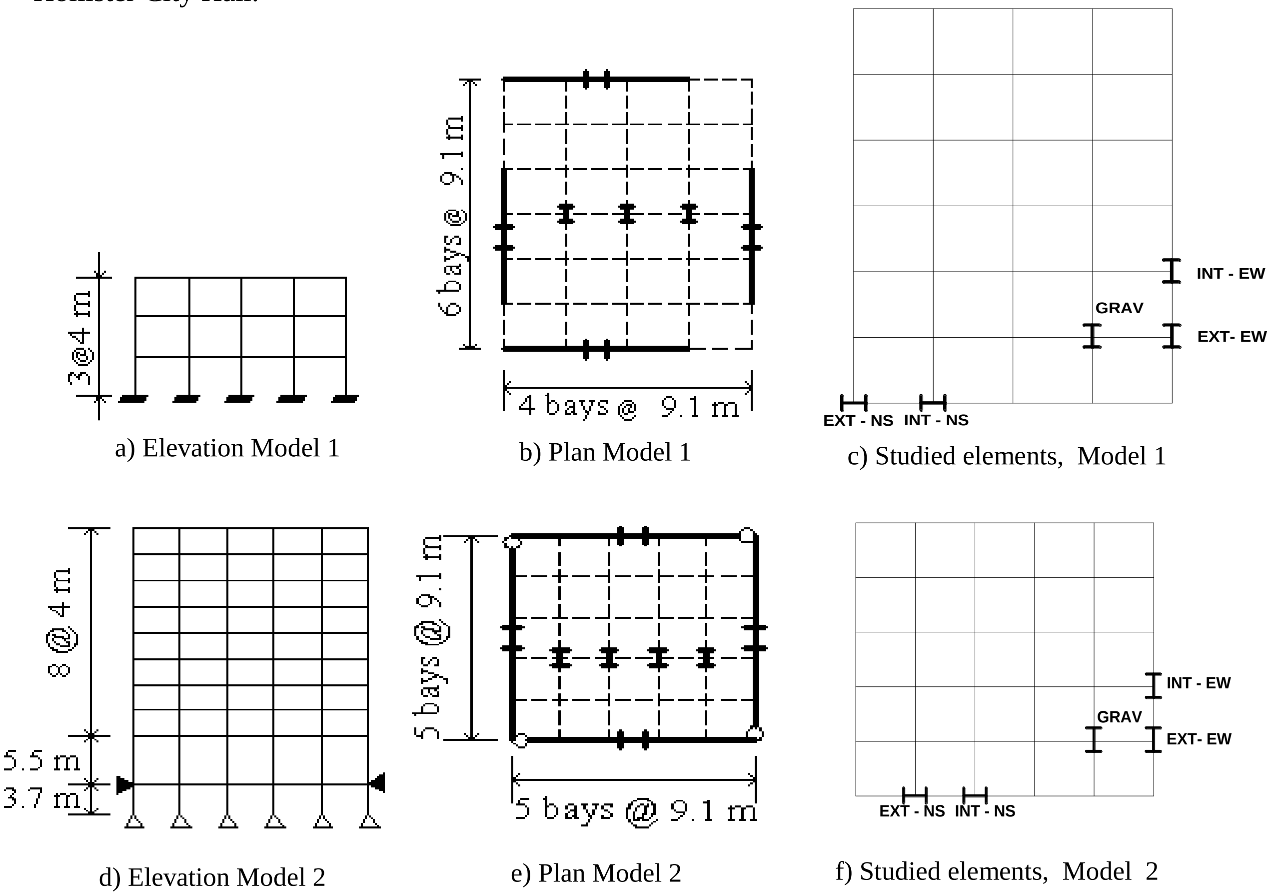


Figure 1. Elevation, plan and element location for Models 1 and 2

3.2. SDOF systems.

The accuracy of the rules is also studied for *equivalent* SDOF systems. One equivalent SDOF model is considered for each MDOF system. These systems have a degree of freedom in each horizontal direction. They will be denoted hereafter as Model 1E and Model 2E. The elevation and plan of these systems are shown in Fig. 2. The weight of an equivalent SDOF system is the same as the total weight of its corresponding MDOF system and its lateral stiffness is selected in such a way that its natural period is the same as the fundamental natural period of its corresponding MDOF system. The damping ratio and the yield strength are selected to be the same for both structural representations. It must be noted that in a strict sense, the simpler models are not the typical SDOF systems studied in the structural dynamics textbooks since axial forces can be developed in the columns under the action of horizontal excitations

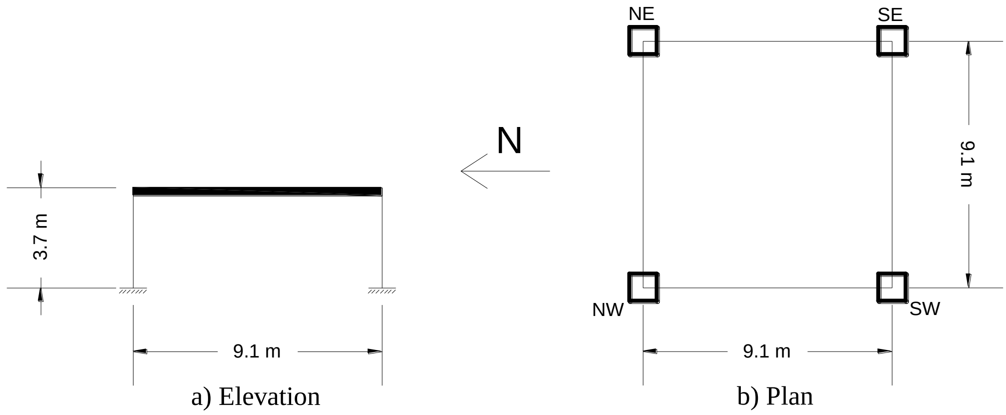


Figura 2. Elevation and plan of the equivalent SDOF models (Models 1E and 2E)

4. COMBINATION RULES

The combination rules are formally defined in this part of the paper. The combination of the effects of the two horizontal components is specifically addressed. For the ease of discussion, R_X will represent hereafter the maximum absolute load effect at a particular location when the structure is excited by the horizontal *X* component of a given earthquake. Similarly, R_Y will denote the corresponding maximum absolute load effect when the structure is excited by the horizontal *Y* component of the earthquake. The load effects produced by each component can be calculated using various methods including the equivalent lateral load procedure, modal analysis, and time history analysis. For time history analysis, elastic and inelastic analysis methods can be used to evaluate the load effects. Using the *Percentage* rule, the combined effect considering the two components can be calculated as:

$$R_{C1} = R_X + \lambda R_Y \quad \text{or} \quad R_{C1} = \lambda R_X + R_Y \tag{1}$$

The *30%* combination rule is represented by λ = 0.3. According to the *SRSS* rule, the combined response is given by

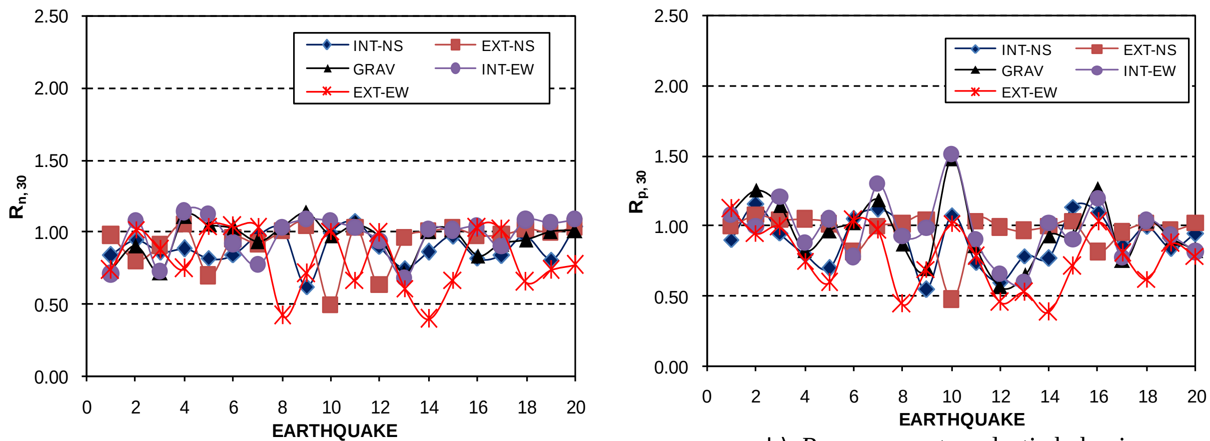
$$R_{C2} = \sqrt{{R_X}^2 + {R_Y}^2} \tag{2}$$

5. APLICABILITY OF THE RULES TO MDOF SYSTEMS

Recorded horizontal time histories will be denoted as normal components. When they are transformed to uncorrelated components following the procedure suggested by Penzien and Watabe (1975) and Clough and Penzien (1993) they will be denoted, as stated earlier, as principal components**.** For any response parameter (axial loads or base shear), the *reference response* (maximum response) for normal components is denoted hereafter as R_n. Similarly, the *reference response* for the principal components is denoted as R_p.

5.1. Accuracy of the 30% and SRSS combination rules

The axial load values, obtained according to the 30% rule, are normalized with respect to the reference responses (R_n or R_p) defined earlier; the resulting random variables are denoted as $R_{n,30}$ and $R_{p,30}$, for normal and principal components, respectively. Typical values of $R_{n,\,30}$ and $R_{p,30}$ are presented in Figs 3a and 3b for elastic behavior and Model 1. It is observed that these parameters vary significantly with the particular earthquake being considered and the locations of the elements without showing any trend. For most of the cases the combined response is underestimated, values smaller than 50% are observed in many cases even for principal components. For the SRSS rule, the corresponding random variables are $R_{n,SRSS}$ and $R_{p,SRSS}$ for normal and principal components, respectively. The major observations made for the 30% rule also apply to this rule. The corresponding statistics are summarized in Table 1. It is observed that both rules underestimate the axial load by about 10% for both, normal and principal components and that the uncertainty (COV) in the underestimation is about 20%. These results indicate that for complex MDOF systems, there is a certain degree of correlation between the effects of individual components of earthquakes, even for the case of uncorrelated components. The statistics for base shear are summarized in Table 1 too. The results indicate that, unlike the case of axial load, both rules reasonably overestimate the combined base shear. The overestimation is about 10% and is observed to be essentially the same for normal and principal components. The uncertainty in the estimation is much larger for axial load than for base shear. The accuracy of the combination rules in the estimation of the combined axial load and base shear for inelastic structural behavior is also studied. The used earthquakes were scaled in such way that the average interstory drift was about 1.8%. Similar plots and tables to those of elastic behavior are also developed but are not shown. All the observation made for elastic behavior essentially remain the same for inelastic behavior. The only additional observation is that the uncertainty in the prediction significantly increases for axial load.



a) $R_{n,30}$ parameter, elastic behavior

b) $R_{p,30}$ parameter, elastic behavior

Figure 3. Accuracy of the 30% rule for MDOF systems and earthquake loading, Model 1

Table 1. Statistics for $R_{n,30}$, $R_{p,30}$, $R_{n,SRSS}$ and $R_{p,SRSS}$ for MDOF systems and earthquake loading, axial load and base shear, elastic behavior

MODEL (1)	COLUMN LOCATION (2)		30% RULE Normal $R_{n,30}$ Mean (3)	COV (4)	30% RULE Principal $R_{p,30}$ Mean (5)	COV (6)	SRSS RULE Normal $R_{n,SRSS}$ Mean (7)	COV (8)	SRSS RULE Principal $R_{p,SRSS}$ Mean (9)	COV (10)	Sample size (11)
1	AXIAL LOAD	INT-NS	0.90	0.12	0.92	0.21	0.89	0.11	0.91	0.19	40
		EXT-NS	0.93	0.16	0.97	0.13	0.90	0.17	0.94	0.13	40
		GRAV	0.96	0.13	0.97	0.24	0.96	0.14	0.96	0.23	40
		INT-EW	0.98	0.14	0.98	0.22	0.97	0.14	0.97	0.22	40
		EXT-EW	0.81	0.26	0.78	0.28	0.78	0.26	0.76	0.28	40
		ALL ELEMENTS	0.91	0.18	0.92	0.23	0.90	0.18	0.91	0.22	200
	BASE SHEAR		1.07	0.07	1.07	0.09	1.09	0.07	1.09	0.09	40
2	AXIAL LOAD	EXT-NS	0.91	0.22	0.92	0.16	0.88	0.22	0.90	0.17	40
		INT-NS	0.95	0.09	0.99	0.12	0.94	0.11	0.97	0.12	40
		GRAV	0.97	0.11	0.99	0.12	0.95	0.12	0.97	0.11	40
		INT-EW	0.77	0.25	0.79	0.24	0.75	0.24	0.78	0.23	40
		EXT-EW	0.95	0.14	0.99	0.13	0.94	0.14	0.97	0.12	40
		ALL ELEMENTS	0.91	0.19	0.94	0.17	0.89	0.18	0.92	0.17	200
	BASE SHEAR		1.09	0.06	1.11	0.06	1.11	0.08	1.12	0.08	40

5.2. Correlation between individual effects.

The basic assumption of the *SRSS* rule is that there is no correlation between the horizontal components. It is implicitly assumed that if there is no correlation between the accelerograms, the corresponding effects will also be uncorrelated. The actual degree of correlation between the individual effects of the horizontal components and the effect of correlation on the accuracy of the rules are discussed in this section of the paper. The correlation coefficients (ρ) are estimated for Models 1 and 2, for normal and principal components, for elastic and inelastic behavior and for collinear (axial load) and non-collinear (base shear) response parameters. However, only a few results in terms of axial loads on some columns and total base shear of Model 2 are presented. The coefficients of correlation between the normal horizontal accelerograms (ρ_{NO}) are given in Column 2 of Table 2. It is observed that normally recorded components may be highly correlated. The corresponding coefficients for the principal accelerograms are obviously zero. The correlation coefficients of the individual effects are given in Columns (3) through (14). It is shown that the correlation values significantly vary from one earthquake to another and from one element to another. Most of the values can be considered negligible (smaller than 0.25). For many cases however, the correlation is significant. Values of ρ larger than 0.5 are observed in many cases. Results indicate that the effects of individual uncorrelated components (principal components) may be highly uncorrelated and that the rules are not always inaccurate in the estimation of the combined response for large values of ρ. On the other hand, small values of the coefficients are not always related to an accurate estimation of the combined response. The implication of this is that there may be other factors that influence the accuracy of the combination rules. It is discussed further in subsequent sections of the paper.

Table 2. Correlation coefficients (**ρ**) of the effect of individual components, MDOF systems and earthquake loading, axial load, Model 2

EARTH	ρ_{NO}	NORMAL COMPONENTS						PRINCIPAL COMPONENTS					
		ELASTIC			INELASTIC			ELASTIC			INELASTIC		
		EXT-NS	INT-NS	SHEAR	EXT-NS	INT-NS	SHEAR	EXT-NS	INT-NS	SHEAR	EXT-NS	INT-NS	SHEAR
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	0.23	-0.74	0.72	0.73	-0.71	-0.69	0.72	0.84	0.84	0.88	0.88	0.90	0.87
2	-0.17	-0.05	0.42	0.50	-0.50	-0.21	0.38	0.17	0.23	0.39	0.35	0.03	0.36
3	0.32	0.08	0.54	0.59	-0.10	-0.21	0.53	-0.19	0.30	0.35	-0.17	0.33	0.50
4	-0.15	0.29	0.11	0.12	0.44	0.04	0.14	0.28	0.10	0.10	0.44	0.08	0.12
5	-0.23	-0.33	0.36	0.35	-0.68	-0.62	0.42	0.12	-0.10	-0.08	-0.53	-0.20	-0.07
6	0.17	0.21	0.37	0.50	0.59	0.39	0.45	0.15	0.24	0.25	-0.23	0.20	0.28
7	0.18	0.18	0.25	0.48	0.26	0.07	0.44	-0.29	0.66	0.68	-0.76	-0.07	0.70
8	0.11	-0.01	0.07	0.26	-0.07	0.26	0.29	-0.01	-0.08	-0.09	0.03	0.24	-0.07
9	0.13	0.38	-0.11	-0.08	0.39	0.16	-0.13	0.24	-0.19	-0.16	0.23	0.06	-0.18
10	0.13	0.16	0.27	0.29	-0.25	0.12	0.27	0.28	0.31	0.32	-0.24	0.05	0.27
11	-0.33	0.09	0.12	0.10	0.10	0.04	0.13	-0.02	-0.02	0.00	0.07	-0.01	-0.01
12	-0.14	-0.07	0.06	0.17	0.40	0.04	0.14	-0.04	-0.12	-0.05	-0.40	0.07	0.00
13	0.11	0.38	0.28	0.46	0.64	0.00	0.38	0.38	0.15	0.36	0.63	-0.02	0.29
14	0.15	-0.01	0.11	0.14	-0.20	-0.23	0.17	-0.11	0.06	0.16	-0.55	-0.20	0.17
15	0.19	0.35	0.71	0.64	0.80	0.33	0.56	0.29	0.51	0.47	0.70	0.18	0.38
16	0.13	0.01	-0.14	-0.12	0.35	-0.10	-0.13	-0.05	-0.11	-0.03	0.07	-0.04	-0.10
17	-0.13	-0.10	0.02	0.07	0.03	0.08	0.10	-0.01	0.28	0.28	-0.09	-0.06	0.32
18	-0.16	0.09	0.13	0.20	0.55	0.07	0.15	0.14	0.19	0.25	0.53	-0.04	0.20
19	0.13	-0.40	0.07	0.07	-0.43	-0.15	0.06	-0.19	-0.03	-0.07	-0.20	-0.03	-0.08
20	0.18	-0.03	-0.07	-0.06	-0.01	0.05	0.72	-0.04	-0.04	-0.02	-0.10	0.00	0.01

6. ACCURACY OF THE RULES FOR SIMPLER SYSTEMS AND LOADING CONDITIONS

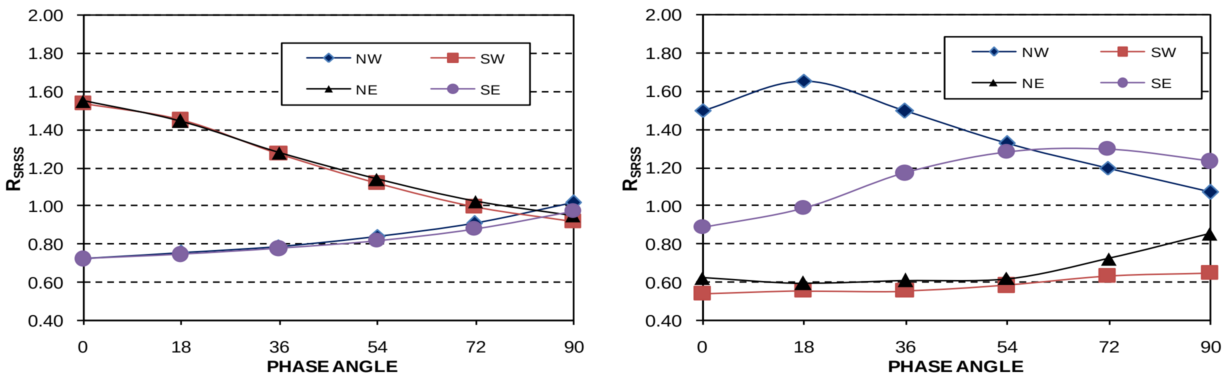
Initially, the *equivalent* SDOF systems defined earlier in Section 3, under the action of harmonic acceleration of the base, are considered. Then, the same SDOF systems are assumed to be acted upon earthquake excitations. Finally, MDOF systems and harmonic excitation are considered.

6.1. SDOF systems and harmonic loading

The accuracy of the rules and the correlation coefficients of the effects of the horizontal components for the *equivalent* SDOF systems subjected to a harmonic acceleration of the base are discussed in this section of the paper. The base accelerations in the *N-S* and *E-W* structural directions are

$$P_X(t) = P_0 \sin \omega t \quad and \quad P_Y(t) = P_0 \sin\,(\omega t + \phi) \tag{3}$$

respectively, where P_0 and ω are the amplitude and the frequency of the harmonic acceleration which are assumed to be 200 mm/sec^2 and 20 rad/sec, respectively. ϕ is the phase angle between the orthogonal horizontal accelerations which defines the degree of correlation of the harmonic components. $\phi = 0^0$ and 90^0 correspond to totally correlated and uncorrelated components, respectively. The R_{30} and R_{SRSS} parameters are used to estimate the accuracy rules for this case. They are essentially the same as $R_{n,30}$ and $R_{n,SRSS}$, but now harmonic loading are used instead. The results for axial loads in the columns of Model 1E are presented in Fig. 4a for the SRSS rule and elastic behavior, the results for the 30% rules are quite similar. It is observed, in general, that if $\phi \leq 72^0$, the rule may underestimate or overestimate the combined response. The level of underestimation or overestimation monotonically increases as the values of the phase angle decrease (increasing correlation). However, the rules accurately estimate the combined axial load for all the columns when the phase angle is 90^0, it is when the horizontal accelerations are totally uncorrelated. The results for the SRSS rule and inelastic behavior are shown in Fig. 4b. Unlike the case of elastic behavior, the values of R_{SRSS} don´t monotonically tend to unity as ϕ varies from 0 to 90^0. It indicates that the elastic response of structures subjected to dynamic loading may be quite different than that of the inelastic response. Even for uncorrelated components there is an important level of underestimation or overestimation. Plots for base shear were also developed but are not shown. However, it is shown that for elastic behavior both rules reasonable overestimate the combined response for both rules and all values of ϕ, the level of overestimation ranges from 5 to 15%. For the case of inelastic behavior the base shear is slightly underestimated (by about 5%) particularly for small values of ϕ.



a) R_{SRSS} parameter, elastic behavior

b) R_{SRSS} parameter, inelastic behavior

Figure 4. Accuracy of the rules for SDOF systems and harmonic loading, Model 1E

Plots for the R_{30} and R_{SRSS}, parameters, for axial load and base shear, are also estimated for Model 2E but the results are not showed. The main observations made for Model 1E also apply to Model 2E. The only differences that can be mentioned are that the values of the underestimation or overestimation for axial load are smaller for Model 2E, and that for base shear, unlike the case of Model 1E, it is reasonably overestimated for all values of ϕ, for elastic and inelastic behavior.

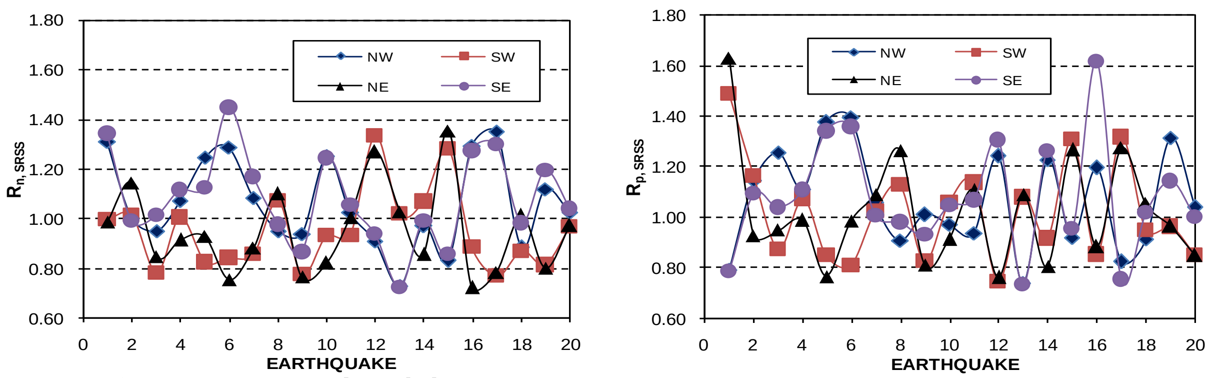
The phase angle (ϕ), correlation coefficients of the harmonic components (ρ_{COMP}), and correlation coefficients of their individual effects are given in Table 3 only for axial loads on columns of Model 1E, for elastic and inelastic behavior. As expected, for this simple loading and structural system, the correlations of the individual effects decrease as the correlation of the horizontal harmonic excitation decreases. The base shear follows a similar trend. The corresponding results for Model 2E were also estimated but are not given. The major conclusions, however, are the same than those of Model 1E.

Table 3. Correlation coefficients (**ρ**) of the effect of individual components, harmonic loading, Model 1E axial load

φ (1)	ρ_{COMP} (2)	ELASTIC				INELASTIC			
		NW (3)	SW (4)	NE (5)	SE (6)	NW (7)	SW (8)	NE (9)	SE (10)
0^0	0.99	-0.97	-0.97	0.97	1.00	-0.74	0.74	-0.86	0.86
18^0	0.91	-0.91	-0.91	0.91	0.95	-0.74	0.74	-0.85	0.85
36^0	0.77	-0.77	-0.77	0.77	0.81	-0.66	0.66	-0.76	0.76
54^0	0.55	-0.55	-0.55	0.55	0.59	-0.53	0.53	-0.59	0.59
72^0	0.27	-0.27	-0.27	0.27	0.31	-0.33	0.33	-0.36	0.36
90^0	-0.01	0.03	0.03	-0.03	0.02	-0.10	0.10	-0.11	0.11

6.2 SDOF systems and earthquake loading

The $R_{p,30}$, $R_{p,SRSS}$, $R_{n,30}$ and $R_{n\ SRSS}$ parameters are used to represent the accuracy of the rules. The results for the SRSS rule and axial loads on columns of Model 1E are given in Fig. 5 for elastic behavior. The results for inelastic behavior are quite similar. As for the MDOF systems and earthquake loading case, the values of $R_{n,SRSS}$ and $R_{p,SRSS}$ vary for one earthquake to another and from one column to another. Unlike the case of MDOF systems both rules on an average basis accurately estimate the combined response. Similar plots are also developed for the 30%, the major observations made for the SRSS rules apply to this case. Results in terms of base shear are also estimated but are not shown either. The only additional observation that can be made is that the level of overestimation is slightly larger for SDOF systems. Model 2E is also studied but, the major conclusions are essentially the same than that of Model 1E. However, they are not shown. From the statistics of $R_{p,30}$, $R_{p\ SRSS}$, $R_{n,30}$, and $R_{n\ SRSS}$, it is observed that, as stated earlier for individual plots, on an average basis, both rules reasonable overestimate the combined response for both, axial loads and base shear. The level of overestimation is, in general, larger for base shear than for axial load and the uncertainty in the estimation is much larger for axial load. For the case of axial load, the overestimation in terms of mean values is larger for principal than for normal components but, for total base shear, it is quite similar for both types of components. The uncertainty in the estimation is similar for the 30% and the SRSS rules but can be quite different for normal and principal components.



a) $R_{n},_{SRSS}$ parameter, elastic behavior

b) $R_{p},_{SRSS}$ parameter, elastic behavior

Figure 5. Accuracy of SRSS rule for SDOF systems and earthquake loading, Model 1E

The correlation coefficients (ρ) for both, axial load and total base shear are discussed next. Only the *NW* and *SW* columns and base shear of Model 2 are considered. The results are given in Table 4. Results indicate that, as for the case of MDOF systems, the ρ values are significant in many of the cases even for principal components. Thus, even for SDOF systems, if the horizontal accelerograms are uncorrelated it does not necessarily imply that their corresponding effects will also be uncorrelated.

Table 4. Correlation coefficients (**ρ**) of the effect of individual components, SDOF systems and earthquake loading, Model 2, axial load, inelastic behavior

EARTH	ρ_{NO} (2)	NORMAL COMPONENTS						PRINCIPAL COMPONENTS					
		ELASTIC			INELASTIC			ELASTIC			INELASTIC		
		NW (3)	SW (4)	SHEAR (5)	NW (6)	SW (7)	SHEAR (8)	NW (9)	SW (10)	SHEAR (11)	NW (12)	NW (13)	SHEAR (14)
1	0.23	-0.70	0.70	0.71	-0.71	0.71	0.72	0.90	-0.90	0.90	0.85	-0.85	0.84
2	-0.17	-0.11	0.10	0.31	-0.01	0.01	0.28	-0.06	0.09	0.33	-0.04	0.04	0.33
3	0.32	-0.30	0.33	0.38	-0.17	0.17	0.39	-0.69	0.72	0.51	-0.70	0.70	0.49
4	-0.15	0.50	-0.53	-0.12	0.50	-0.50	-0.10	0.38	-0.42	-0.34	0.37	-0.37	-0.35
5	-0.23	-0.76	0.76	0.44	-0.76	0.76	0.44	-0.65	0.65	-0.06	-0.57	0.57	-0.11
6	0.17	0.59	-0.59	0.61	0.57	-0.57	0.58	-0.19	0.16	0.25	-0.05	0.05	0.21
7	0.18	0.35	-0.36	0.43	0.36	-0.36	0.43	-0.79	0.81	0.74	-0.77	0.77	0.69
8	0.11	-0.20	0.20	0.41	-0.22	0.22	0.42	-0.05	0.04	0.00	-0.02	0.01	-0.01
9	0.13	0.25	-0.26	-0.11	0.26	-0.26	-0.10	0.09	-0.10	-0.07	0.10	-0.10	-0.05
10	0.13	-0.49	0.51	0.44	-0.45	0.45	0.47	-0.57	0.58	0.59	-0.60	0.59	0.63
11	-0.33	0.09	-0.09	0.10	0.11	-0.11	0.08	-0.08	0.08	0.09	-0.04	0.04	0.08
12	-0.14	0.35	-0.33	0.29	0.40	-0.40	0.31	-0.54	0.50	0.33	-0.51	0.51	0.33
13	0.11	0.11	-0.15	0.19	0.17	-0.17	0.20	0.11	-0.14	0.22	0.13	-0.13	0.21
14	0.15	-0.53	0.55	0.33	-0.59	0.60	0.37	-0.70	0.72	0.43	-0.74	0.74	0.42
15	0.19	0.55	-0.56	0.25	0.56	-0.56	0.25	0.43	-0.44	0.10	0.43	-0.43	0.09
16	0.13	0.10	-0.16	0.25	0.16	-0.15	0.26	-0.23	0.22	0.18	-0.02	0.02	0.19
17	-0.13	-0.09	0.11	-0.15	-0.01	0.01	-0.18	0.10	-0.12	-0.02	0.07	-0.07	0.05
18	-0.16	0.77	-0.75	0.01	0.76	-0.75	0.00	0.76	-0.73	0.11	0.75	-0.75	0.10
19	0.13	-0.51	0.50	0.16	-0.42	0.42	0.10	-0.46	0.46	0.01	-0.37	0.37	-0.04
20	0.18	-0.13	0.15	0.14	-0.17	0.17	0.15	-0.14	0.16	0.09	-0.17	0.17	0.10

6.3 MDOF systems and harmonic loading

Plots and tables for the R_{30} and R_{SRSS} parameters for this case are also developed but are not presented. The major observations made before for SDOF systems and harmonic loading apply to this case: the 30% and SRSS rules may underestimate or overestimate the combined elastic axial load for highly correlated components. For totally uncorrelated components, the rules accurately estimate the elastic axial load. However, for inelastic behavior, the rules may underestimate or overestimate the combined axial load even for high values of the phase angle. The combined base shear is reasonably estimated practically in all the cases. The values of coefficients of correlation are also estimated but are not shown. They presented a similar trend as that of SDOF and harmonic loading.

7. CONCLUSIONS

Results of the study indicate that, for complex MDOF systems and normal and principal components, both combination rules underestimate the axial load by about 10% and the COV of the underestimation is about 20%. Both rules overestimate the base shear by about 10%. The uncertainty in the estimation is much larger for axial load than for base shear. The mean axial loads and base shear values are essentially the same for elastic and inelastic behavior. However, the uncertainty in the prediction of axial load goes up significantly when inelastic behavior is considered. It is observed that the effect of individual components may be highly uncorrelated, not only for normal components, but also for totally uncorrelated (principal) components, contradicting what stated in earlier investigations. Moreover, the rules are not always inaccurate in the estimation of the combined response for large values of correlation coefficients of the individual effects, and small values of such coefficients are not always related to an accurate estimation of the combined response. Only for the case of perfectly uncorrelated harmonic excitations and elastic analysis of SDOF systems, the individual effects of the components are uncorrelated and the 30% and SRSS rules accurately estimate the combined response. It is the authors' belief that the combination rules under consideration were developed for SDOF systems. In the general case, the level of underestimation or overestimation of the response depends on the level of correlation of the components, the type of structural systems, the type of response parameter, the location of the structural member under consideration and the level of structural deformation. The codes should be more specific regarding the applications of the mentioned commonly used combination rules.

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