Structural damage identification based on continuous ant colony optimization for cantilever-type structures

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SUMMARY:
In this paper, the damage detection problem is formulated as an optimization problem such as to obtain the minimum difference between the numerical and experimental variables, and then a modified ant colony optimization (ACO) algorithm is proposed for solving this optimization problem.
The structural damage in the shear buildings are detected by using dynamically measured flexibility matrix, since the flexibility matrix of the structure can be estimated from only the first few modes. The continuous version of ACO is employed as a probabilistic technique for solving this computational problem, which can be reduced to find good paths through solution graphs. Compared to classical methods, one of the main strengths of this meta-heuristic method is the generally better robustness in achieving global optimum. The efficiency of the proposed algorithm is illustrated by numerical example. The proposed method enables the deduction of the extent and location of structural damage, while using short computational time and resulting good accuracy.

Keywords: Damage detection, Ant colony algorithm, Modal approach, Flexibility matrix

1. INTRODUCTION

The possibility of detecting structural damage has generated a wide interest in earthquake engineering over the last two decades. In general, the experimental techniques for damage detection are expensive and require that the vicinity of the damage is known and readily accessible; therefore several methods intend to detect damage based on numerical model and by means of minimum experimental data about dynamic properties or response of damaged structures.

These methods have different approaches to choose damage-sensitive properties. The modal properties such as resonant frequencies and mode shapes are the most common features used for damage detection, since these parameters are functions of the physical properties of the structure (mass, damping, and stiffness). Therefore, changes in the physical properties, such as reductions in stiffness resulting from the onset of cracks or loosening of a connection, will cause detectable changes in these modal properties.

Extensive overviews for methods to detect, locate and characterize damage in structural and mechanical systems by examining changes in measured vibration response can be found in Doebling et al. [1]. Damage detection methods through changes in frequencies have been proposed by Salawu [2]. Although he showed the efficiency of these methods in the most cases, he has concluded that the mode shapes are also required, since it is not possible to detect any damage situation accurately only by using frequencies. Two criteria are commonly defined based on mode shapes [3]. The modal assurance criterion (MAC) indicates correlation between two sets of mode shapes while the coordinate modal assurance criterion (COMAC) indicates the correlation between mode shapes at selected points on the structure. Ko, et al. have presented a method that uses a combination of MAC, COMAC and sensitivity analysis to detect damage in steel framed structures [4]. The results demonstrate that particular mode pairs can indicate damage, but when all mode pairs are used, the indication of damage deviates due to the modes that are not sensitive to the damage.

It can be shown that the effectiveness of mentioned methods is highly dependent on the number of modes used in the calculation; Lin [5] has observed that higher modal frequencies contribute to the stiffness matrix values to a greater extent. Therefore, to obtain a good estimate of the stiffness matrix, one needs to measure almost all the modes of the structure, especially the high frequency modes. From a testing standpoint, it is more difficult to excite the higher frequency response of a structure, as more energy is required to produce measurable response at these higher frequencies than at the lower frequencies. To overcome this problem, the approach based on flexibility changes has drawn wide attention, since the flexibility matrix can be accurately calculated using only a few lower modes and it is very sensitive to damage. The efficiency of modal flexibilities method in comparison with measured frequencies or mode shapes methods are shown by Zhao and De Wolf [6].

Regardless of which properties are chosen as a criteria for structural identification and damage detection, different computational approaches are observed in literature reviews. The structural identification can be treated as an optimization problem in terms of developing a model which correlates the numerical data with experimental data from the structure, and then the variables are updated to obtain the minimum difference between the numerical and experimental data. Ling and Haldar applied a least squares method to solve their proposed optimization method [7], and Liu and Chen presented an optimization model solved by Newton's method [8]. Both of these methods perform point to point search. Applying such classical methods for optimization problems has some disadvantages. The classical methods often require initial guess, which is difficult to determine for damage detection problems, and more importantly they may converge falsely to a local optimal point rather than the global optimum solution. Contrary to classical optimization methods, applying heuristic algorithms show better robustness in achieving global optimum, while they can be applied for large scale examples with many unknown parameters. Koh and Perry presented a detailed uniformly sampled genetic algorithm with gradient search to detect structural damage [9]. They considered the fitness function as mean-square error between the measured acceleration and the estimated response from the mathematical model. The proposed strategy involved multi-species exploration, adaptive search space reduction and quasi-random sequence sampling exploration, adaptive search space reduction and quasi-random sequence sampling.

In this paper, the flexibility matrix is used to specify an objective function for an optimization procedure which is then implemented by Ant Colony Optimization (ACO) algorithm, since there is a significant interest in the development of an algorithm that uses as few measurements as possible to obtain the physical characteristics of the system, without a priori knowledge of the system, and through less computational time consuming. The efficiency of the present method is illustrated through examples.

2. DEFINITION OF DAMAGE PARAMETERS

A cantilever-type structure can be modelled as a lumped mass system for analysis. The damage is represented by a stiffness reduction factor α_i, defined as the ratio between the damaged stiffness to the initial stiffness. The stiffness matrix of the damage structure $[K_d]$ is expressed as a sum of element matrices multiplied by reduction factors,

$$[K]_d = \sum_{i=1}^{n} \alpha_i [K_i] \qquad (0 \le \alpha_i \le 1), \tag{2.1}$$

The value $\alpha_i = 1$ indicates that the element is undamaged whereas $0 < \alpha_i < 1$ implies partial damage.

2.1. Composition of the Flexibility Matrix

The motion equation of a system with n degrees of freedom can be expressed as Eqn. 2.2. , where $[K]$ and $[M]$ are the stiffness matrix and mass matrix, respectively, and $[\phi] = \{\phi_1, \phi_2, ..., \phi_n\}$ represents the non-mass normalized mode shape matrix, where ϕ_i represents the i th mode shape, and $[\Omega] = diag\{\omega_1^2, \omega_2^2, ..., \omega_n^2\}$ represents the diagonal matrix of squared natural frequency.

$$[M][\phi][\Omega] = [K][\phi] \tag{2.2}$$

With the mass-normalized mode shape matrix $[\Phi], \left([\Phi]^T[M][\Phi] = [I]\right)$, the stiffness $[K]$ and the flexibility $[F]$ matrices are related to the modal data as follows:

$$[K] = [M][\Phi][\Omega][\Phi]^T[M] = [M]\left(\sum_{i=1}^{n} \omega_i^2 \phi_i \phi_i^T\right)[M] \tag{2.3}$$

$$[F] = [\Phi][\Omega]^{-1}[\Phi]^T = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \phi_i \phi_i^T \tag{2.4}$$

According to Eqn. 2.3., the stiffness matrix is proportional to the natural frequency, therefore the modal contribution to the stiffness matrix increases as frequency increases. Besides, the mass matrix needs to be determined. On the other hand, Eqn. 2.4. shows that the flexibility matrix is more affected by low natural frequencies, since the modal contribution to the flexibility matrix decreases as frequency increases, and therefore the flexibility rapidly converges to a good approximation with a few low frequency modes.

Therefore, to calculate a precise stiffness matrix, it requires high natural frequencies, while a good estimate of the flexibility matrix can be obtained only by a few of the lower frequencies. It shall be mentioned that it is more difficult to determine the high natural frequencies through the experiments, and in the most cases, only a few low-frequency modes can be measured. According to Eqn. 2.4., the number of modes measured through the experiment n_e does not affect the size of the flexibility matrix, since the size of $[\Phi]$ and $[\Omega]$ equal to $dof \times n_e$ and $n_e \times n_e$, respectively, and therefore, the flexibility matrix $[F]$ can be derived in size of $dof \times dof$.

Based on the above discussion, in this paper, the flexibility matrix is applied in the objective function of the proposed optimization model to detect the structural damage.

3. FORMULATION OF DAMAGE DETECTION AS AN OPTIMIZATION PROBLEM

In this paper, the problem of damage detection is modelled as a constrained nonlinear optimization problem, where the stiffness reduction factors α_i are considered as decision variables (updating parameters), and the error between the flexibility matrix of the reference state and damaged state of the structure is applied in the objective function as follow:

$$\Delta = \{\delta_{ij}\} = [F]_d - [F]_r \tag{3.1}$$

The flexibility matrix of the damaged structure $[F]_d$ is obtained from natural frequencies and mode shapes of the damaged structure (based on structural response monitoring), by Eqn. 2.4., while the updating stiffness matrix of the damage structure (Eqn. 2.1.) is applied for formation of the flexibility matrix of the reference state $[F]_r$.

If the stiffness reduction factors can be accurately deduced, then the flexibility matrix of the reference state and damaged state will be equal. The more accurate stiffness reduction factors result that the calculated matrix Δ approximates to zero matrix. Therefore, the objective function f is defined as a function that minimizes the matrix $\{\delta_{ij}\}$, as follow:

$$\min \qquad f = \left(\sum_{i,j=1}^{n} \delta_{ij}^2 \right)^{1/2} \tag{3.2}$$

4. IMPLEMENTATION OF CONTINUOUS DOMAINS ANT COLONY OPTIMIZATION FOR PROPOSED DAMAGE DETECTION MODEL

A meta-heuristic algorithm based on the ants' behaviour was developed in early 1990s by Dorigo and Gambardella [10]. This algorithm was called ant colony optimization (ACO) because it was motivated by social behaviour of ants. It has been initially proposed for solving combinatorial optimization problems. Since the proposed model for the damage detection problem requires choosing decision variables α_i from continuous domain $(0 \le \alpha_i \le 1)$, a version of ACO for continuous domains (ACO_R) which are presented by Socha and Dorigo [11] is applied in this paper. The implementation steps of this algorithm for the proposed damage detection model are expressed below.

In ACO_R, the ants' solutions are ordered in an archive according to their objective function values (i.e. $f_1 \le f_2 \le ... \le f_k$). The size of this archive k is a constant parameter of the algorithm, and it may not be smaller than the number of dimensions of the problem being solved, therefore it shall be considered larger than the number of degrees of freedom of the structure (n).
Also, each solution of the archive has an associated weight w. For lth solution (i.e. lth row of the archive), w_l is created in the following way:

$$w_l = \frac{1}{qk\sqrt{2\pi}} e^{-\frac{(l-1)^2}{2q^2k^2}} \tag{4.1}$$

In Eqn. 4.1, q is a constant parameter of the algorithm. When q is small, the best-ranked solutions are strongly preferred, and when it is large, the probability becomes more uniform in the ants' searches.

Table 4.1. The Structure of Solution Archive Kept in Algorithm

number of DOFs →

Archive size ↓

	1	2	.	.	.	i	.	.	.	n	objective function	associated weight
1	α_1^1	α_1^2	.	.	.	α_1^i	.	.	.	α_1^n	f_1	w_1
2	α_2^1	α_2^2	.	.	.	α_2^i	.	.	.	α_2^n	f_2	w_2
.	.	.				.				.	.	.
.	.	.				.				.	.	.
.	.	.				.				.	.	.
l	α_l^1	α_l^2	.	.	.	α_l^i	.	.	.	α_l^n	f_l	w_l
.	.	.				.				.	.	.
.	.	.				.				.	.	.
.	.	.				.				.	.	.
k	α_k^1	α_k^2	.	.	.	α_k^i	.	.	.	α_k^n	f_n	w_n

At the start of the algorithm, the solution archive is initialized generating k solutions by uniform random sampling, but in the next iterations the sampling process is accomplished in two phases. Phase one consists of choosing a solution from archive to form a Gaussian function.
Since the solution belongs to a continuous domain $(0 \le \alpha_i \le 1)$, a probability density will be applied instead of discrete probability distribution to choose a component of solution, and one of the most popular functions that is used as a probability density is the Gaussian function (g). In the phase one, the probability p_l of choosing the l th Gaussian function is given by:

$$p_l = \frac{w_l}{\sum_{j=1}^{k} w_j} \tag{4.2}$$

To consider the correlation between the variables, this choice is done only once per ant, per iteration. If the l th solution is chosen, the Gaussian functions $(g_l^i, i = 1,2,...,n)$ are applied for constructing the whole solution (i.e. $\alpha_l^i, i = 1,2,...,n$).
Phase two consists of sampling from the chosen Gaussian function. This will be done by applying a random number generator that is able to generate random numbers according to a parameterized normal distribution. The Gaussian function $g_l^i(x)$ is defined by following equation:

$$g_l^i(x) = \frac{1}{\sigma_l^i \sqrt{2\pi}} e^{-\frac{(x-\mu_l^i)^2}{2\sigma_l^{i\,2}}} \tag{4.3}$$

According to Eqn. 4.3, in order to define the g_l^i, the values of μ_l^i and σ_l^i shall be defined. μ_l^i is the mean value of the distribution, and the i th variable of the chosen solution is assigned to it (i.e. $\mu_l^i = \alpha_l^i$). σ_l^i is the standard deviation of the distribution, which is calculated as follows:

$$\sigma_l^i = \xi \sum_{j=1}^{k} \frac{\left|\alpha_j^i - \alpha_l^i\right|}{k-1} \tag{4.4}$$

The parameter ξ acts as pheromone evaporation rate, it influences the way the long term memory is used.

After each iteration, the set of newly generated solutions will be added to the archive, and then the archive will be ordered and the same number of worst solutions will be removed, therefore the total size of the archive does not change. This process will be repeated until the stopping condition is met.

5. NUMERICAL EXAMPLES

To verify the feasibility of the proposed damage detection method using the continuous ant colony algorithm, the proposed method was applied to two numerical examples.

5.1. Lumped Mass System of 3 Degrees of Freedom (DOF)

This example shows the ability of the proposed algorithm in detecting the location and the intensity of the damage for the different cases of the damage scenarios. It includes a two-dimensional shear frame type structure with structural properties as given in Table 5.1.

Table 5.1. Structural Properties of 3-DOF Structure

	Level 1	***Level 2***	***Level 3***
Stiffness (Kn/m)	*5000*	*4000*	3000
Mass (Kg)	*6000*	*50000*	4000
Mode Shape (ϕ)	1.000	-1.000	0.475
	0.747	0.509	-1.000
	0.380	0.921	0.805
Natural frequency (ω)	0.435	1.064	1.526

The structure consists of rigid beams and flexible columns, effectively reducing the motion to a single translational degree of freedom at each floor level as shown in Fig. 5.1. The mass of the structure is lumped at each floor level.

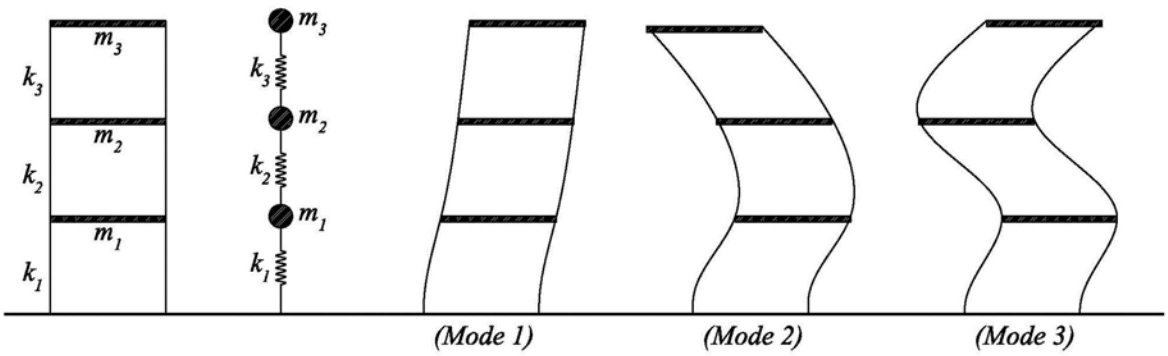

Figure 5.1. 3-DOF structure

To investigate the proposed algorithm, three scenarios of damage are defined as indicated by Table 5.2. In case 1, it is assumed a numerically generated damaged at the second floor with 2% of extent (the stiffness of the second floor k_2 is reduced by this amount. i.e. $\alpha_2 = 0.02$). Case 1 demonstrates the performance of the method to determine the location and the extent of a single damage with low damage extent; in case 2 the damage are considered in two different locations which one of the damage extent is more higher than the other one, and in case 3 the partial damage is allowed to occur in all elements.

Table 5.2. Analyzed Damage Scenarios

Scenario	***Case 1***	***Case 2***	***Case 3***
Level No. - Damage Extent (%)	2 – 2	1 – 50 2 – 10	1 – 20 2 – 20 3 – 20

A numerical analysis is performed for the 3 stories shear building, and the natural frequencies and mode shapes for the damaged state are calculated to derive the flexibility matrix of the damaged structure $[F]_d$, which is then applied to the proposed algorithm to deduce the stiffness of damaged structure.

As described in section 4, in the ACO_R search procedure, the size of the archive, the number of ants in each iteration, the pheromone evaporation rate and the constant parameter of q play important role in convergence speed of the algorithm. After some trials and according to the suggestions published by Socha and Dorigo [11], the constant parameters of the proposed algorithm have been set up as Table 5.3. The initial archive solution is randomly generated within the specified parameter range; no good guess of initial parameter values is needed.

Table 5.3. Summary of the Parameters Used in the Proposed Algorithm for 3-DOF Structure

Parameter	***Value***
No. of ants in an iteration	6
Evaporation Rate	0.85
Locality of the search	0.2
Archive Size	10

The identification results are presented in Table 5.4. It shows that the proposed method can detect the structural damage successfully in all three cases. In terms of fine-tuning efficiency, the maximum absolute error is a good indicator. In this regard, the proposed method gives a good solution. As it is shown in Table 5.4., the proposed algorithm can predict the stiffness reduction factor for single and low value of damage extent (2%) as good as for multiple damage scenario (case 2 & 3) and large extent damage.

Table 5.4. Identification Results by the Proposed Algorithm for 3-DOF Structure

	α_1		α_2		α_3		**Mean absolute error**	**Max. absolute error**
	Value	Error (%)	Value	Error (%)	Value	Error (%)		
Case 1	0.9696	-3.04	0.0200	0	0.9789	-2.11	1.72	3.04
Case 2	0.4901	-1.98	0.1005	0.5	0.9793	-2.07	1.52	2.07
Case 3	0.2046	2.3	0.1947	-2.65	0.1971	-1.45	2.13	2.65

To further test the applicability of the proposed method, a large system is studied in the next section.

5.2. Lumped Mass System of 30-DOF Structure

In this example, the proposed algorithm is applied for a 30 stories shear building with a single damage in the stiffness of each floor, hence a total of 30 unknown parameters, i.e. 30 stiffness reduction factors are to be identified. For the undamaged structure, the exact value of stiffness and mass is $7000\,KN/m$ and $6000Kg$ respectively for each level. It is assumed that the stiffness of half of the floors are reduced 25% i.e. $\alpha = 0.25$. Clearly, the numerical parameters used in the ACO_R method play an influential role with regards to the accuracy and efficiency of the proposed algorithm. For this large example, the archive size and number of ants in each iteration are set as 100 and 10 respectively. Error value of the estimated stiffness reduction factor is shown in Fig. 5.2.

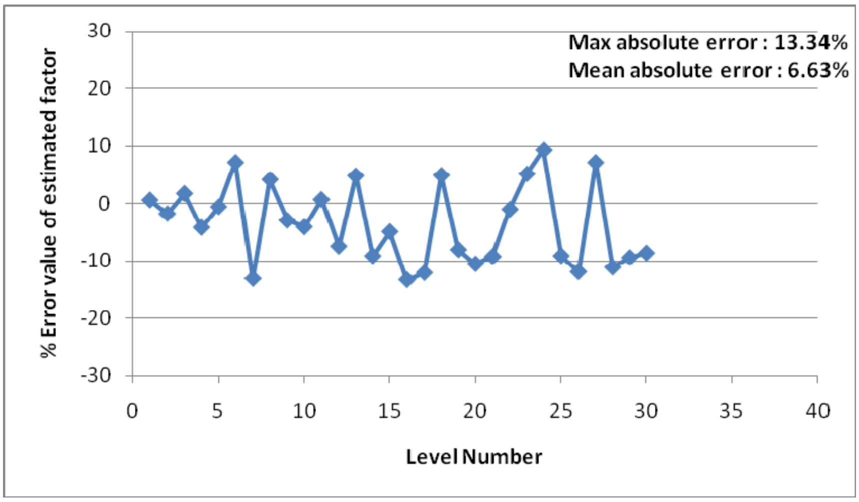

Figure 5.2. Identification results by the proposed algorithm for 30-DOF structure

As it is shown in Fig. 5.2., the proposed algorithm has detected the location and the intensity of the damage with a reasonable degree of accuracy. The mean absolute error of the identification results is 6.63%. figure.

6. CONCLUSIONS

This paper has presented an algorithm to detect damage extent and location by the continuous ant colony optimization. Its implementation is relatively straightforward, relying on forward analysis instead of the otherwise ill-conditioned inverse analysis, and is not required good initial guess. Furthermore, the flexibility matrix as an objective function can be defined only by a few of the lower frequencies. Regard less of the damage extent or location; the proposed algorithm can detect the structural damages. It has been shown by definition different damage scenarios for a 3-DOF structure. The damage extent and location has been also detected successfully for large scale problem.

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