Depolarization and Modified Conversions of Seismic Waves by Site-City Effects : A Theoretical Model

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SUMMARY:
This article is devoted to the propagation of obliquely-incident homogeneous elastic waves in presence of a city resting on a homogeneous half-space. The city is modeled as a periodic distribution of oscillators which overall behavior was reduced with the homogenization method into a frequency-dependant surface impedance by Boutin and Roussillon (2006). It is hereby shown that soil-city interactions during oscillators' resonance lead to the apparent stiffening of the surface that induce around the oscillators' eigen-frequency (i) a reduction of conversion between P and SV waves ; (ii) the depolarization of normally-incident shear waves ; (iii) the conversion between SH waves and P and SV waves. Soil-city interactions also result in an apparent radiative damping for the resonators associated to the emission of waves by the oscillators in the soil.

Keywords: Resonant surface, Boundary layer, Homogenization, Wave propagation, Impedance

1. INTRODUCTION

Seismic engineering practice usually considers that the ground motion only results from the seismic source and the feature of the substratum; it disregards the densely urbanized surroundings. However, some observations made during the 1985 Michoacan earthquake, such as beatings and long codas, remained unexplained even using 3D refined models (Chavez-Garcia and Bard, 1994). This led Wirgin and Bard (Wirgin and Bard, 1996) to suggest that ground motion could also result from structure-soil-structure interactions with the "sur-stratum" made up by the city. Calculations based on several methods (Clouteau and Aubry, 2001; Guéguen *et al.*, 2002; Tsogka and Wirgin, 2003), supported the idea that for specific situations, global city-scale soil-structure interactions can occur: the so-called Site-City effect can be significant. But what are the signatures of these global interactions on the seismic waves and what are their consequences on the dynamics of the buildings?

The problem is treated analytically considering two simple observations: (i) the dynamical soil-structure interactions occur because the seismic frequency range matches the eigenfrequencies of the buildings; (ii) urban landscapes often look like grids with similar buildings among large districts. These observations lead to consider the city during a seismic event as (i) a resonant surface with (ii) a periodic distribution of resonators.

Provided that the period is much smaller than the wavelength in the soil, Boutin and Roussillon (Boutin and Roussillon, 2004; Boutin and Roussillon, 2006) homogenized the boundary layer of the local fields emitted by the resonators and reduced the macroscopic behaviour of the city into a frequency-dependant surface impedance depending on the dynamic of the oscillators.

The physical principles leading to the macroscopic surface impedance are presented in the first part of the paper. In the following parts, its effects on the polarization and the conversions of elastic waves are exposed.

2. FROM THE HOMOGENIZATION OF THE RESONANT SURFACE TO ITS MACROSCOPIC IMPEDANCE

2.1. Statement of the problem

The study focuses on the propagation of harmonic elastic waves in a linear, elastic, isotropic, homogeneous half-space on which lies a resonant surface. The half-space is characterised by its density ρ and its Lamé coefficients λ and μ. The resonant surface is viewed as the infinite periodic repetition of the same representative surface element Σ. *Along the whole paper, we assume that only one oscillator stands on the period and that it oscillates only in the horizontal direction e_o with one degree of freedom* (see Fig. 2.1).

The reflection of elastic waves on the border between two media is usually analysed comparing the impedances of the media on each side of the border. As well, the macroscopic impedance Z_O of the resonant surface is established and then compared to the impedance of the homogeneous half-space for shear waves $Z_S=(\rho\mu)^{1/2}$. The reflecting waves generate a surface displacement that sets the oscillators in motion. While oscillating, they exert on the surface a distribution of local stresses. Assuming the distance between the oscillators is small before the wavelength Λ (hypothesis of scale separation) this distribution is seen by the waves as a homogeneous distribution of stress at the leading order. The surface impedance Z_O is defined as the ratio between this macroscopic stress and the surface velocity.

The macroscopic surface impedance Z_O is therefore established in two steps: firstly, the macroscopic stress imposed by the resonant surface is expressed by the homogenization method as a function of the local stresses exerted by the oscillator on the period; secondly, the equation of motion of the oscillator links these local stresses to the surface displacement that set the oscillator in motion.

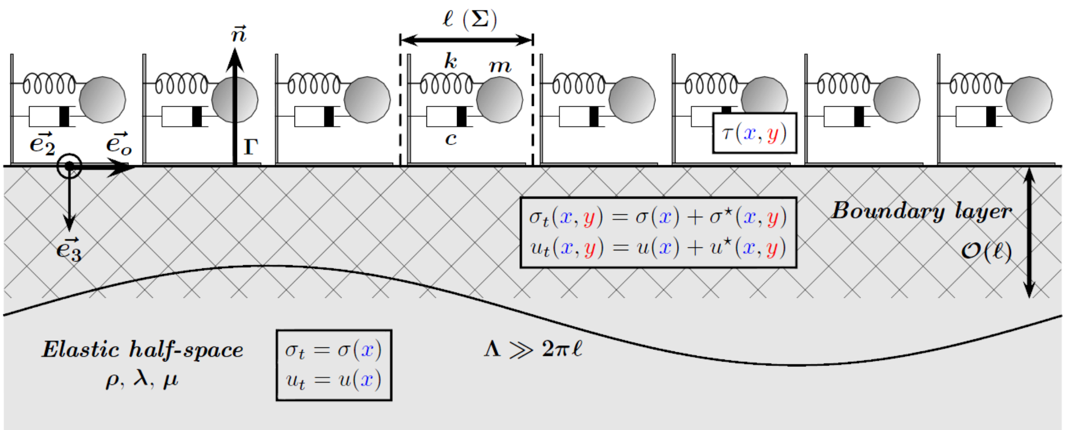

Figure 2.1 Elastic half-space loaded by periodic distribution of horizontal one-degree-of-freedom-oscillators

2.2. Homogenization of the stresses exerted on the surface by the oscillators

The key of the homogenization process is the geometric hypothesis of "scale separation": it is assumed that the length ℓ of the period Σ is small compared to the wavelength Λ of the incident wave. This condition is quantified by the scale ratio $\varepsilon=2\pi\ell/\Lambda<<1$. Denoting ω the pulsation and $c_S=(\mu/\rho)^{1/2}$ the shear waves velocity, the scale separation means that the phase difference $\varepsilon=\ell\omega/c_S$ of the incident wave over the length ℓ is small before 1 : neighbouring resonators oscillate in phase.

Under this scale separation condition, the dynamic has two scales: far from the surface, the waves propagate as macroscopic fields σ(x) and u(x) which fluctuate over a characteristic length Λ; in the vicinity of the surface, the oscillators emit Σ-periodic fields σ*(x,y) and u*(x,y) located in a boundary layer. The local stresses τ exerted by the oscillators on the surface are balanced by both macroscopic fields and boundary layer fields (see Fig. 2.1).

Following the homogenization process, the macroscopic fields are described by the macroscopic space variable x while boundary layer fields are furthermore described by the microscopic space variable $y=\varepsilon^{-1}x$ adapted to their characteristic length of variation $\ell \ll \Lambda$. That leads to modify the differential operators so that they take account of the local perturbation at the macroscopic scale. The stresses and displacements are then expanded in powers of ε and reported in the balance of the surface, the boundary layer and the half-space. The balances are successively solved for each power of ε.

At the leading order, the resonant surface exerts the macroscopic stress $<\tau>$, which is the average of the local stresses τ exerted by the oscillator on one period while set in motion by the waves.

$$<\tau>(x) = \frac{1}{|\Sigma|} \int_\Sigma \tau(x,y)\ dS_y \qquad (2.1)$$

2.3. The macroscopic impedance of the resonant surface

In this paper we consider one-degree-of-freedom resonators that oscillate in the horizontal direction eo only. While set in motion by the waves they exert the stress τ on the surface in the resonant direction e_o and do not exert stresses in any other direction.

In the resonant direction e_o the force $|\Sigma|<\tau>$ undergone by the period Σ at the leading order is balanced by the force $-m\omega^2 U_m$ undergone by the mass m of the oscillator on the period. The surface impedance Z_O in the resonant direction e_o is defined as the ratio between $<\tau>$ and the surface velocity $-i\omega U_\Gamma$:

$$Z_o = \frac{<\tau>}{-i\omega U_\Gamma} = \frac{1}{|\Sigma|}\frac{m\ \omega^2\ Um}{-i\omega U_\Gamma} \qquad (2.2)$$

Besides, the displacement U_m of the mass of the oscillator is linked to the surface displacement U_Γ by the equation of motion of the weakly-damped oscillator. Denoting ω_o the eigen-pulsation of the oscillator and $\xi \ll 1$ its damping, the equation of motion leads to the dynamical response function

$$\frac{Um}{U_\Gamma} = \frac{1 - i\,2\xi\frac{\omega}{\omega_o}}{1 - i\,2\xi\frac{\omega}{\omega_o} - \frac{\omega^2}{\omega_o^2}} \qquad (2.3)$$

As a result, the ratio between the surface impedance Z_O and the impedance of the media beneath Z_S is made up of a parameter η and of a dynamical function:

$$\frac{Z_o}{Z_S} = -\eta\,\frac{-i\frac{\omega}{\omega_o}\left(1 - i\,2\xi\frac{\omega}{\omega_o}\right)}{1 - i\,2\xi\frac{\omega}{\omega_o} - \frac{\omega^2}{\omega_o^2}} \qquad \eta = \frac{\ell\omega_o}{c_S}\,\frac{m}{\rho|\Sigma|\ell} \qquad (2.4)$$

The parameter η is made up of two terms: the first one $\varepsilon_o = \ell\omega_o/c_S$ is the scale ratio evaluated at the eigen pulsation ω_o of the oscillator. For the resonance to occur in the pulsation range of the scale separation, ε_o is small before 1. The second term in η is the ratio between the mass m of the oscillator and the mass $M_\Sigma = \rho|\Sigma|\ell$ of the medium under one period Σ on a depth ℓ (the characteristic size of Σ). As a consequence, the parameter η is at best of the first order (in ε_o) if the resonating mass m is of the same order as the mass of soil M_Σ. Keeping in mind the order of magnitude of η, we can find the asymptotic dynamical behaviours of the ratio of impedance Z_O/Z_S and estimate the analogous surface conditions imposed by the oscillators to the waves for a given pulsation (see Fig. 2.2).

In quasi-static ($\omega \ll \omega_o$) or inertial ($\omega \gg \omega_o$) regime, the limits of the dynamical function lead to a surface impedance Z_O much smaller than the impedance of the medium Z_S: this contrast of impedances is analogous to a free surface condition.

$$\omega \ll \omega_o \Rightarrow \left|\frac{Z_o}{Z_S}\right| \sim \left|i\,\eta\,\frac{\omega}{\omega_o}\right| \ll 1 \qquad \omega \gg \omega_o \Rightarrow \left|\frac{Z_o}{Z_S}\right| \sim |-2\xi\,\eta| \ll 1 \qquad (2.5)$$

But at the resonance of the weakly-damped oscillator ($\omega=\omega_o$) the ratio of impedances tends to $-\eta/2\xi$.

If the oscillators are perfectly elastic ($\xi=0$), the ratio of impedances Z_O/Z_S becomes infinite: this contrast of impedances is analogous to a rigid condition. Note that the resonance of the oscillators leads to the apparent "stiffening" of the surface in the direction of the oscillations only. Hence, anisotropic resonators which have different eigen pulsations in the different directions lead to anisotropic surface conditions: apparently free in the directions in quasi-static or inertial regime, apparently stiffened in the directions of resonance.

In the case of weakly-damped oscillators, the ratio $\eta/2\xi$ must not be small before 1 for the site-city effect to be significant at the resonance under the scale separation condition. That happens if the resonating mass m is at least of the same order as the mass $M_{\Sigma}=\rho|\Sigma|\ell$ of the medium under one period Σ on a depth ℓ. For numerical applications, we will use $\eta=2\xi=10\%$ which stands for a dense urban area built on soft soil.

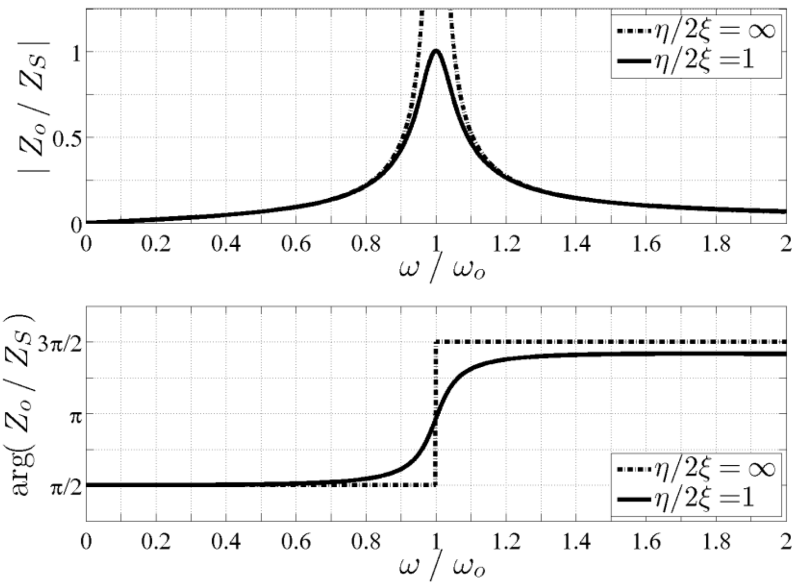

Figure 2.2 Ratio between the surface impedance Z_O in the resonant direction and the impedance Z_S of the medium for shear waves. Calculations performed with $\eta=10\%$ and $2\xi=0\%$ or 10%

3. INCIDENT WAVES POLARIZED IN THE RESONANT DIRECTION: THE CASE OF THE HORIZONTALLY-ISOTROPIC RESONANT SURFACE

In this section, we study the propagation of elastic waves which horizontal displacement is polarized in the resonant direction e_o. These results are also applicable if the resonators have the same dynamical behavior in both the horizontal direction e_o and e_2 and the waves have any horizontal polarization: this section is devoted to horizontally-isotropic resonant surfaces.

3.1 Structural damping versus apparent radiative damping: illustration in the case of SH waves

Let us consider a shear wave in incidence on the surface with an angle θ_S and which amplitude U_i is polarized in the resonant direction as shown in Fig. 3.1. It gives rise to a reflected wave with respect to the Descartes' relation; we note U_r its amplitude. The stress generated by the waves at the surface must equal the stress exerted by the oscillators set in motion: it leads to the coefficient of reflection

$$\mathrm{i}\omega Z_S(U_i - U_r)\cos\theta_S = -\mathrm{i}\omega Z_O(U_i + U_r) \quad \Rightarrow \quad R = \frac{U_r}{U_i} = \frac{\cos\theta_S + Z_O/Z_S}{\cos\theta_S - Z_O/Z_S} \tag{3.1}$$

The surface displacement U_Γ in the resonant direction is the sum of the incident and the reflected displacements $U_\Gamma=U_i+RU_i$ and the oscillators' displacement U_m is given by Eqn. 2.3

$$\frac{U_\Gamma}{U_i} = 2\frac{1-i\,2\xi\frac{\omega}{\omega_o}-\frac{\omega^2}{\omega_o^2}}{1-i\,2\zeta\frac{\omega}{\Omega_o}-\frac{\omega^2}{\Omega_o^2}} \qquad \frac{U_m}{U_i} = 2\frac{1-i\,2\xi\frac{\omega}{\omega_o}}{1-i\,2\zeta\frac{\omega}{\Omega_o}-\frac{\omega^2}{\Omega_o^2}} \tag{3.2a}$$

$$2\zeta = \frac{2\xi+\eta/\cos\theta_S}{\sqrt{1+2\xi\eta/\cos\theta_S}} \qquad \Omega_o = \frac{\omega_o}{\sqrt{1+2\xi\eta/\cos\theta_S}} \tag{3.2b}$$

The coupled system behaves with the apparent damping 2ζ and the apparent eigen-pulsation Ω_o. As shown in Fig. 3.1, the surface displacement U_Γ is twice the incident displacement U_i when the oscillators are quasi-static ($\omega<<\omega_o$) or in inertial regime ($\omega>>\omega_o$). This situation is analogous to a free surface condition and is coherent with the contrast of impedances $Z_O/Z_S<<1$ (see Eqn. 2.5).

At the resonance $\omega=\omega_o$ of perfectly elastic oscillators ($\xi=0$) the surface does not move ($U_\Gamma=0$): this situation is analogous to a rigid surface condition and is coherent with the contrast of impedances $Z_O/Z_S=\infty$ (see Fig. 2.2). However, the oscillators' displacement U_m is finite at the resonance even though they are perfectly elastic. The Eqn. 3.2 show that they behave as damped oscillators with an apparent damping $\eta/2\cos\theta_S$. The system being non-dissipative, the energy lost by the oscillators is gained by the half-space: this apparent damping is due to the emission of waves by the oscillators.

In the case of weakly-damped oscillators, the finite contrast of impedances $Z_O/Z_S=-\eta/2\xi\neq\infty$ at the resonance leads to the surface displacement $4\xi\cos\theta_S/\eta\neq0$ for a unitary incident wave: the surface condition is no longer analogous to a rigid surface condition but the surface is apparently "stiffened".

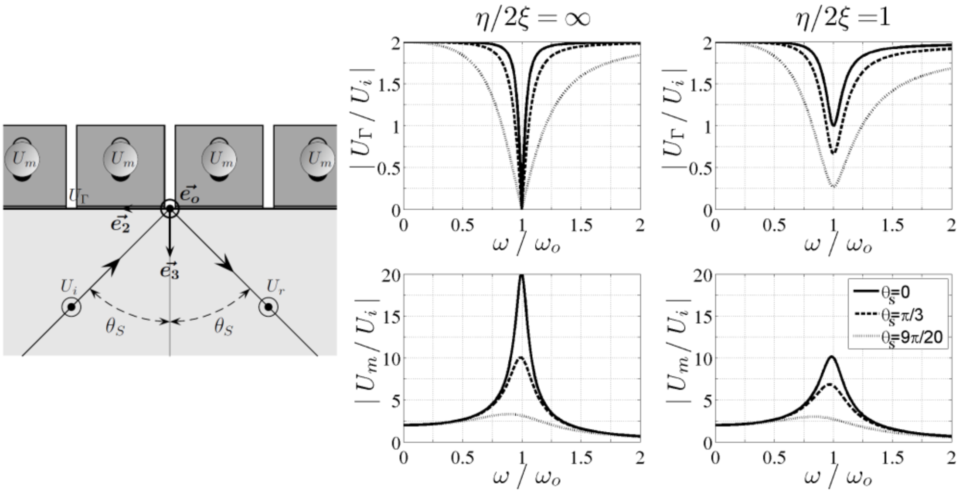

Figure 3.1 Surface displacement (top) and oscillators' displacement (bottom) in the resonant direction for a unitary incident SH wave polarized along the resonant direction.
Calculations performed with $\eta=10\%$ and $2\xi=0\%$ or 10%

To conclude this paragraph, let us make two remarks. Firstly, the contrast of impedances $Z_O/Z_S=-\eta/2\xi$ at the resonance shows a competition between the structural damping 2ξ and the apparent radiative damping η: isolating the oscillators, the ratio $\eta/2\xi$ indicates if the energy is preferentially structurally-lost or turned back into waves, keeping in mind that the apparent radiative damping η depends on the scale separation and the ratio between the resonating mass m and the mass $M_\Sigma=\rho|\Sigma|\ell$ of the medium (see Eqn. 2.4). Secondly, note that no reflected wave is produced in the very particular case $\eta=2\xi\cos\theta_S$ (R=0 in Eqn. 3.1): the incident energy stays in the resonant surface for these parameters.

3.2 Modified conversion of incident P waves

Let us consider one incident P wave which incident plane contains the resonant direction e_o of the oscillators (see Fig. 3.4). It generates displacements in the non-resonant vertical direction e_3 and in the horizontal resonant direction e_o. As a result, the oscillators impose a stress-free condition along e_3 and the dynamical stress $< \tau >$ along e_o. No out of plane stresses are imposed: reflected P and SV waves are sufficient to fulfill these surface conditions.

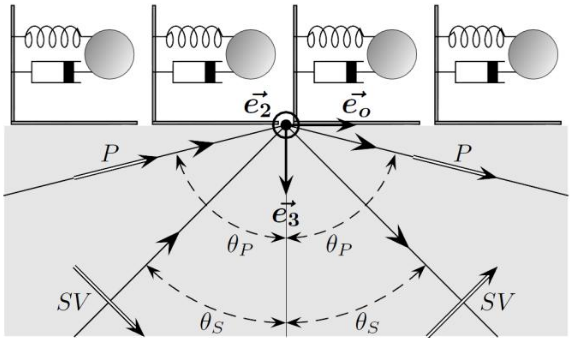

Figure 3.4 Reflection of SV and P waves which incident plane contains the resonant direction e_o

In quasi-static and inertial regimes, the surface condition is analogous to a free surface condition: for some incident angles θ_P the amplitude of the SV wave is greater than the amplitude of the reflected P wave (see Fig. 3.5 in the case of the static resonant surface ω=0). The incident P wave is substantially converted into a reflected SV wave (the conversion depends on the Poisson's ratio ν)

At the resonance of perfectly elastic oscillators (ξ=0) the surface condition is analogous to the mixed condition: free in the vertical non-resonant direction e_3 and rigid in the resonant direction e_o. To fulfill this surface condition, one reflected P wave is sufficient: the incident P wave is no longer converted into a SV wave (see Fig. 3.5 in the case ω=1 and $\eta/2\xi=\infty$).

At the resonance of weakly damped oscillators, the incident P wave gives rise to both P and SV waves but the P wave is amplified (compared to the quasi-static and inertial regimes) and the SV wave is attenuated. As a result, the modulus of the SV wave remains smaller than the modulus of the reflected

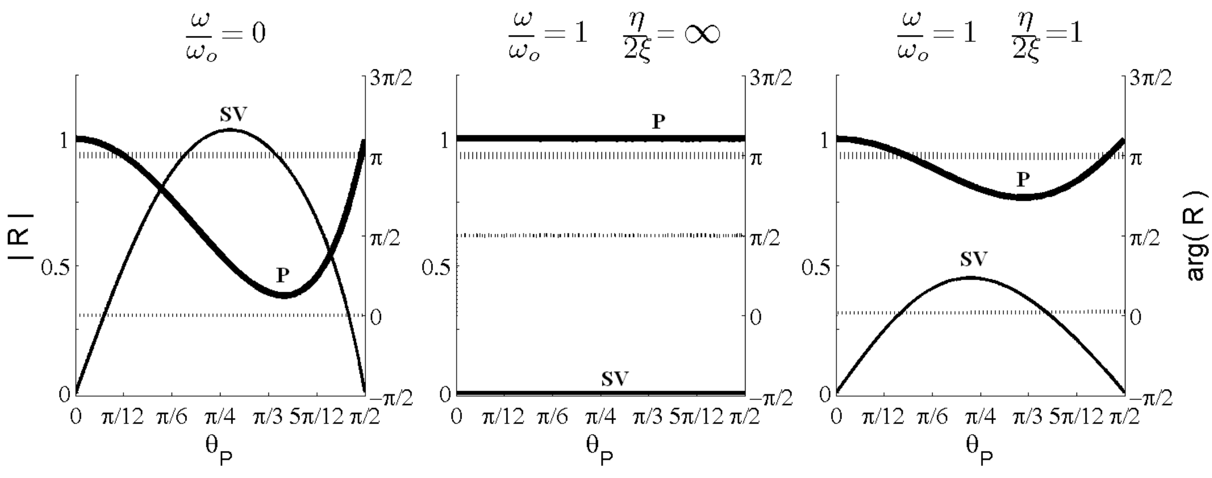

Figure 3.5 Modulus of the coefficient of reflection in the case of an incident P wave which incident plane contains the resonant direction e_o. Calculations performed with ν =1/3, η=10% and 2ξ=0% or 10%

3.3 Modified conversion of incident SV waves

Let us consider one incident SV wave which incident plane contains the resonant direction e_o of the oscillators (see Fig. 3.4). As for an incident P wave, no out of plane stresses are imposed by the oscillators at the surface and reflected P and SV waves are sufficient to fulfill the surface conditions.

In quasi-static and inertial regimes, the surface condition is analogous to a free surface condition: there exists a critical incident angle ($\pi/6$ for a Poisson's ratio $\nu = 1/3$) beyond which the P wave becomes inhomogeneous (Descartes' relation). Moreover, depending on the Poisson's ratio, the incident SV wave is converted into a P wave which can have an amplitude greater than the reflected SV wave (see Fig. 3.6 in the case of the static resonant surface $\omega=0$).

At the resonance of perfectly elastic oscillators ($\xi=0$) the surface condition is analogous to a vertically-free and horizontally-rigid condition. To fulfill this surface condition, one reflected SV wave is sufficient: the incident SV wave is no longer converted into a P wave (see Fig. 3.6 in the case $\omega=1$ and $\eta/2\xi=\infty$).

At the resonance of weakly damped oscillators, the conversion is less important than in quasi-static or inertial regimes (see Fig. 3.6 in the case $\omega=1$ and $\eta/2\xi=1$). Note that for quasi-normal incident waves, the reflected SV wave can be substantially attenuated at the resonance, depending on the ratio $\eta/2\xi$. In the very particular case $\eta/2\xi=1$, no normal shear waves is reflected at the resonance (R=0 in Eqn. 3.1).

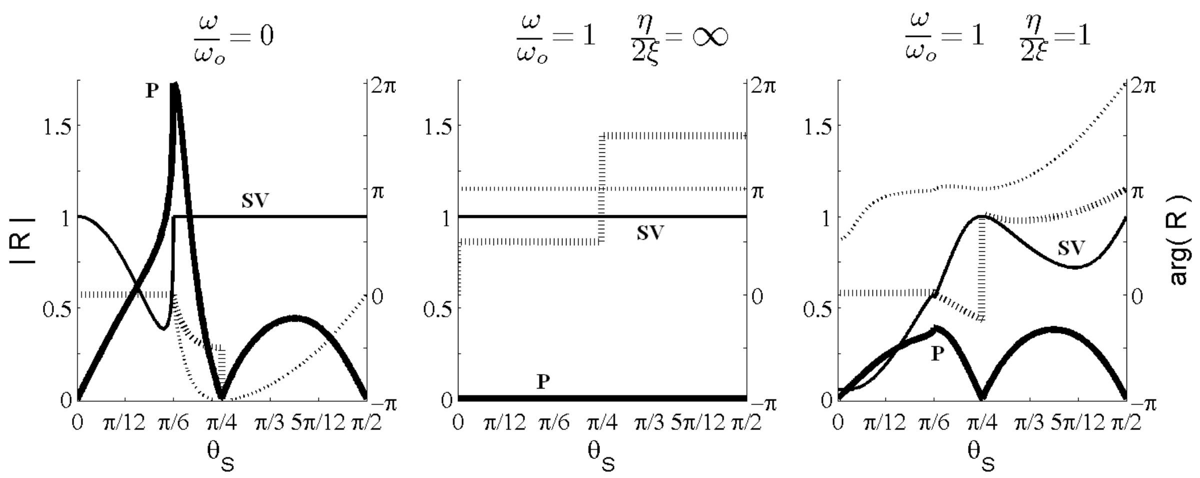

Figure 3.6 Modulus of the coefficient of reflection in the case of an incident SV wave which incident plane contains the resonant direction e_o. Calculations performed with $\nu = 1/3$, $\eta=10\%$ and $2\xi=0\%$ or 10%

4. INCIDENT WAVES POLARIZED OUT OF THE RESONANT DIRECTION: THE CASE OF HORIZONTALLY-ANISOTROPIC RESONANT SURFACES

4.1 Depolarization of normally-incident shear wave

Let us consider a normally-incident shear wave which polarization makes an angle α with the resonant direction e_o(see Fig. 4.1). Its displacement is projected on e_o and on the non-resonant direction e_2. The component $\cos\alpha$ along e_o is reflected with the coefficient of reflection R given in equation Eqn. 3.1 with $\theta_S=0$ and represented in Fig. 4.2; the component $\sin\alpha$ along the non-resonant direction e_2 is reflected in phase as under a free surface condition. The displacement of the reflected wave reads

$$\overrightarrow{u_r} = |R| \cos\alpha \cos(\omega t - \arg(R)) \overrightarrow{e_o} + \sin\alpha \cos(\omega t) \overrightarrow{e_2} \qquad (4.1)$$

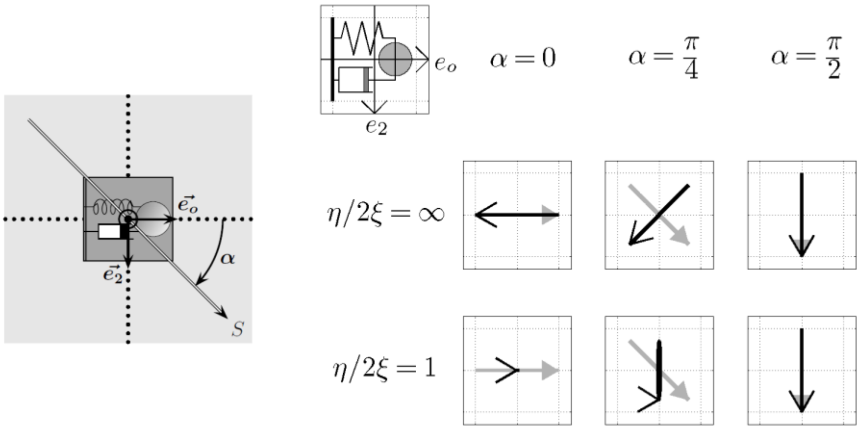

Figure 4.1 Depolarization of a normally-incident shear wave which polarization makes an angle α with the resonant direction e_o in the case of perfectly elastic oscillators (η/2ξ=∞) and damped oscillators (η/2ξ=1) at the resonance. Grey arrows are the incident wave. The polarization of the reflected waves is in black: the arrow gives the orientation of the polarization when the incident wave is at its maximum.
Calculations performed with η=10% and 2ξ=0% or 10%

In the case of perfectly elastic oscillators, the modulus of R remains unitary but its argument varies from 0 to 2π (see Fig. 4.2): the reflected wave is depolarized. At the resonance the component of the reflected wave in the resonant direction e_o is opposed to the component of the incident wave along e_o (arg R= π at the resonance); meanwhile their components in the non-resonant direction e_2 are equal. For α=π/4, the reflected wave is therefore perpendicular to the incident wave (see Fig. 4.1).

At the resonance of weakly-damped oscillators, the depolarization is less marked as the contrast of impedance Z_O/Z_S between the resonant surface and the half-space is no longer infinite but equal to η/2ξ. In the very particular case η=2ξ, the coefficient of reflexion R along the resonant direction vanishes at the resonance (see Fig. 4.2): the reflected wave is totally polarized along the non-resonant direction (see Fig. 4.1).

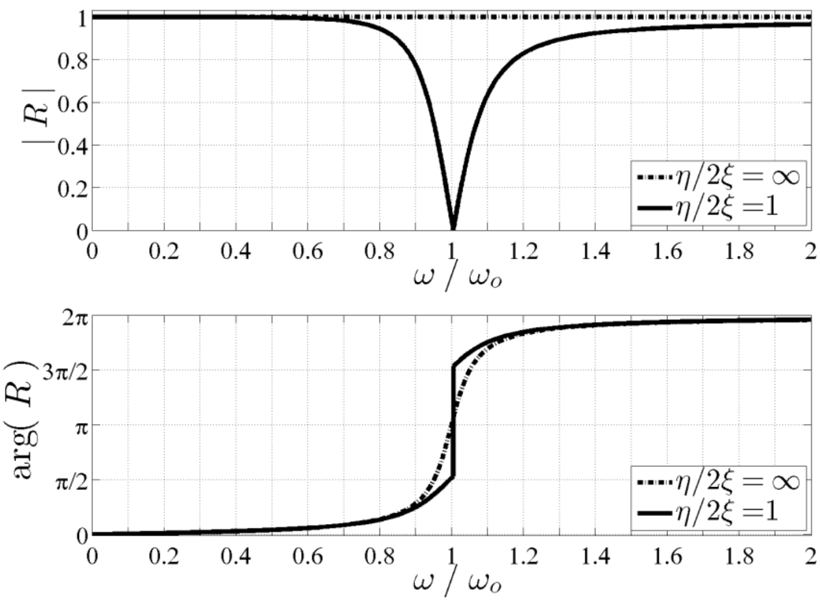

Figure 4.2 Coefficient of reflection R of a normally-incident shear wave polarized in the resonant direction.
Calculations performed with η=10% and 2ξ=0% or 10%

4.2 Conversion of SH into SV and P waves

Let us consider an incident SH wave which polarization makes an angle α with the resonant direction e_o and incoming with the angle θ_S (see Fig. 4.3).

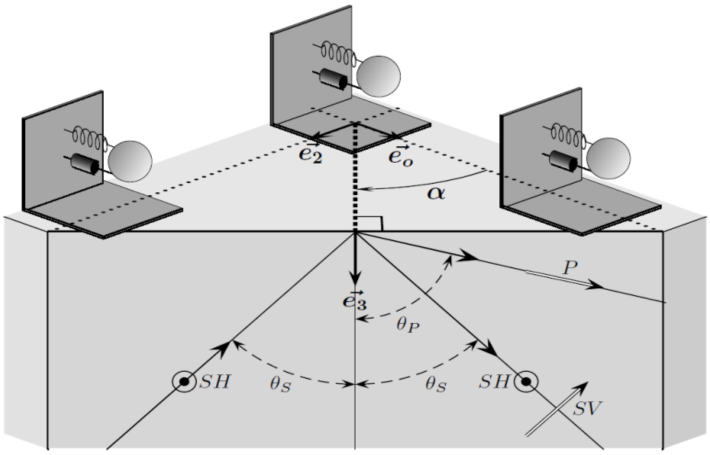

Figure 4.3 SH wave which polarisation makes an angle α with the resonant direction e_o and incoming with an incidence θ_S

In quasi-static and inertial regimes, the surface condition is analogous to a free surface condition: the incident SH wave gives rise to one SH wave only as the stresses generated at the surface by the waves are along the direction of polarization and compensate.

At the resonance, the component cosα of the incident wave in the resonant direction set the oscillators in motion, which impose stresses in the resonant direction e_o. One reflected SH wave cannot compensate in the same time the stresses generated by the oscillators along e_o and the stresses generated by the incident wave in the non-resonant direction e_2: P and SV waves have to be involved (see Fig. 4.4). The vertical stress created at the surface by the SV and P waves will balance one another to fulfill the vertically-free condition. And the three reflected waves will balance one another to fulfill the horizontal conditions: the incident SH wave is converted into P and SV waves.

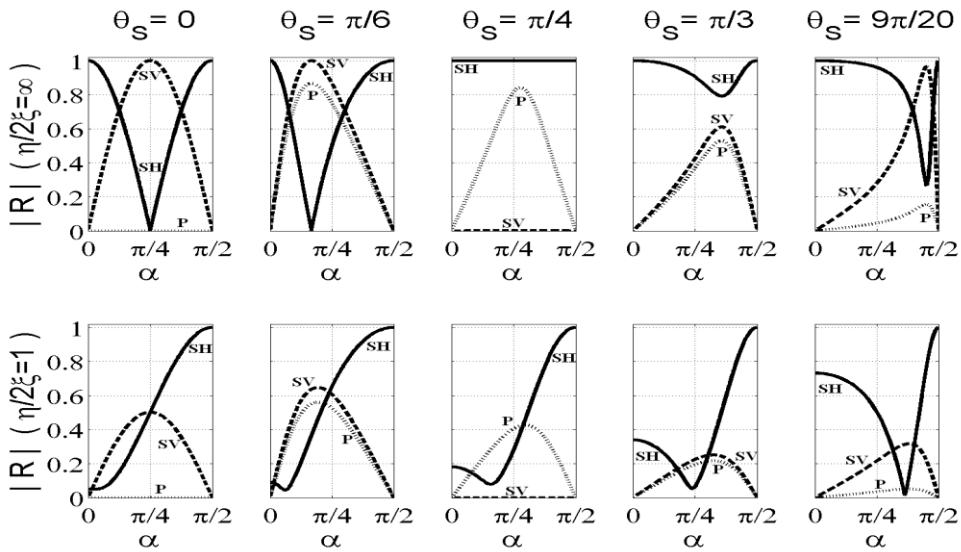

Figure 4.4 Modulus of the coefficients of reflection at the resonance of the oscillators for a θ_S-incident SH wave which polarization makes an angle α with the resonant direction e_o. Calculations performed with ν =1/3, η=10% and 2ξ=0% or 10%

Reciprocally, incident SV or P waves which incident plane makes an angle with the resonant direction set the oscillators in motion which impose out-of-plane stresses. Incident SV or P waves give rise to one reflected SH wave to compensate the out-of-plane stresses.

5. CONCLUSION

Providing a scale separation between the length and the distance between buildings, the periodic model of the city can be homogenized. At low and high pulsations, the surface condition is analogous to a free surface. But the resonance of the buildings leads to the apparent stiffening of the surface in the resonating direction only. Anisotropic resonators enable us to build in a practical way anisotropic surfaces, free in the non-resonant directions while apparent stiffened in the resonant direction.

It has been shown that an isotropic resonant surface leads to (i) the attenuation of the displacements of both the surface and the resonators around the eigen-frequency of the oscillator ; (ii) the modification of the conversions between P and SV waves. Besides, an anisotropic resonant surface leads to (i) the depolarization of horizontal shear waves ; (ii) a conversion between SH waves and SV and P waves.

An analogous experimental program within the SERIES project is being conducted with a resonant surface made up of over thirty bending beams resting on a soft layer. The specimen is designed so that the oscillators' eigenfrequency matches the fundamental frequency of the layer. It shall validate the theoretical results in the case of normally-incident shear waves and show the attenuation of the displacement at the resonance and the depolarization effect.

As an opening, what have been done on homogeneous waves is applicable to surface waves. For example, consider a Rayleigh wave propagating in the resonant direction e_o. Equalizing the stress generated by the P and SV inhomogeneous waves at the surface and the stress imposed by the oscillators while set in motion, the equation of dispersion will depend on the dynamical ratio between the surface impedance Z_O and the medium impedance Z_S. As a consequence, the velocity of the Rayleigh wave will depend on the pulsation resulting in the dispersion of the field. What is more, damped oscillators will attenuate the wave along its propagation, even though the medium is perfectly elastic.

REFERENCES

Boutin, C. and Roussillon, P. (2004). Assessment of the urbanization effect on seismic response. *Bulletin of the Seismological Society of America* **94:1**,251-268.

Boutin, C. and Roussillon, P. (2006). Wave propagation in presence of oscillators on the free surface. I*nternational Journal of Engineering Science* **44:1**,180-204.

Chavez-Garcia, F.J. and Bard, P.Y. (1994). Site effects in Mexico City eight years after the September 1985 Michoacan earthquakes. *Soil Dynamics and Earthquake Engineering* **13:1**,229-247.

Clouteau, D. and Aubry, D. (2001). Modification of ground motion in dense urban areas. *Journal of Computational Acoustics* **9:4**, 1659-1675

Guéguen, P., Bard, P.Y. and Chavez-Garcia, F.J. (2002). Site-city seismic interaction in Mexico City-like environments; an analytical study. *Bulletin of the Seismological Society of America* **92:2**, 794-811

Tsogka, C. and Wirgin, A. (2003). Simulation of seismic response in a idealized city. *Soil Dynamics and Earthquake Engineering* **23:1**, 391-402

Wirgin, A. and Bard, P.Y. (1996). Effects of buildings on the duration and the amplitude of ground motion in Mexico City. *Bulletin of the Seismological Society of America* **86:3**, 914-920