Damping factors for seismic design of structures with energy hysteretic dampers, located in the valley of Mexico.

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SUMMARY:
A simple mathematical expression is presented herein to estimate spectra reduction damping factors for seismic design of systems with hysteretic dampers. The factors are obtained from the ratios between Uniform Failure Rate Spectra (UFRSs) corresponding to different zones within the Valley of Mexico associated to various dominant ground vibration periods. The equation proposed is applicable to the limiting state near collapse of structural systems. The damping factor expression depends on the dominant ground period, the structural period, and on the parameters of the hysteretic dampers.

Keywords: Damping factor, Uniform Failure Rate Spectra, Hysteretic damper, Seismic design, Seismic codes

1. INTRODUCTION

Spectral design ordinates specified in most of the seismic design codes throughout the world can be generally reduced by ductility factors, resistant factors, and damping factors. In this study reduction damping factors are obtained from the ratios between Uniform Failure Rate Spectra (UFRSs) corresponding to different zones within the valley of Mexico associated to various dominant ground vibration periods.

A seismic hazard analysis is initially performed corresponding to each of the zones into which the valley of Mexico has been subdivided. For the purpose of defining the empirical transfer functions correlation was made of the spectral ordinates belonging to accelerograms recorded simultaneously in two stations (one of them in hard soil and the other in soft soil) and seismic hazard curves are obtained containing values of the exceedance rates for different seismic intensities of a site, for various structural periods. Then, demand hazard curves are calculated for different parameters of systems with hysteretic dampers (Castillo Cruz 2012).

Subsequently, a value of the exceedance rate, $\nu = 0.008$ is used to define the limiting state near collapse (associated to pseudo-accelerations with an expected return period of 125 years). Using the demand hazard curves, the value of the spectral acceleration is obtained and Uniform Failure Rate Spectra (UFRS) curves are plotted for different zones of the valley of Mexico and for various parameters for the systems with hysteretic dampers.

2. CHARACTERIZATION OF SYSTEMS WITH HYSTERETIC-TYPE DAMPERS

For purposes of analysis of structures with hysteretic-type dampers a linear and structural characterization should be made by using two properties: stiffness and strength. Such properties are defined in this study through parameters α and γ.

Parameter α is defined as the ratio existing between the stiffness of the damper and the stiffness of the base system (structure without energy dissipating devices), and γ is defined as the ratio between the yield force of the damper and the total force of the combined system:

$$\alpha = \frac{K_d}{K_c} \quad ; \quad \gamma = \frac{F_{yd}}{F_T} \tag{1}$$

where K_d is the stiffness of the damper, and K_c is the stiffness of the base system, F_{yd} is the yield force of the damper and F_T is the total force acting on the structure-damper system.

The structure-damper system is modeled as a one-degree-of-freedom system with critical damping of $\xi = 5\%$ plus a damping element as shown in figure 1.

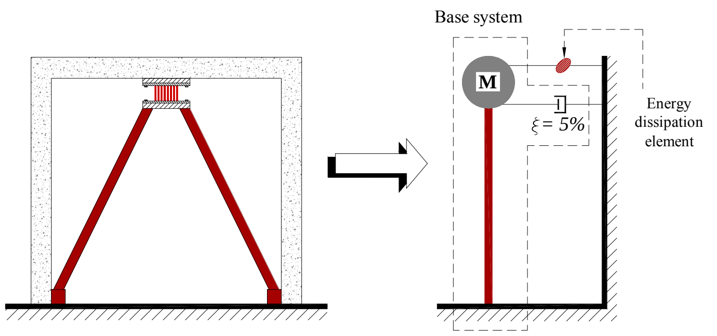

Figure1. Model of the structure-energy dissipating device system

Figure 2 depicts the behavior of a structure-damper system subjected to a monotonically increasing load. The curves correspond to the base system and to the energy dissipation element. The base system shows an elastic linear behavior whereas the damper presents an elasto-plastic behavior. The sum of the ordinates of the curves corresponding to the base system and to the damper gives place the bi-linear behavior of the structure-damper system.

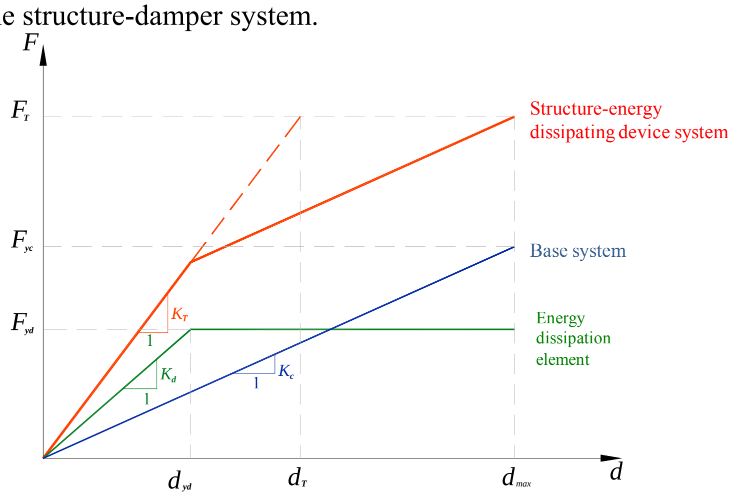

Figure2. Force-displacement curves of the parts constituting the structure-damper system

3. CURVES OF SEISMIC HAZARD AND CALCULATION OF UNIFORM FAILURE RATE SPECTRA

For the purpose of integrating a data base that takes into account the dynamic characteristics of the valley of Mexico selection was made of 334 seismic motions recorded by the accelerometer network of the valley (see Figure 3). A set of earthquakes with similar epicenter distances were chosen, subduction earthquakes with magnitudes exceeding 6.9 were selected. The response spectra of pseudo-acceleration of each record were calculated and their dominant periods were determined (depending on the type of soil where the motion was recorded).

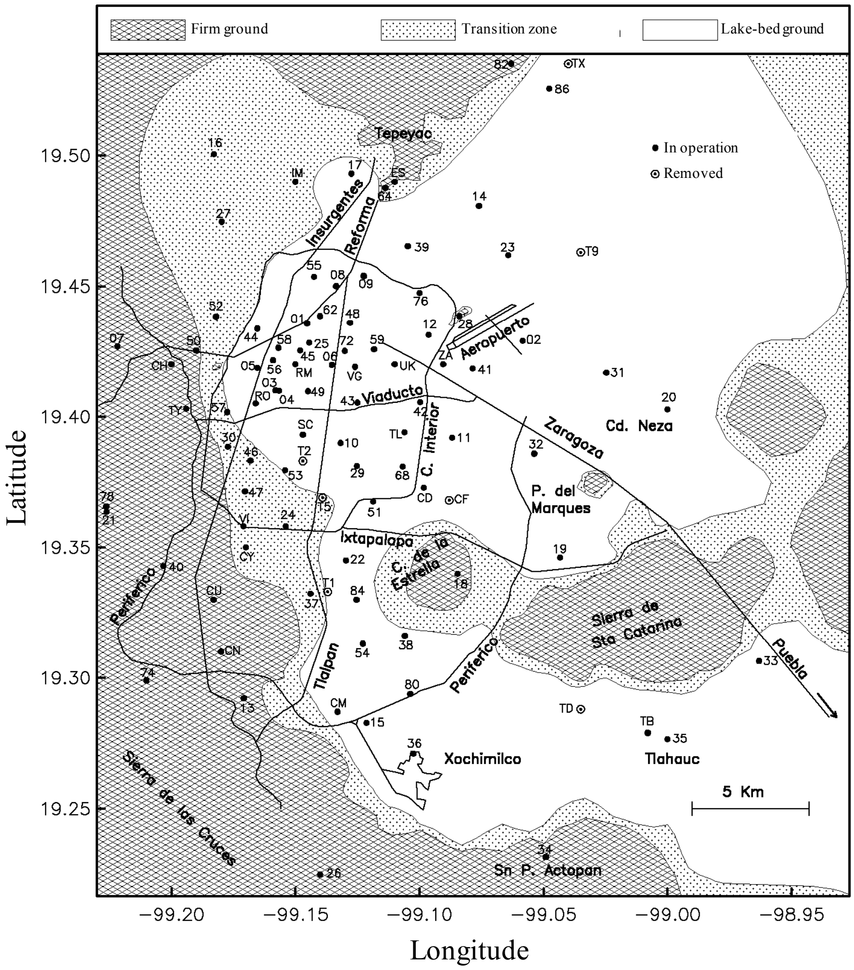

Figure 3. Accelerometer network of the valley of Mexico (Mexican Strong Earthquakes Ddata Base, 1999)

The seismic motions were classified into seven zones within the valley of Mexico, depending on the period where the peak spectral pseudo-acceleration took place and on the location of the station within the accelerometer network. Table 1 contains the list of the seven zones (A to G) and their corresponding interval of dominant soil periods (T_s).

Table 1. Zones in the valley of México

Zone	T_s [s]
Zone A	≤ 0.5
Zone B	$0.5< T_s \leq 1.0$
Zone C	$1.0< T_s \leq 1.5$
Zone D	$1.5< T_s \leq 2.0$
Zone E	$2.0< T_s \leq 2.5$
Zone F	$2.5< T_s \leq 3.0$
Zone G	$3.0< T_s \leq 4.0$

The Uniform Failure Rate Spectra (UFRS) contain the maximum ordinates that can occur in a particular site. These ordinate have the same probability of failure of the system, per unit time. The methodology to calculate the UFRS is described in the following (Rivera and Ruiz, 2007):

The total ductility of the combined system, μ_a , is defined as:

$$\mu_a = \frac{d_{MAX}}{d_y} \quad (2)$$

where d_y is the yield displacement of the base system and d_{MAX} is the maximum displacement of the combined system.

The yield displacement, d_y , of the combined system can also be defined as follows:

$$d_y = \frac{F_T}{K_T} = \frac{F_{yc}(1+\gamma)}{K_c(1+\alpha)} = d_{MAX} \frac{(1+\gamma)}{(1+\alpha)} \quad (3)$$

replacing Eqn. 4 in Eqn. 3, the total ductility of the combined system is expresed in terms of the parameters of the hysteretic damper:

$$\mu_a = \frac{(1+\alpha)}{(1+\gamma)} \quad (4)$$

On the other hand, the ductility of the hysteretic damper is defined as follows:

$$\mu_d = \frac{d_{MAX}}{d_{yd}} \quad (5)$$

It is possible to demonstrate that the total ductility of the combined syst can be expresed in terms of the ductility of the hysteretic damper and its characteristic parameters by the Eqn. 6:

$$\mu_a = \mu_d \left[\frac{\gamma d_{\max}}{\alpha d_y} \right] \quad (6)$$

The stiffness of the base system is defined as follows:

$$K_C = \frac{4\pi^2 M}{T^2} \quad (7)$$

With these last properties and the characteristic parameters of the hysteretic damper, the structural systems are excited with the records clasified in the seismic data base.

For each structural response the ratio between the maximum displacement and the the yield displacement is obtained. Once the demanded ductility ($\mu_{demanded}$) and the allowed ductility ($\mu_{allowed}$) are known, the parameter Q is defined as follows:

$$Q = \frac{\mu_{demanded}}{\mu_{allowed}} \quad (8)$$

In this study when the parameter Q is greater than unity a condition of failure is considered.

To calculate the seismic demand hazard curves the formulation suggested by Esteva (1968) was used. The annual rate of structural failure is calculated by means of the following equation (Esteva and Ruiz, 1989):

$$\nu_F = \int \left| \frac{d\nu}{dS_a} \right| P\left(Q \geq 1 \middle| S_a\right) dS_a \tag{9}$$

where $\left| d\nu / ds_a \right|$ is the absolute value of the derivative of the seismic hazard curve, and $P\left(Q \geq 1 \middle| S_a\right)$ is the probability of occurrence of the structural failure when subjected to a seismic intensity S_a.

The curves of seismic hazard (ν) were calculated in terms of the ratios between the response spectra for each zone and the response spectra of the station *Ciudad Universitaria* that is founded upon hard soil of the valley of Mexico.

Curves of seismic demand hazard were calculated for systems located in the different zones and with several values of the parameters α and γ. From these curves the values of the UFRS were obtained for each zone associated to a given exceedence rate, and given values of α and γ.

Figures 4 to 6 show Uniform Failure Rate Spectra (UFRS) corresponding to combined systems for zones A, D, and G of the valley of Mexico, with hysteretic-type dampers with different values of α and γ.

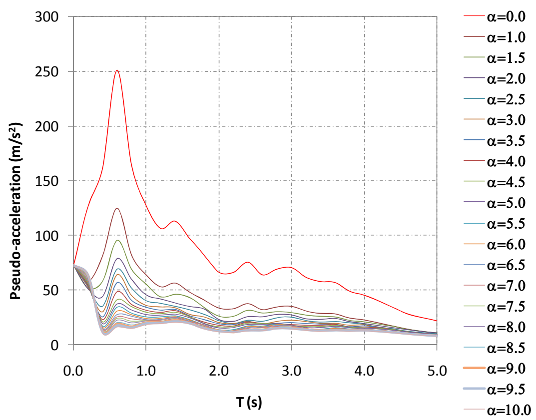

Figure 4. UFRS for the zone A, $\gamma = 0.50$ and different values of α

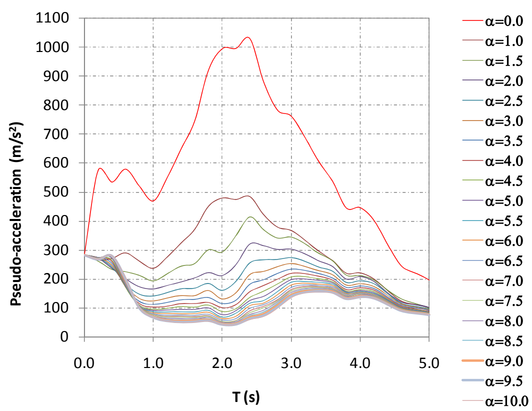

Figure 5. UFRS for the zone D, $\gamma = 0.50$ and different values of α

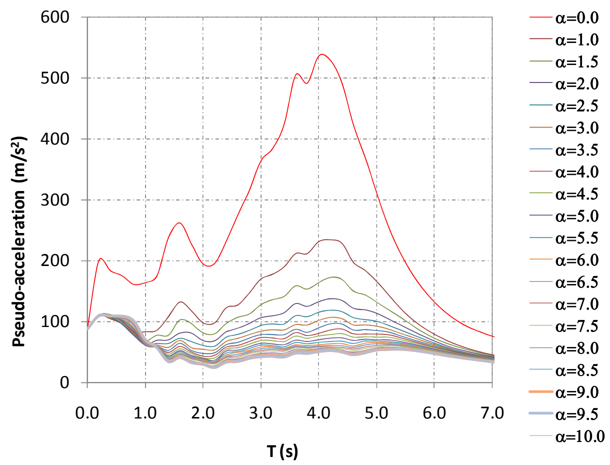

Figure 6. UFRS for the zone G, $\gamma = 0.50$ and different values of α

4. REDUCTION FACTOR β_h FOR STRUCTURES WITH HYSTERETIC-TYPE DAMPERS

The reduction damping factor (β_h) due to the presence of hysteretic dampers for each zone was obtained by dividing the UFRS corresponding to systems with hysteretic dampers by the value of UFRS corresponding to systems with no dampers ($\alpha = \gamma = 0$), as follows:

$$\beta_h = \frac{S_a(T,\alpha,\gamma)}{S_a(T,\xi = 5\%)} \tag{10}$$

where $S_a(T,\alpha,\gamma)$ is the UFRS corresponding to systems with hysteretic dampers and $S_a(T,\xi = 5\%)$ is the UFRS for conventional systems.

Figures 7 to 9 show the graphical representation of the spectral ratios for zones A, D and G and different values of α and γ.

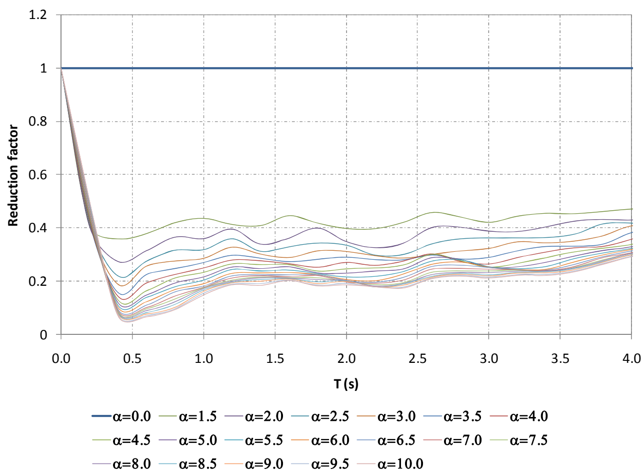

Figure 7. Spectral ratio for the Zone A, $\gamma = 0.5$ and different values of α

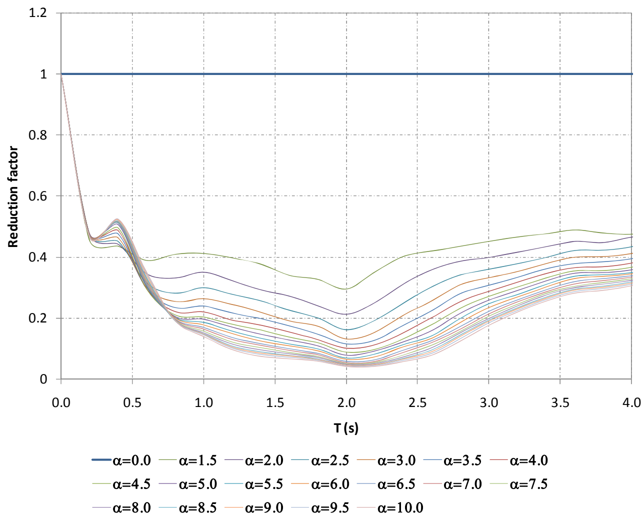

Figure 8. Spectral ratio for the Zone D, $\gamma = 0.5$ and different values of α

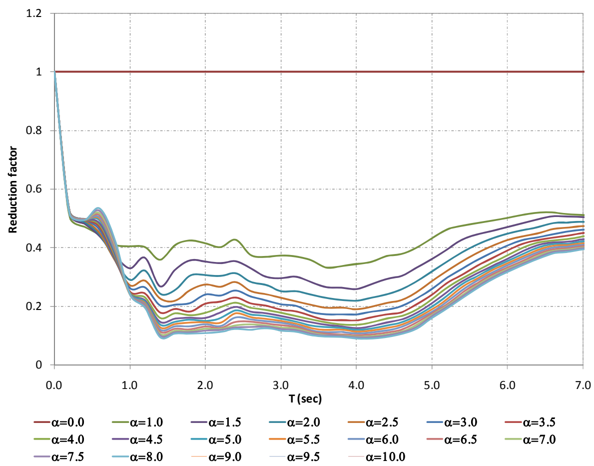

Figure 9. Spectral ratio for the Zone G, $\gamma = 0.5$ and different values of α

5. MATHEMATICAL EXPRESSION PROPOSED FOR THE DAMPING FACTOR β_h

The spectral ratios of each zone were fitted using the minimum square method applied to an equation that describes their behavior as a function of the structural period, the damper parameters α and γ as well as of the dominant ground period of each zone.

The equation proposed herein is divided into three parts: the first one corresponds to the envelope of the reduction factor β_h, the second part takes into account the characteristics of the damper as well as the ratio between the soil period and the structural period, and the last part considers the parameters that depend on the period of the soil. The equation proposed for determining the reduction factor β_h is the following:

$$\beta_h = \left[1 + \left[\frac{(\psi \cdot e)^{T_s}}{T_s} \cdot T_b \cdot \gamma \cdot \alpha \right]^{\frac{0.07 \cdot \alpha \cdot T_s}{\lambda \cdot T_b}} \right]^{-\Delta} \tag{11}$$

where λ, ε y Δ are parameters that depend on the parameters of the hysteretic device and the ratio between the structural period (*T*) and the dominant period of the ground (T_S), as follows:

$$\lambda = \eta_1 \gamma + \eta_2 \quad ; \quad \varepsilon = \eta_3 - \eta_4 \gamma \tag{12}$$

$$\Delta = \begin{cases} \lambda & ; \text{ if } T < F T_b \\ \lambda \left(\frac{F T_b}{T} \right)^{Z \cdot \varepsilon \cdot \sqrt{\frac{T_s}{T}} \cdot \ln\left(\frac{\alpha}{\gamma}\right)} & ; \text{ if } T \geq F T_b \end{cases} \tag{13}$$

F and Z depend only of the period of the ground:

$$F = \begin{cases} 2.5 \; ; & if \;\; T_S < 1 \\ 1 \; ; & if \;\; T_S \geq 1 \end{cases} \tag{14}$$

$$Z = e^{-2.5\,(T_s - 0.25)} + 0.4 \tag{15}$$

The values of the parameters $\eta_1, \eta_2, \eta_3, \eta_4$ and ψ are indicated in the table 2 and depend on each particular zone.

Table 2. Values of the parameters $\eta_1, \eta_2, \eta_3, \eta_4$ y ψ **.**

Zone	η_1	η_2	η_3	η_4	ψ
A	1.103	0.651	1.5	1	$1.2/\gamma$
B	0.931	0.671	2	1	$0.15/\gamma$
C	0.45	1.157	1.5	1	$0.4/\gamma$
D	1.143	0.953	2.63	2.17	$0.4/\gamma$
E	0.491	1.223	3.807	3.971	$\sqrt{0.2/\gamma}$
F	0.6	1.2	5	5.29	$\sqrt{0.2/\gamma}$
G	0.811	1.04	3.22	2.22	0.6

As it can be seen, Eqn. 11 is a function of the structural period, of the damper parameters α and γ, and of the dominant period of the ground for each particular zone.

Figures 19 to 21 illustrate the fitting of the function proposed and the reduction factors β_h for zones A, D and G, respectively, and different values of the parameters α and γ.

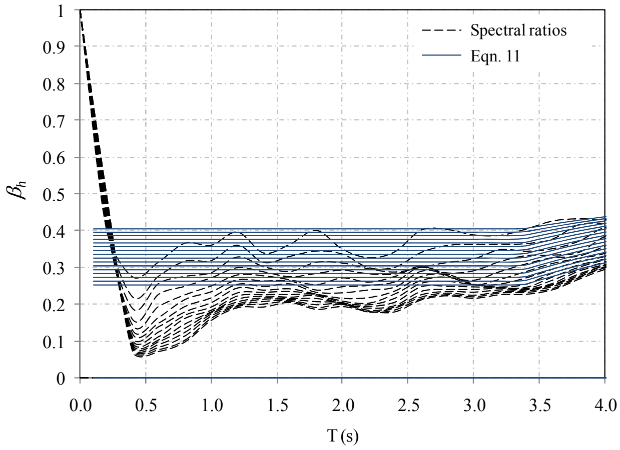

Figure 10. Fit for the Zone A, $\gamma = 0.50$

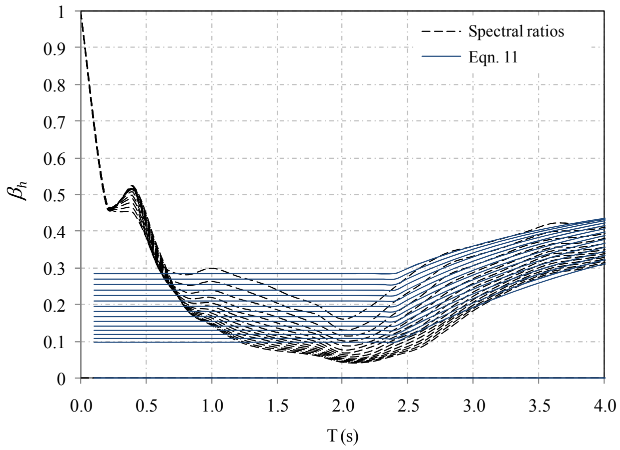

Figure 11. Fit for the Zone D, $\gamma = 0.50$

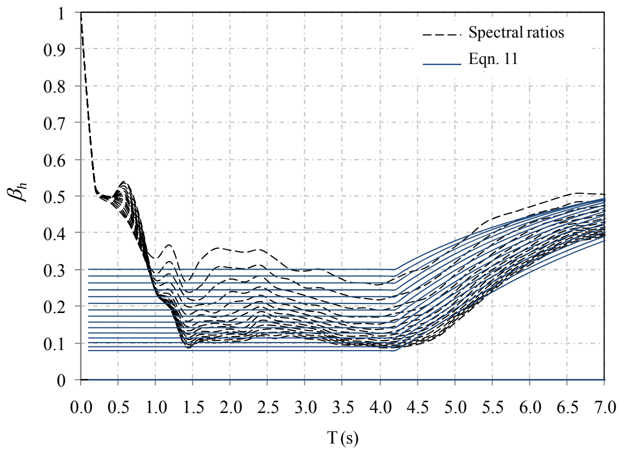

Figure 12. Fit for the Zone G, $\gamma = 0.50$

CONCLUSIONS

A simple mathematical expression is presented herein to reduce the spectral ordinates for purposes of design of structures with hysteretic energy dissipation devices. The equation is a function of the structural period, of the characteristics parameters of the hysteretic dampers and of the dominant period of the ground where the structure is located within the valley of Mexico.

As opposed to other papers presented on this subject matter, in this study the rule of reduction proposed was obtained from ratios between uniform failure rate spectra.

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