# Calculation of Response Spectrum Directly from Bedrock to Free Field using Equivalent Linear Method

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**SUMMARY:**
A stable relationship between the peak ground velocity (PGV) and the maximum value of 5%-damped pseudo-spectral velocity (PSV) is presented based on analysis of 818 far-field horizontal records selected from 16 large earthquakes in CESMD and PEER strong motion database. The validity and applicability of this relationship is inspected using 226 down-hole array records. As for the near-field ground motions, this relationship also shows good agreement by changing the regression parameter. A new approach to calculate the free field response spectrum of horizontal layered site directly from that of the bed rock is presented with the following iterative procedure: 1) Using the standard equivalent linear method model and the equivalent strain level (with assumed value for the first iteration) at the middle of each layer, determining the transfer functions for a unit input specified at the bedrock. 2) Treating the earthquake motions as a stationary random process with the power spectral density (PSD) function which is converted from the bed rock response spectrum, calculating the PSD functions of every layer using the transfer functions. 3) By converting the PSD functions to PSV, evaluating the PGV from the PSV, and calculating strain using the relationship between the shear strain and particle velocity of homogeneous medium, obtain the equivalent strain of every layer, which is used to get new shear modulus and damping to determine new transfer functions. After the iteration is converged, free field response spectrum is derived from PSD function of the surface layer. The usefulness and validity of the proposed method is demonstrated by comparing the results with those of conventional methods for different project sites.

*Keywords: Free field response spectrum; peak ground velocity; equivalent linear method; horizontal layered site*

## 1. INTRODUCTION

There are mainly two methods to derive free field response spectrum (spectral acceleration (SA) is used in this paper) from the bedrock SA. One is the conventional method (CM) that several artificial accelerograms, generated in accord with the given bedrock RS, serves as bedrock input. The average free field SA is obtained with a site response analysis procedure such as SHAKE(Schnabel et al.,1972). This is the most commonly way in practice(GB 17741-2005, 2005) but it is very cumbersome. The other one is to calculate the free field SA directly from the bedrock SA. Studies on this method are firstly reviewed at the beginning of this paper. Then the relationship between PGV and maximum value of PSV is confirmed by regression of the strong earthquake motion. Based on this relationship and the theory of equivalent linear method, a new method for calculation of free field SA directly from bedrock SA is proposed. To demonstrate the validity of the new method, several examples are presented including five sites used in the former research.

Su had proposed a method using the assumption that the input motion is stationary Gaussian process and the soil profile can be modeled as multi-degree-of-freedom(MDOF) system (Su et al.,1992). With the relationship between power spectral density (PSD) function and the spectral acceleration (SA), the free field SA is calculated from the bedrock SA. Furthermore, equivalent linear theory is used to consider dynamic soil properties. But the complicated parameters hinder the wide application of this method.

Based on the elastic assumption and the relationship between SA and PSD of the earthquake records, Afra and Pecker(Afra and Pecker,2002)proposed a method with the following process: calculating the PSD from the given bedrock SA as the input; calculating the transfer function of the soil which is 1 or 2 layers at the top of the rock base; multiplying the input PSD and the transfer function and converting it into the SA. But this method will be unfeasible when the nonlinear properties of the soil could not be negligible during strong earthquakes.

Miura (Miura et al.,2001) convert the bedrock SA into Fourier amplitude spectrum using the relationship between SA and the 0-damping velocity spectrum which is assumed to approximate the Fourier amplitude spectrum. The site is simplified as a single layer and the response of the free field is calculated with the harmonic input motion. Then the site is idealized as a lumped-mass system with the mass at the central of each layer and the transfer functions of the first and second model are determined. The strain of each layer is calculated using the lumped mass model by supposing the response of the field is identical and the site response is controlled by the first mode. New dynamic soil properties are refined using the strain as well as the function of dynamic shear moduli and damping ratio versus strain amplitude. Then the stiffness and damping of the lumped mass model are also renewed based on the new dynamic soil properties. The iteration is ended when the period of the first mode is converged. This method is proposed in the new version of building standard law of Japan. But there are two main shortcomings of this method. One is the assumption that the basic model of the MDOF dominates the site response and the response of soil strata is equivalent to a single layer on half space. The other one is that only two specific values of SA corresponding to the first and second periods are worked out while the main part of the SA is constructed with empirical methods.

## 2. RELATIONSHIP BETWEEN PGV AND SPECTRA

Like the peak ground acceleration (PGA), PGV is a key parameter indicating the characteristics of the ground motions. A widely used relationship, proposed by Newmark(Newmark and Hall,1982), is given as: PGV can be calculated from the 5%-damped PSV at 1 second( PSV(T=1.0)),

$$PGV = PSV\left(T = 1.0\right)/1.65 \tag{2.1}$$

Based on analysis of the published ground-motion prediction equations, stochastic simulations and strong motion data, Bommer(Bommer and Alarcon,2006) educed empirical correlations between PGV and SA at 0.5 second,

$$PGV = SA\left(T = 0.5\right)/20 \tag{2.2}$$

To derive a new predictive equation between the PGV and spectrum for the purpose of accuracy and stability, this paper performs regression analysis of the far-field strong motion records selected from the database of the Pacific Earthquake Engineering Research Center (PEER)(http://peer.berkeley.edu) and Center for Engineering Strong Motion Data(CESMD)(http://strongmotioncenter.org). Only one horizontal component among the three components at a station is used. The stations of all records selected are either free field sites or ground and basement of one or two stories buildings in order to reduce the influence of soil-structure interaction. Totally 818 samples from 16 large earthquakes listed in table 1 are analyzed. With regression analysis, a kind of stable linear correlations between PGV and the maximum PSV of the far-field strong motions is,

$$PGV = \max\left(PSV\right)/R_1 \tag{2.3}$$

The parameter $R_1$ can be identified with the simple least-squares optimal fit and $R_1$=3.0 for far-field records. Fig. 1 shows the comparison of the results calculated with different equations and those of the records. The results of the proposed equation are in good agreement with the records than the other two.

**Table 2.1.** Parameters of Earthquake Records Selected for Prediction of PGV

| No | Earthquake Name | Time (mm/dd/yy) | Mag. ($M_L$) | Epicenter Location (Long., Lat.) | Far-field records | | Near-field records | |
|---|---|---|---|---|---|---|---|---|
| | | | | | Epicentral Dist.(km) | Sum | Epicentral Dist.(km) | Sum. |
| 1 | Coalinga | 05/02/1983 | 6.5 | 36.25N ,120.30W | 31.7~67.7 | 42 | - | - |
| 2 | Morgan Hill | 04/24/1984 | 6.2 | 37.32N,121.68W | 32~79 | 15 | 3.8 | 2 |
| 3 | Palm Springs | 07/08/1986 | 5.9 | 34.00N,116.61W | 29~83 | 15 | 10.0~21.0 | 4 |
| 4 | Whittier Narrows | 10/01/1987 | 6.1 | 34.06N ,118.07W | 29.3~101.9 | 26 | 13.9~19.4 | 10 |
| 5 | Loma Prieta | 10/18/1989 | 7.0 | 37.04N ,121.88W | 28~106.8 | 45 | 2.8~21.0 | 28 |
| 6 | Landers | 06/28/1992 | 7.3 | 34.22N, 116.43W | 31~186.1 | 43 | 14.0 | 2 |
| 7 | Bigbear | 06/28/1992 | 6.5 | 34.20N, 116.83W | 37.9~157.2 | 34 | 11.0 | 2 |
| 8 | Northridge | 10/17/1994 | 6.7 | 34.21N,118.54W | 30~147.1 | 54 | 5.0~20.0 | 22 |
| 9 | Hector Mine | 10/16/1999 | 7.1 | 34.60N, 116.27W | 48.4~216.5 | 58 | — | — |
| 10 | Anza | 10/31/2001 | 5.1 | 33.50N ,16.52W | 29~108 | 38 | — | — |
| 11 | Big Bear City | 02/22/2003 | 5.4 | 34.31N,116.85W | 31~100 | 52 | 8.0~17.0 | 8 |
| 12 | San Simeon | 12/22/2003 | 6.5 | 35.71N,121.10W | 48~442 | 42 | 12.0 | 2 |
| 13 | Parkfield | 09/28/2004 | 6.0 | 35.81N,120.37W | 31~151 | 11 | 0.5~19.0 | 102 |
| 14 | Anza | 06/12/2005 | 5.6 | 33.53N,116.57W | 29~102 | 54 | 5.1~19.5 | 18 |
| 15 | Yucaipa | 06/16/2005 | 5.3 | 34.06N,117.01W | 28~108 | 60 | 2.9~20 | 18 |
| 16 | Chi-Chi Taiwan | 09/20/1999 | 7.6 | 23.86N,120.80E | 29.3~185 | 228 | 2.2~21.9 | 86 |

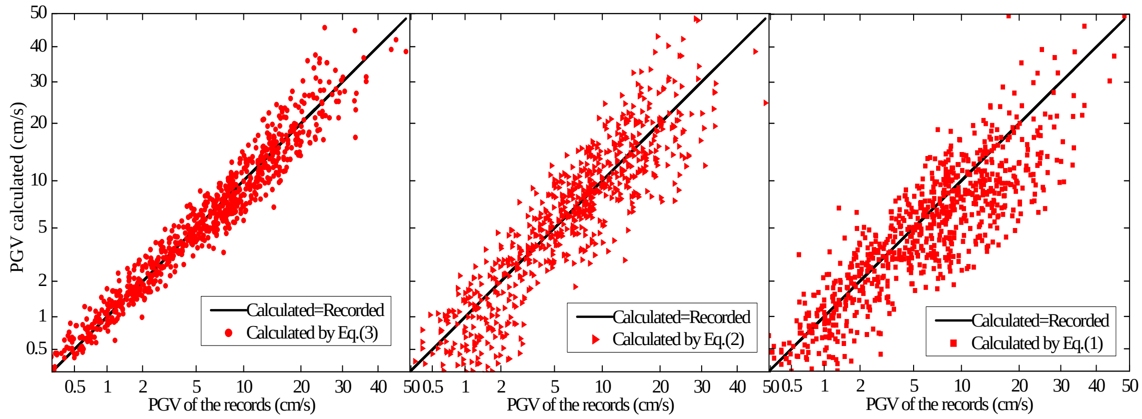



**Figure 1.** Comparison of PGV with different equations for the far-field records

Near-field records with epicentral distance less than 22km are selected from those large earthquakes in Table.2.1. Two horizontal components of the same station are regarded as two independent random processes and totally 304 records are used. Regression result shows that the relationship between PGV and the maximum value of PSV can also be expressed with Eq.(2.3) with parameter $R_1 = 2.4$, as illustrated in Fig. 2. Furthermore, comparison in Fig.2 indicates that with increase of PGV, the deviation of Eq.(2.3) will increase. There is a suggestion that the predictive equation should be carefully handled for sites close to active faults where near-source directivity effects may be expected. It is recommended that $R_1$ in predictive Eq.(2.3) equal to 3.0 for far-field earthquake sites while for near-source sites a smaller value of 2.4 is proposed in this paper.

Another comparison is made of the above-mentioned records with data recorded by down-hole arrays which provide detailed information for studies of local site effects. The acceleration time series whose

PGA are not less than 0.01g are selected from CSMIP database of four down-hole arrays: Treasure Island, La Cienega, Eureka and El Centro-Meloland. Two horizontal components of each station, regarded as independent samples, are all selected and total records are 226. Detailed information of the down-hole arrays and earthquakes are described by Graizer et al.(2000). Regression results of the down-hole array records are shown in Fig.3, which shows that the results obtained with Eq.(2.3) when $R_1$ is equal to 3.0 are in good agreement with the records.

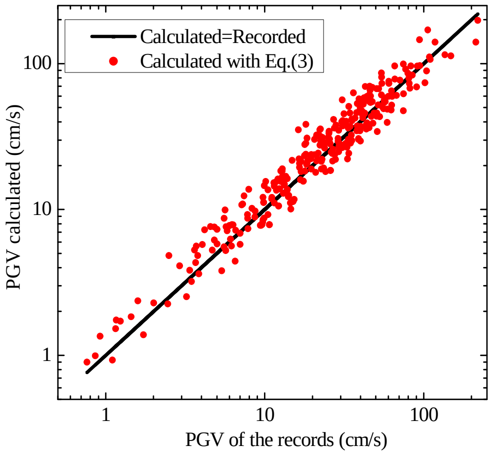



**Figure 2.** Near-field records results

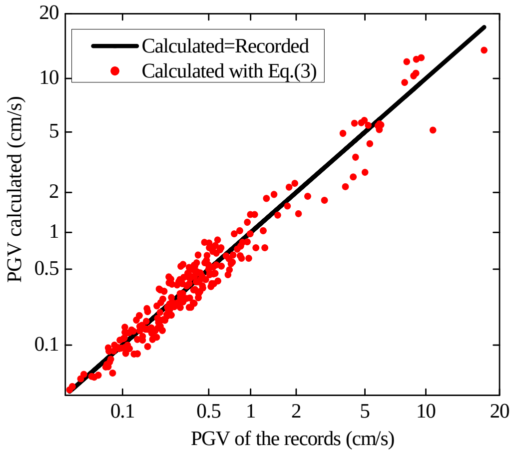



**Figure 3.** Dow-nhole array records results

## 3. PROPOSED METHOD

### 3.1. Derivation of SA from PSD and vice versa

The correlation between SA and PSD is significant in the simulation of strong ground motion. Park(1995) and Paskalov and Reese(2003) presented simple approach to convert SA from PSD and vice versa, respectively. Inasmuch as the approach of Park needs no numerical iterations, it is feasible for discrete SA and PSD. Forward equation of derivation of SA from PSD is,

$$SA^2(\omega_k,\lambda) \simeq \sum_{j=1}^{m} p_j R_{k,j}^2(\omega_k,\omega_j,\lambda) \quad k=1,2,\ldots,n \tag{3.1}$$

Where $p_j = G(\omega_j)\Delta\omega_j; G(\omega_j)$ is the discrete PSD and $\lambda$ is damping ratio. Using $v_j = \omega_j/2\pi$ and $\chi = \omega_j/\omega_k$, $R_{k,j}^2(\omega_k,\omega_j,\lambda)$ in Eq.(3.1) can be expressed as,

$$R_{k,j}^2(\omega_k,\omega_j,\lambda) = \left\{\sqrt{2\ln(v_jT_e)} + \frac{5.772}{\sqrt{2\ln(v_jT_e)}}\right\}\sqrt{\frac{1+4\lambda^2\chi^2}{(1-\chi^2)^2+4\lambda^2\chi^2}}$$

Where $T_e$ is the effective duration of ground motion and can be determined from the given envelope function. Backward calculation of PSD from SA is obtained by solving the following equation,

$$\begin{aligned} &\min \sum_{k=1}^{n}\left\{SA^2(\omega_k,\lambda) - \sum_{j=1}^{m} p_j R_{k,j}^2(\omega_k,\omega_j,\lambda)\right\} \\ &p_j \ge 0; \quad j=1,2\ldots,m \end{aligned} \tag{3.2}$$

### 3.2. Introduction of Equivalent Linear Method

The significance of the local site effect has long been recognized by earthquake engineers and seismologists which triggers the development of equivalent linear method for 1D seismic response analysis since the later 1960s(Idriss and Seed,1968). SHAKE is the most popular program on the list as it was among the first to be developed, and even today it is still widely used and studied. By idealizing the soil deposit as a stratified, viscoelastic medium and the seismic excitation as vertical propagation shear waves, the equivalent linear method adjusts the stiffness and damping ratio of soil layers in each iteration until they are compatible with the strain level induced by the earthquake loading. In frequency domain, harmonic displacements $u(z,t)$ with frequency $\omega$ can be written as $u(z,t)=U(z)\exp(i\omega t)$. The amplitude of the incident and reflected wave at depth $z$ is $U(z)=E(\omega)\exp(ikz)+F(\omega)\exp(-ikz)$, where $k=\omega\big/\sqrt{G(1+2i\lambda)/\rho}$; $G$ and $\rho$ are shear modulus and mass density, respectively. $E(\omega)$ and $F(\omega)$ can be determined with the boundary conditions: displacements and shear stress must be continuous at the layer interfaces as well as the stress must be zero at the free surface. Thus the transfer function $H_{n,N}(\omega)$ between the displacements at layer $n$ and the input layer $N$ can be defined as,

$$H_{n,N}(\omega)=\frac{E_n(\omega)+F_n(\omega)}{E_N(\omega)+F_N(\omega)} \tag{3.3}$$

Where $E_n(\omega)$ and $F_n(\omega)$ are amplitude of incident and reflected waves can be determined with the boundary conditions. Dynamic shear modulus $G_n$ and damping ratio $\lambda_n$ at layer $n$ are defined as a function of the equivalent shear strain $\overline{\gamma}_n$ as following,

$$G_n=G(\overline{\gamma}_n) \qquad \lambda_n=\lambda(\overline{\gamma}_n) \tag{3.4}$$

Here $\overline{\gamma}_n=\alpha\gamma_{n,\max}$, $\alpha=0.65$ and $\gamma_{n,\max}$ is the maximum strain at layer $n$. As strain is unknown beforehand, iteration is need.

### 3.3. Algorithm of the Proposed Method

With the assumption that the earthquake motion is a stationary Gaussian random process and the theory of the equivalent linear method, site response under narrow band frequency can be determined by idealizing the soil strata as linear system. According to the stochastic vibration theory, the relationship of PSD function between of layer $n$ and input layer $N$ is given as,

$$G_n(\omega)=\left|H_{n,N}(\omega)\right|^2 G_N(\omega) \tag{3.5}$$

The maximum strain $\gamma_{n,\max}$ is needed to determine the equivalent strain $\overline{\gamma}_n$ when the transfer function is calculated with equivalent linear method. When plane shear waves propagate in a homogeneous soil, the shear strain $\gamma$ at some arbitrary point $z$ is directly proportional to the particle velocity $\dot{u}$ at that point:

$$\gamma=\frac{\partial u(t-z/V_s)}{\partial z}=-\frac{1}{V_s}\frac{\partial u}{\partial t}=-\frac{1}{V_s}\dot{u} \tag{3.6}$$

Where $V_s$ represents shear velocity of the soil layer. Thus the maximum strain of the soil under stationary process input is,

$$\gamma_{\max} = \dot{u}_{\max} / V_s \simeq PGV / V_s \tag{3.7}$$

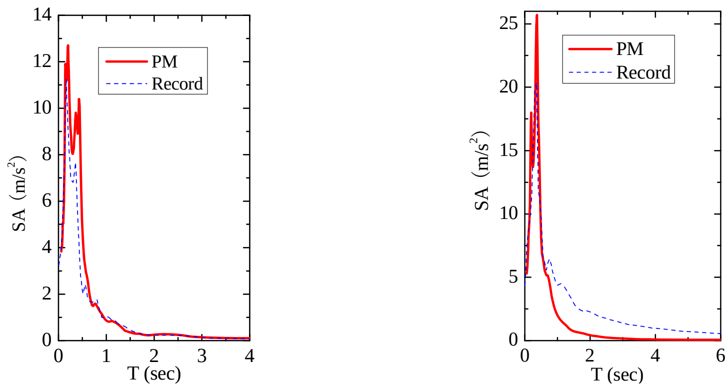
The following iteration steps are presented for the calculation of free-field SA from the bedrock SA:

- Discrete bedrock PSD function $G_N(\omega)$ is derived from bedrock response spectra $SA_N(\omega)$ with Eq. (3.2) (here that 100 discrete points will be sufficient for $SA_N(\omega)$ and $G_N(\omega)$). Set initial equivalent shear strains $\bar{\gamma}_n^0$ to each layer.
- Determine dynamic material properties $G_n$ and $\lambda_n$ of each layer based on Eq. (3.4) and equivalent shear strain $\bar{\gamma}_n$. Obtain the transfer function $H_{n,N}(\omega)$ with Eq.(3.3) and the recursion formulas for amplitude $E_n(\omega)$ and $F_n(\omega)$.
- Calculate PSD function $G_n(\omega)$ in each layer under the input $G_N(\omega)$ with Eq.(3.5) and then convert it into the response spectrum $SA_n(\omega)$ by Eq.(3.1). Determine $PSV_n(\omega)$ using the relationship between PSV and SA( $PSV_n(\omega) = SA_n(\omega)/\omega$ ), then get the maximum value $\max(PSV_n)$. $PGV$ is estimated with Eq.(2.3).
- Define the maximum strain $\gamma_{n,\max}$ by Eq.(3.7) in each layer. Multiply $\gamma_{n,\max}$ by factor $\alpha$ to get the equivalent strain $\bar{\gamma}_n$.
- Compare the equivalent strain with their values in the previous iteration. Iterate as necessary.

## 4. CASE STUDIES

### 4.1. Example 1

The down-hole array records obtained at Chiba site of Industrial Institute of Science, Tokyo University, during the 1987 Chiba-ken Toho-oki earthquake (M=6.7) and Shin-Fuji transformer station during 1983 Kanagawaken-Yamanashiken-Kenkyo earthquake (M=6.0) are used to testified the propose method. Soil profiles, dynamic soil properties, accelerometer locations and the acceleration records are given by Yoshida(Yoshida et al.,2002) and Jiang(Jiang and Xing,2007). Response spectra can be directly calculated from the acceleration time histories. The 5% damped SA obtained from the accelerations of the bottom accelerometers (48m depth at Chiba site and 28m depth at Shin-Fuji respectively) are used as input to evaluate the surface response spectra with factor $R_1 = 2.4$. The SA of the proposed method and the records are very similar, as illustrated in Fig.4.

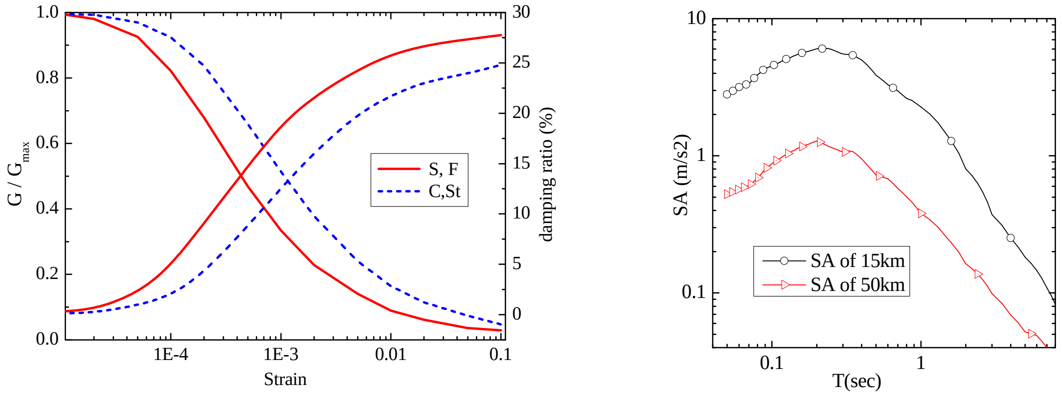



(a) Chiba site (b) Shin-Fuji site

**Figure 4.** Comparison of free field SA between the results of the proposed method( red line denoted as PM) and those of the down-hole array records(dash line)

### 4.2. Example 2

Five soil profiles with detailed description of soil properties(Fig.5(a)), depths and shear wave velocities(Fig.6(a)), had been given by Miura et al(2001). Response spectra were established for earthquake magnitude M=6 with a near-field distance 15km and a far-field distance 50km using attenuation relationships for North China region(GB 50267-97,1998), as illustrated in Fig.5(b). The free field SA was calculated with the proposed method and the conversional equivalent linear method. It was noted that factors for evaluating PGV in Eq.(2.3) were 3.0 for far-field earthquakes and 2.4 for near-field ones. Comparing the free field response spectra evaluated by the proposed method and the conventional way in Fig. 6(b) and (c), it was observed that results for far-field earthquakes were in excellent agreement while spectra predicted by the proposed method were a little higher at site 1 and 3 for near-field earthquakes. A smaller value of factor $R_1 = 1.6$ will improve the results of the proposed method, as is shown in Fig. 6(c). This example indicates that the scaling factor $R_1$ in Eq.(2.3) is a key parameter of the proposed method. It is recommended that the value of $R_1$ is 3.0 for small to moderate earthquakes and is 2.4 or smaller for large earthquakes.




(a) Soil properties (b) Input SA

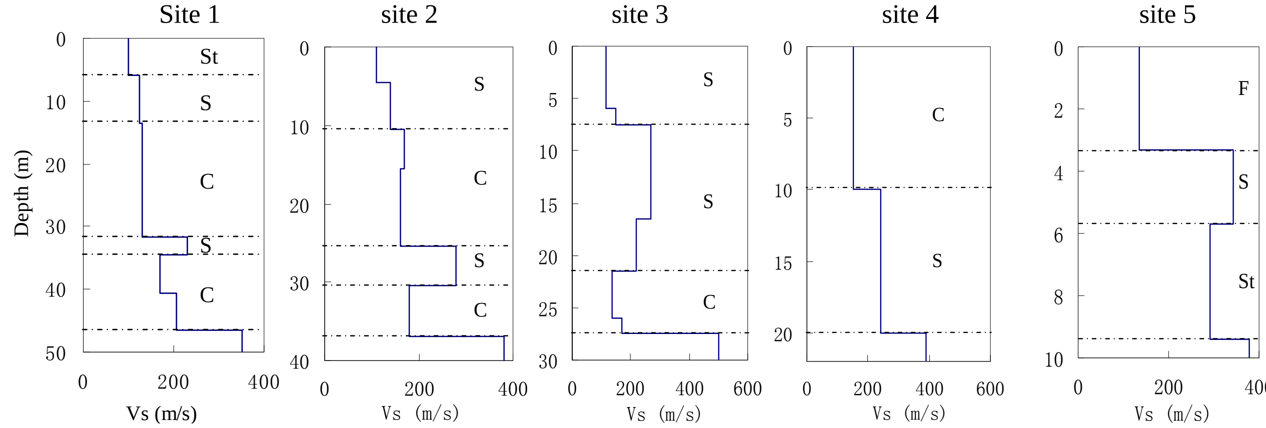
**Figure 5.** Soil properties and input SA for five typical sites

## 5. DISCUSSION AND CONCLUSION

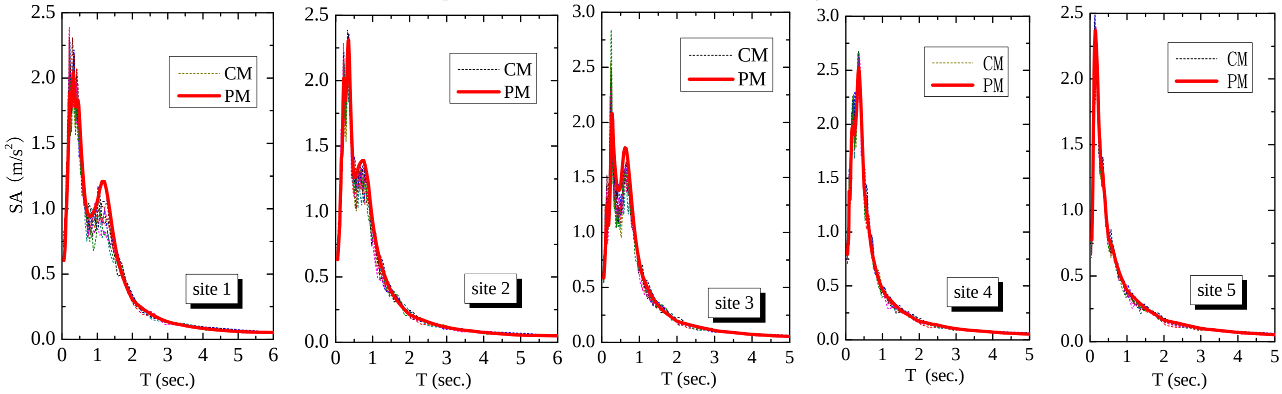
Empirical relationship between PGV and the maximum value of 5% damped PSV is regressed from more than 800 far-field strong motion records obtained at West America and Taiwan region. The validity of this relationship is testified with down-hole array records, which indicates that it can be used for site response analysis.

A simple method by which the free surface response spectra can be directly calculated from that of the bedrock is proposed based on the theory of equivalent linear method and the relationship between PGV and response spectra. Case studies show that the results of the propose method are always in good agreement with those of the conventional way as well as those derived from the down-hole array records.

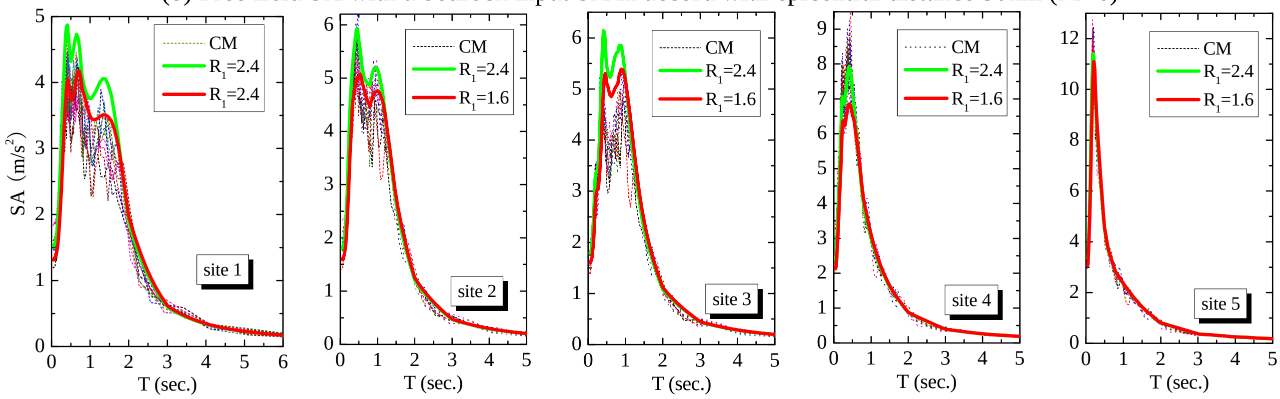
Two key factors that influence the accuracy of the proposed method are the correlation between SA and PSD and the approximate relationship between PGV and the response spectrum. The approach to convert the SA into PSD and vice versa, as propose in section 3.1, does not influence validity of the propose method if the approach has sufficient accuracy. PGV can be estimated by PSV or SA with Eq.(2.1) and (2.2) respectively provided that those equations satisfy the required precision for specified site conditions. Because the relationship of Eq.(2.3) is regressed with far-field strong motion records, a smaller value of the scaling factor $R_1$ is recommended for near-field earthquakes to take account of evident nonlinearity of soil properties. It is noted that using smaller value of $R_1$ in the proposed method is solely for the sake of accuracy, and not indicates the invalidity of Eq.(2.3). Furthermore, the proposed method has the same advantages and deficiencies as the equivalent linear method.




(a) soil profiles (S:sand;; F:fill;; C:clay;; St:silt)




(b) Free field SA with a bedrock input SA in accord with epicentral distance 50km (M=6)




(c) Free field SA with a bedrock input SA in accord with epicentral distance 15km (M=6)

**Figure 6.** Comparison of response spectra calculated with SHAKE(CM) and the proposed method(PM)

## REFERENCES


China Earthquake Administration, GB 50267-97, (1998), Code for seismic design of nuclear power plants, Beijing: China Planning Press.(in Chinese)

General Administration of Quality Supervision, Inspection and Quaratine of the People's Republic of China, GB 17741-2005, (2005), Evaluation of seismic safety for engineering sites, Beijing: Standards Press of China.(in Chinese)

Afra, H., and Pecker, A.,(2002), Calculation of free response spectrum of a non-homogeneous soil deposit from bed rock response spectrum. *Soil Dynamics and Earthquake Engineering*, **22:** 157-165.

Bommer, J. J., and Alarcon, J. E., (2006), The prediction and use of peak ground motion. *Journal of Earthquake Engineering*, **10:1,** 1-31.
Graizer, V., Shakel, A., and Hipley, P., (2000), Recent data recorded from down-hole geotechnical arrays. *Proceedings of CSMIP2000 Seminar on Utilization of Strong-motion Data*, Sacramento, California, 23-38.
Idriss, I. M., and Seed, H. B., (1968), Seismic response of horizontal soil layers, *Journal of the Soil Mechanics and Foundations Division, ASCE*, **94:SM4**,1003-1031.
Jiang, T., and Xing, H., (2007), An equivalent linear method considering frequency-dependent soil properties for seismic response analysis. *Chinese Journal of Geotechnical Engineering,* **29:2,** 218-224. (in Chinese)
Miura, K., Koyamada, K., and Iiba, M.,(2001), Response spectrum method for evaluating nonlinear amplification of surface strata, *Journal of Structural Engineering, AIJ*, **539,**57-62.
Newmark, N. M., and Hall, W. J.,(1982), Earthquake spectra and design, *Earthquake Engineering Research Institute, Berkeley, California*.
Park Y. J., (1995), New conversion method from response spectrum to PSD functions. *Journal of Engineering Mechanics, ASCE*, **121:12,**1391-1392.
Paskalov, A., and Reese S.,(2003), Deterministic and probabilistic floor response spectra, *Soil Dynamics and Earthquake Engineering*, **23,** 605-618.
Schnabel, P. B., Lysmer, J., and Seed H. B., (1972), SHAKE: a computer program for earthquake response analysis of horizontally layered sites, *Report No. EERC72-12,* University of California, Berkeley.
Su, J., Zhou, X., and Fan, S., (1992), A practical method for calculating ground response spectrum directly from base rock response spectrum, *Chinese Journal of Geotechnical Engineering*, **14:5,** 27-35.(in Chinese)
Yoshida, N., Kobayashi, S., Suetomi, I., al. et., (2002), Equivalent linear method considering frequency dependent characteristics of stiffness and damping, *Soil Dynamics and Earthquake Engineering*, **22:,** 205-222.