Force demands on circular piles subjected to the passage of seismic waves

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SUMMARY: (10 pt)
Several authors have reported that bending moments generated at the fixed base of a pile, or at the interface between a soil layer and a stiffer bottom one, by the passage of seismic waves through the soil can exceed the moment at the head of the pile. The present work explores the influence that has each of the variables affecting the problem (soil properties and structural properties) by considering first a harmonic motion at the base and determining the changes of the bending moments with frequency for a single soil layer and a pier fixed at the base, and for a soil deposit with 2 soil layers. The results show that the kinematic moments in the pile at the interface between two layers with different properties can in some cases exceed the moment at the top of the pile if the ratio between the shear wave velocities is large enough.

Keywords: Passage of seismic waves, Piles, Soil Structure Interaction

1. INTRODUCTION

The dynamic stiffness of a single pile and the effects of kinematic interaction due to seismic waves on the motion at the head of the pile were studied by Blaney (1974) using a consistent boundary matrix developed by Blaney et al (1974) to reproduce a layered soil deposit surrounding the pile and adding the dynamic stiffness matrix of the pile itself. A number of studies have been conducted and published since then by Fan et al (1991), Kaynia and Kausel (1982), Kaynia (1991) and Kaynia and Mahzooni (1997), Dobry and O'Rourke (1983), Mylonakis (2001) and Maiorano et al (2009) among others investigating inertial and kinematic effects for foundations on single piles and pile groups. In those studies emphasis has been placed not only in the filtering of high frequency components of motion by the piles, but also on the forces induced in the piles by the seismic waves where it was found that the pile's forces are mostly due to the kinematic interaction effects, showing a strongly dependence on the piles' rigidity, with exception of frequencies close to the natural frequency of the soil-structure system where the inertial interaction is more significant.

There are in the literature several recommendations where it has been reported that the SSI effects due to pile-group configuration, number of piles in the group, and relative spacing between piles are not of great significance for lateral displacements, but for pile-cap rotation, as the soil's properties are; further, the kinematic bending effects in a pile group can be accurately predicted through a single pile response analysis (Fan et al, 1991, Kaynia and Kausel, 1982, Nikolaou et al, 2001, and Maiorano et al, 2009). Another important finding is that the pile-head fixidity conditions and the elastic modulus between pile and soil have an important and similar effect on the seismic response of both single pile and pile group (Fan et al, 1991, Kaynia and Kausel, 1982). On the other hand, it has been recommended for small and intermediate size pile groups that are not closely spaced to ignore the pile-soil interaction and to treat the individual piles in the group as fixed-head single piles (Kaynia and Mahzooni, 1997). Others works have pointed out that, kinematic bending strains are larger between layers with different stiffness and at the head of the caped pile (Dobry and O'Rourke, 1983, Nikolaou et al, 2001, and Maiorano et al, 2009). Contrary to the belief that the Winkler representation is always approximate, Mylonakis (2001) showed that it accurately describes the pile-soil interaction when dividing the vertical soil shear tractions and the corresponding settlements along the pile, and also proposed, from a nonlinear regression, an expression to evaluate a depth-independent Winkler modulus that ranges between 2.7Gs and 1.8Gs for the most practical cases of interest.

The fact that in the seismic design of a pile foundation it is necessary to consider not only the bending moments due to the forces transmitted by the structure to the piles but also those induced by the seismic waves travelling through the soil, had been recognized and mentioned repeatedly (Nikolaou et al, 2001). Some of those studies developed rigorous and simplified expressions, simplified charts or different analytical models to evaluate the maximum forces imposed on the piles by the kinematic effects (Dobry and O'Rourke, 1983, Nikolaou et al, 2001, Maiorano et al, 2009, and Mylonakis, 2001), but most of them considered the case of a pile, or a pile group, penetrating a hard layer in a layered medium, and in particular a soil deposit with two layers of different stiffness, and indicating that the moments due to the seismic waves could in fact be larger than those caused by the structural vibrations. The objective of this work is to investigate under what conditions this may happen. To achieve the goal a harmonic motion at the base rock with unit acceleration (1m/sec^2) is considered to study the variation of the bending moments with frequency. The results of the study reported here should help to understand the importance of this relationship.

The case of a single soil layer of finite thickness with uniform properties is studied first assuming that the pile penetrates the underlying, infinitely rigid rock, and is therefore fixed at its base. This case should provide an upper limit for the value of the moment at the base of the pile or at the interface between two soil layers with different properties. The results studied are the bending moment at the bottom of the pile BM_b, the moment at the base of the pier (top of the pile) BM when considering a single degree of freedom system consisting of a single column pier of height h with the same properties of the pile and a mass M at the top, and less importantly the moment that would have occurred at the top of the pile with its head fixed against rotation and without a mass or structure on top BM_t.

The case of a soil deposit with two soil layers of different stiffness and the pile penetrating into the bottom, stiffer one, is considered next varying the ratio between the values of the modulus (or shear wave velocities) of the two layers. In this case a more accurate model of the pile and the surrounding soil, based on Blaney's (1974a and 1974b) formulation is used, accounting for the full coupling between the pile and the soil along the pile's length and the diameter of the soil cavity, ignored with a Winkler foundation.

2. SINGLE SOIL LAYER

Considering a soil layer with a shear wave velocity v_s, a mass density ρ_s, a shear modulus G, an internal soil damping ratio D, and a thickness L, the motions at any depth due to a unit acceleration at the base with a circular frequency Ω is given by

$$u_s = -\frac{1}{\Omega^2}\frac{\cos\alpha z}{\cos\alpha L} \tag{2.1}$$

where $z=0$ at the surface and $z=L$ at the bottom, with

$$\alpha = \frac{\Omega}{v_s\sqrt{(1+2iD)}} \tag{2.2}$$

Considering the pile as a beam on elastic (Winkler) foundation with a constant k and defining

$$\lambda^4 = \frac{(k-m\Omega^2)}{4EI} = \frac{k^*}{4EI} \tag{2.3}$$

and

$$A = \frac{\left(k/_{k^*}\right)}{\left(1+\alpha^4/_{4\lambda^4}\right)} \tag{2.4}$$

with *m* the mass of the pile per unit length, *E* the modulus of elasticity of the pile multiplied by $\left(1+2iD_p\right)$, D_p the internal damping in the pile, and *I* its moment of inertia, the displacements along the pile, assuming a long pile (a small value of λ) can be written as

$$u_p = e^{\lambda z}(E_1 \cos\lambda z + E_2 \sin\lambda z) + Au_s \tag{2.5}$$

where E_1 and E_2 are coefficients to be determined imposing the boundary conditions. The solution of the differential equation considering the boundary conditions applicable at each case of interest, and assuming the value of the stiffness *k* of the Winkler foundation as $k = 4\,G\,(1+2\,i\,D\,)$ and the internal damping for both the soil and the pile as 5% leads to.

$$BM_b = EIu_p''(L) = -\left(\frac{2EI\lambda^2}{\Omega^2}\right)\left(-1 + A\left(1-\frac{\alpha^2}{2\lambda^2}\right) - \frac{iC\alpha}{\lambda}\right) \tag{2.6}$$

$$BM_t = EIu_p''(0) = \left(\frac{2\alpha^2 EIB}{\Omega^2}\right) \tag{2.7}$$

where i is the imaginary number.

Considering now a pile that extends above the surface of the soil by a height *h*, representing a pier of a bridge bent with a single column, and with a mass *M* concentrated at the top, representing a tributary mass of the deck, neglecting the mass per unit length of the pier in relation to the top mass, if we consider a horizontal force (shear force in the pier) *H* as the inertia force $H = M\,\Omega^2\,(u_t + h\,\phi_t\,)$ and moment at the base of the pier $BM = Hh$, and solving a system of 2 equations with 2 unknowns the moment at the base of the pier, top of the pile, can be written as

$$BM = \frac{Rh(u_{s0}+h\phi_{s0})}{\left(1-\frac{R(1+2\lambda h+2\lambda^2 h^2)}{2EI\lambda^3}\right)} \tag{2.7}$$

In the following, the values of the bending moment at the base of a fixed pile BM_b and at the base of the pier *BM* are determined for different combinations of the soil properties, the thickness of the soil deposit and the radius of the pile as functions of the frequency. The values of the moment that would occur at the top of the pile without structure or mass if the head was fixed BM_t are also shown for comparison. Both BM_b and BM_t reach their maximum values at the natural frequency of the soil, being the second always smaller.

2.1. Effect of the height

The maximum moment at the base of the pier (top of the pile), once the structure is placed will be the product of the mass concentrated at the top by its acceleration and the pier height. For a given soil deposit, a given mass and a given set of pile properties, as the pier height increases the moment should increase linearly if the acceleration were constant. The acceleration will be affected, however, by the natural frequency of the soil-structure system (natural frequency of the structure accounting for the flexibility of the foundation) and its relation to the natural frequency of the soil layer. It will reach a maximum when they coincide. For a soil deposit with a shear wave velocity $v = 100$ m/sec and a thickness $L = 10$ m, a concrete pile with a radius of 0.5 m and a top mass of 50,000 Kg, the natural frequency of the structure will be equal to that of the soil deposit for a pier height of approximately 3.5 m. One would have then the maximum acceleration. For smaller natural frequencies of the structure (higher values of the pier height) the acceleration would decrease rapidly and eventually become nearly constant. With increasing pier height the moment would start to decrease, reach a minimum (at about 7 m), and then start to increase linearly. This is illustrated in figure 1 where one can see a sharp peak at a height of about 3.5 m and a linear variation beyond a certain point. For this set of soil properties and pile dimensions the bending moment at the fixed base of the pile has a value of 0.6E+7 N-m. There is clearly a range of values of the pier height h (from approximately 6 m to a little over 8 m) where the kinematic moment in the pile exceeds the moment at the base of the pier resulting from the combined kinematic and inertial interaction effects. For all other values of h the latter will exceed the former.

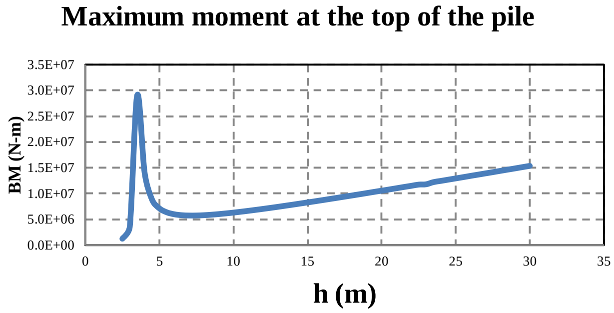

Figure 1. Variation of the bending moment at the base of a pier with pier height for v=100 m/s, L =10 m, r =0.5 m and M=50000 kg

2.2. Effect of the top of the mass

A similar reasoning could be applied to the effect of the mass on the moment at the base of the pier. For a fixed set of values of the other parameters as the mass increases the moment will reach a peak when the natural frequency of the system coincides with that of the soil, will decrease after until it reaches a minimum and will eventually increase linearly. Figure 2 shows the variation of the moment at the base of the pier (top of the pile) as a function of the top mass for a pile with a radius of 0.5 m, a pier height of 5 m, and the same soil profile used before. It can be seen that the moment reaches a minimum for a mass of approximately 60,000 kg and then starts to increase linearly with increasing value of the mass. In this case the moment is never less than 6.0E+6 N-m and thus it is always higher than the kinematic interaction moment at the fixed base of the pile, unaffected by the value of the mass. For a pier height of 10 m on the other hand the moment at the base of the pier would be smaller than the one at the fixed base of the pile for masses below 40,000 kg.

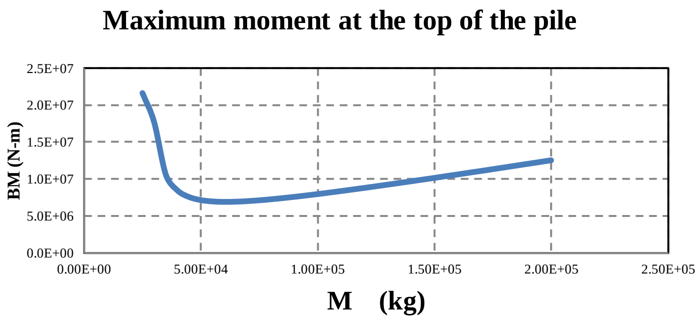

Figure 2. Variation of the bending moment at the base of a pier with top mass for v=100 m/s, L =10 m, r =0.5 m and h=5 m

2.3. Effect of the radius

The radius (or moment of inertia) of the pile affects the value of the moment at the base of the pier mainly through the natural frequency of the structure (accounting for the flexibility of the foundation) and only slightly through the acceleration at the top of the pile without structure (kinematic interaction effects on the motion at the top of the pile with respect to that at the free surface of the soil, a very small effect over the range of frequencies considered). The comparison with the moment at the fixed base of the pile is, however, more difficult because this will change with the radius as previously discussed. Figure 3b shows the variation of the 3 moments (bottom of pier, base of pile and top of a fixed head pile without structure on top) as a function of the radius of the pile for the same soil profile, a pier height of 5 m and a mass of 50,000 kg. The maximum moment at the base of the pier is reached for a radius of 0.63 m. For values of the radius larger than about 0.75 m the kinematic interaction moment, at the fixed base of the pile would be larger than that at the base of the pier. For values larger than about 0.85 m even the moment at the top of a fixed head pile without structure due to kinematic interaction exceeds the one obtained with the structure on top. Figure 3(b) shows the corresponding results with a pier height of 10 m and the same other parameters. Now the peak value of the moment at the base of the pier occurs for a radius of nearly 1.0 m, but in this case the moment at the base of the pier exceeds that at the fixed base of the pile over the complete range of values of the radius of practical interest.

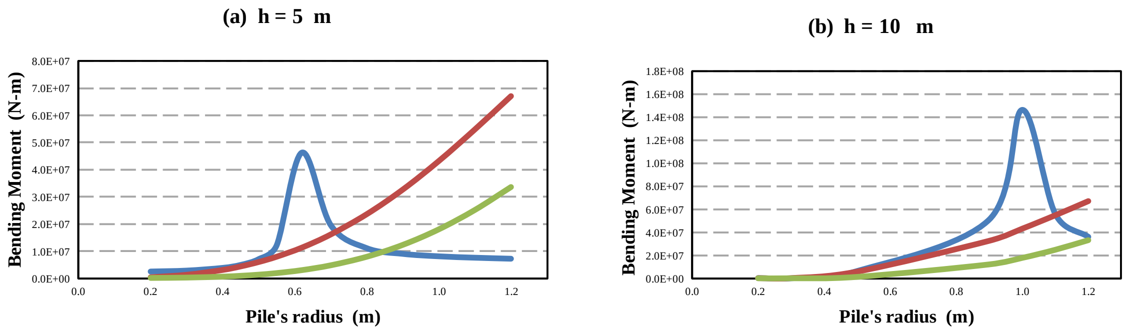

Figure 3. Variation of the bending moment with pile radius for M =50000 kg, continuous, dash-dot and dashed lines are respectively for MB, BM_b and BM_t.

2.3. Effect of soil properties

An increase in the thickness of the soil layer would result in a longer period (smaller natural frequency) of the deposit and a proportional increase in the value of the kinematic interaction moment at the fixed base of the pile. It would have no effect on the natural frequency of the structure including the flexibility of the foundation since the lateral stiffness of the pile will not be affected by the soil properties below 10 m. This would decrease the frequency at which the maximum response due to kinematic interaction takes place at thus the values of the acceleration at the free surface of the soil and the acceleration of the mass, affecting therefore the moment at the base of the pier. Since the moment at the fixed base of the pile is directly proportional to L it could exceed the one at the base of the pier. This is illustrated in figure 4(a) for the case with L = 20 m, h = 5 m and M = 50000 kg. It can be seen that BM_b has now a value of 1.2E+7 which is twice that obtained for the same case with L =10 m, but at a frequency of 1.25 Hz rather than 2.5 Hz. The moment at the base of the pier BM has now a first peak at the natural frequency of the soil layer and a second one at the structure's frequency (same as before). The value of the first peak is slightly smaller than that of the previous first peak (at the structure's natural frequency) because the frequency is smaller, and the value of the peak at the structure's frequency has decreased because the amplification of the ground motion is smaller. On the other hand at this frequency (about 1.8 Hz) the moment at the base of the pier is higher than that at the base of the pile. Figure 4(b) shows the results for a pier height of 10 m and again a soil thickness of 20 m. One can see in this case that the moment at the base of the pile is the same as for the previous one (since it is not affected by the structure). The moment at the base of the pier has now its first peak at the structure's frequency but it is larger than in the case with L = 10 m because the soil amplification at that frequency is now larger. At the natural frequency of the soil layer the moment is higher than for L = 10 m due to the increase in the acceleration but it is still smaller than the fixed base moment for the pile.

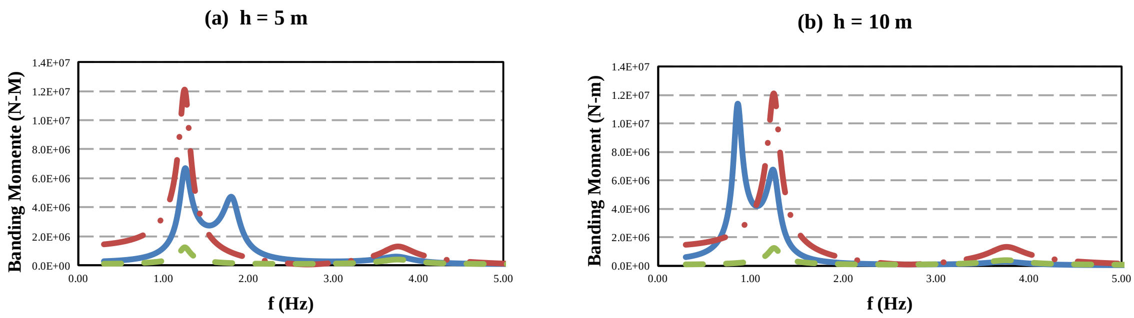

Figure 4. Variation of the bending moment with the normalized frequency for ν=100 m/s, L =20 m, r =0.5 m and h=5 m M =50000 kg, continuous, dash-dot and dashed lines are respectively for MB, BM_b and BM_t.

A decrease in the shear wave velocity of the soil layer (due perhaps to the nonlinear behavior of the soil during the seismic motions), would be equivalent to an increase in the thickness of the soil layer as far as the natural frequency of the soil layer is concerned and the effects of this change in the value at which the maximum kinematic moments would occur. The amplitude of the moment at the fixed base of the pile is however inversely proportional to the value of the shear wave velocity raised to the 3/2 power and therefore the increase would be larger than that resulting from an increase in the layer thickness by the same factor. So for instance if the shear wave velocity was reduced by a factor of 2 the maximum moment would increase by a factor of about 2.8 rather than 2. The effects on the maximum moment at the base of the pier are due in part to the change in the natural frequency of the soil, same as for the case of an increase of thickness, but also by the resulting change in the lateral stiffness of the pile that would increase the inertial interaction effects and decrease the natural frequency of the structure accounting for the flexibility of the foundation. Depending on the original frequency of the structure on a rigid base and the amount of reduction due to inertial interaction this might result in a decrease or an increase in the moment. Figure 5(a) shows the results for the case with L = 10 m and h = 5 m but a shear wave velocity of only 50 m/sec. In this case the shift has increased considerably the moment at the base of the pier. Figure 5(b) shows on the other hand the results for the same case but with h = 10 m. In this case the increases in the moments at the base of the pier are smaller.

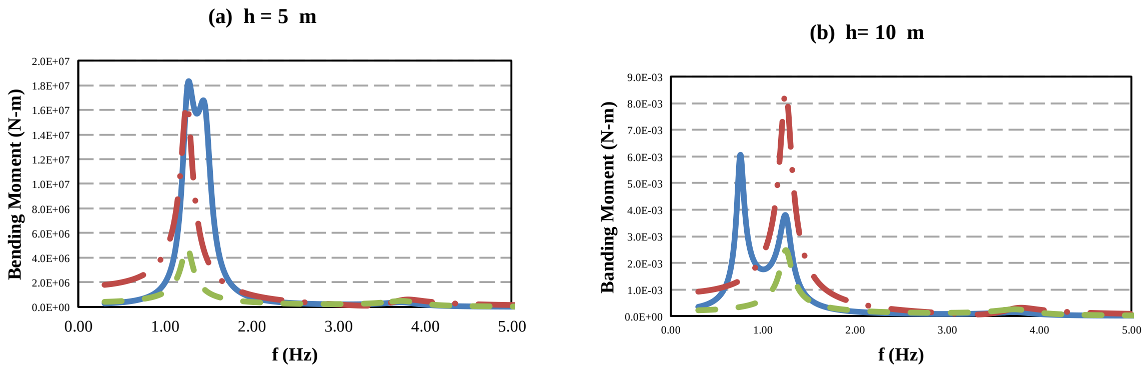

Figure 5. Variation of the bending moment with the normalized frequency for ν=50 m/s, L =10 m, r =0.5 m and M =50000 kg, continuous, dash-dot and dashed lines are respectively for MB, BM_b and BM_t.

An increase in the shear wave velocity of the soil would result in a proportional increase in the natural frequency of the soil layer. For a value of the shear wave velocity of 200 m/sec and a thickness of the soil layer of still 10 m, the natural frequency would become 5 Hz that is beyond the range of practical interest for most earthquakes. The maximum moment at the base of the pile would decrease now by a factor of 2.8 instead of 2. Inertial interaction effects would decrease and the natural frequency of the structure accounting for the foundation would be closer to that of the structure on a rigid base.

Figure 6 shows the results for a shear wave velocity of 200 m/sec, a thickness L = 10 m, and h = 5 m and 10 m, respectively charts (a) and (b). In both cases the moments at the base of the pile are the same since only the structure is changed and they are smaller than those with a wave velocity of 100 m/sec by a factor of 2.828. The moment at the base of the pier has been considerably reduced (by a factor of about 2) due to the change in the value of the surface acceleration at that frequency whereas the value for h = 10 m is only a little smaller than that for ν = 100 m/sec.

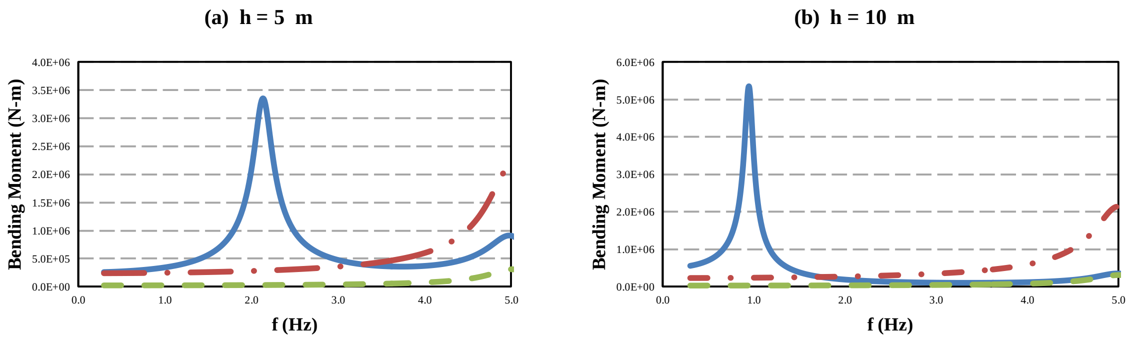

Figure 6. Variation of the bending moment with the normalized frequency for ν=200 m/s, L =10 m, r =0.5 m and h=5 m M =50000 kg, continuous, dash-dot and dashed lines are respectively for MB, BM_b and BM_t.

3. SOIL DEPOSIT WITH 2 SOIL LAYERS

Considering now a soil deposit with two layers, the second (bottom) one stiffer, and a pile penetrating into the bottom layer, the results of interest for a unit harmonic base acceleration at bedrock would be the moment at the interface between the two layers "BMB" and the moment at the base of the pier (top of the pile) "BM". The cases considered, table 3.1, correspond to values of the different parameters for which the moment at the base of the pile exceeded the moment at the base of the pier over a range of frequencies of potential practical interest.

Table 3.1. Variables considered in each case of study

Case	L (m)	v_1 (m/sec)	r (m)	M (kg)	h (m)
1	10	100	0.5	50000	7
2	10	100	0.5	50000	10
3	20	100	0.5	50000	5
4	20	100	0.5	50000	10
5	10	50	0.5	50000	5

For each one of these cases the ratio of the shear wave velocity of the second layer to that of the first was taken with values of 2, 3, 5, 7.5, 10, 20, and 100. In all cases it was assumed that the bottom layer had a thickness of 10 m and that the pile penetrated 5 m into it. The results obtained corresponded to the maximum moments at the base of the pier (top of the pile) and at the interface between the 2 layers, and the frequencies at which these maxima occurred.

The results showed that for $\mathrm{v}_2 = 2\mathrm{v}_1$ the bending moment at the interface (kinematic interaction) is smaller than that at the top of the pile over the complete range of frequencies. The maximum value of this moment becomes larger than that at the top of the pile at the natural frequency of the soil deposit for a value of $\mathrm{v}_2/\mathrm{v}_1$ approximately equal to 5. For a ratio of the velocities of 20 (a shear wave velocity of the bottom layer of 2000 m/sec) the moment at the interface (now at almost exactly 2.5 Hz) is larger than at the top but it still has not reached the value corresponding to a rigid base. Figure 7(a) shows the variation of the maximum moment at the top of the pile versus the velocity ratio. It can be seen that the maximum value decreases steadily as the velocity of the bottom layer increases. This maximum occurs always at a frequency of 1.3 Hz. Figure 7(b) shows the variation of the moment at the interface with the velocity ratio. It increases as the velocity of the bottom layer increases and it tends to the value obtained for a rigid base (6.0E+6) but it is still smaller by about 20% for a ratio of 20. Table 3.2 shows the variation of the natural frequency of the soil deposit with the velocity ratio where it can be observed that the frequency starts at 1.98 Hz for $v_2 = 200$ m/sec but approaches fast the value of 2.5 Hz.

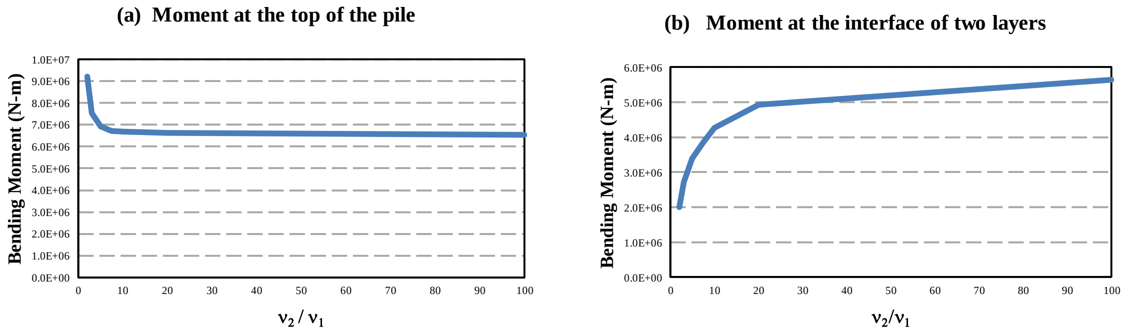

Figure 7. Variation of the bending moment at the top of the pile vs. v_2 / v_1

Table 3.2. Maximum moments and frequencies at which they occur for v_1 =100 m/s, $L_1 = L_2$ =10 m, h =5 m and M =50000 kg

v_2/v_1	*BM*	f_1	*BM*	F_2
2	9.20E+6	1.30	2.00E+6	1.98
3	7.55E+6	1.30	2.73E+6	2.24
5	6.91E+6	1.30	3.39E+6	2.41
7.5	6.73E+6	1.30	3.87E+6	2.44
10	6.68E+6	1.30	4.27E+6	2.47
20	6.63E+6	1.30	4.93E+6	2.50
100	6.54E+6	1.30	5.64E+6	2.50

Table 3.3 lists the values of the moments versus the ratio of shear wave velocities of the two layers. In this case the natural frequency of the structure remains at 0.85 Hz in all cases, and its value decreases again as the stiffness of the bottom layer increases but it reaches faster a limit value of 7.49E+6 N-m which is higher than the one obtained with the other model. The results for the moment at the interface are identical to those of the previous case confirming that this moment is not affected by the characteristics of the structure.

The variation of the two moments with frequency for the case with $L_1 = 20$ m, $h = 5$ m and the same other properties as in the previous two cases, and values of $v_2 = 200$ and 2000 m/sec is listed on table 3.4. The results for this case exhibit a very different behavior from that reported for the previous two cases. Because the natural frequency of the structure accounting for the foundation and the natural frequency of the soil deposit are very close to each other both moments have their peak values at the same frequency. The frequency at which the maximum moment at the base of the pier (top of the pile) takes place changes now with the impedance ratio between the two layers and the value of the moment increases with increasing frequency instead of decreasing. The behavior of the moment in the pile at the interface between the two layers behaves as before. The maximum moment at the interface exceeds now that at the top for a ratio of shear wave velocities of 5 or larger.

Table 3.3. Maximum moments and frequencies at which they occur for v_1 =100 m/s, L_1 = L_2 =10 m, h =10 m and M =50000 kg

v_2/v_1	*BM*	f_1	*BM*	F_2
2	8.38E+6	0.85	2.00E+6	1.98
3	7.84E+6	0.85	2.73E+6	2.24
5	7.60E+6	0.85	3.39E+6	2.41
7.5	7.53E+6	0.85	3.87E+6	2.44
10	7.51E+6	0.85	4.27E+6	2.47
20	7.49E+6	0.85	4.93E+6	2.50
100	7.49E+6	0.85	5.64E+6	2.50

Table 3.4. Maximum moments and frequencies at which they occur for v_1 =100 m/s, L_1 =20m, L_2 =10 m, r =0.5 m and M =50000 kg

v_2/v_1	*BM*	f_1	*BM*	F_2
2	5.87E+6	1.11	3.99E+6	1.11
3	6.24E+6	1.14	5.24E+6	1.14
5	6.56E+6	1.23	6.83E+6	1.23
7.5	6.64E+6	1.24	7.99E+6	1.24
10	6.67E+6	1.24	8.75E+6	1.24
20	6.68E+6	1.25	1.03E+7	1.25
100	6.70E+6	1.25	1.17E+7	1.25

4. CONCLUSIONS

The objective of this work was to assess the parameters values, involved in the kinematic interaction moments induced in a pile by the seismic waves propagating through the soil could exceed the moment at the top of the pile due to the forces exerted by the vibrations of the supported structure (accounting for both kinematic and inertial interaction effects). For that, a large number of parametric studies changing the soil properties (thickness of the layer and shear modulus), and the radius of the pile showed that with the simplifying assumptions above outlined and for logical ranges of the parameters the value of the bending moment BM_b at the fixed base of the pile was indeed directly proportional to the length of the pile under the ground (thickness of the soil layer) and proportional to the ratio EI/G raised to the ¾ power while the moment BM_t that would happen at the top of a fixed head pile without any mass or structure on top would be directly proportional to that ratio. The results of the study confirm, as previously reported by several other authors, that the kinematic moments in the pile at the interface between two layers with different properties can in some cases exceed the moment at the top of the pile if the ratio between the shear wave velocity of the bottom layer to that of the top layer is large enough (larger than 5 in most cases but only 3 or more in a few ones). This does not seem however to be the general case, but even if the kinematic moments do not exceed the ones computed at the top due to combined kinematic and inertial effects, they are likely to be of the same order of magnitude, pointing out the need to consider them in the seismic design of the pile.

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