Nonstationary Frequency Content of Ground Motions From The 2011 Tohoku Earthquake in Japan

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SUMMARY:
The 2011 Tohoku earthquake (Mw9.0) and tsunami caused severe casualties，loss of property and destroy of natural environment. After this earthquake, nearly 700 strong ground motions were released by the K-NET of National Research Institute for Earth Science and Disaster Prevention (NIED). For earthquake engineering, timely and effectively processing these ground motion records and comprehensively summarizing their characteristics and rules, can not only reveal the source mechanism of this earthquake, but also help to renew our existing knowledge of devastating earthquake. In this paper, the non-stationary frequency content of ground motions is investigated based on the instantaneous frequency of ground motion. First, an exponential decay model is used to describe the instantaneous frequency and optimization algorithm is used to determine the model parameters. Then, the multidimensional correlation and attenuation relations of model parameters are analysed. Lastly, the analysis results are compared with those of 2008 Wenchuan earthquake (Mw7.9) to demonstrate some unique properties of these mega earthquakes.

Keywords: earthquake, ground motion, nonstationarity, frequency content, instantaneous frequency

1. INTRODUCTION

On 11 March 2011，a magnitude 9.0 (Mw) undersea megathrust earthquake (known as the 2011 Tohoku earthquake for short) struck Japan. It was the most powerful known earthquake ever to have hit Japan, and one of the five most powerful earthquakes in the world since modern record-keeping began in 1900. The earthquake and the powerful tsunami triggered caused severe casualties，loss of property and destroy of natural environment. According to a report by the Japanese National Police Agency (2012), 15,854 people were killed, 26,992 were injured, and 3,155 were missing across twenty prefectures. 129,225 buildings totally collapsed, 254,204 buildings 'half collapsed', and 691,766 buildings were partially damaged. The economic cost estimated by the World Bank was up to US$235 billion, making it the most expensive natural disaster in modern history (Zhang 2011).

Immediately after this earthquake, nearly 700 strong ground motions were released by the K-NET of National Research Institute for Earth Science and Disaster Prevention (NIED). Undoubtedly, it’s important to process these ground motion records timely and effectively and to summarize their characteristics and rules comprehensively, which can not only reveal the source mechanism of this earthquake but also help to renew our existing knowledge of devastating earthquake.

As for the engineering properties of strong ground motions, the nonstationarities in amplitude and frequency contents have been regarded as important properties besides the conventional properties in amplitude, frequency and duration. Since last two decades, considerable research has been reported on the nonstationary property in amplitude, but not much work has been done on the nonstationary properties in frequency contents. Recent research has shown that the nonstationary property in frequency contents of strong ground motions have non-negligible influence on elastic-plastic response of structures under certain conditions (Dong 2010). Though several notions have been proposed to describe the nonstationarity in frequency contents, most of them cannot be easily used for practical purpose due to their nonparametric nature, which has blocked their further applications in selection and synthesis of proper ground motions for seismic design. The modelling, statistical properties and attenuation laws of nonstationarity in frequency contents of earthquake ground motions have been the open issues in earthquake engineering.

To solve the difficulties mentioned above, instantaneous frequency (IF) is used as a simple but effective approach to describe the nonstationary property in frequency contents. Then, an exponential decay model is proposed to represent the IF and the downhill simplex method is employed to estimate the corresponding model (IF) parameters. Finally, the method is applied to the strong ground motions recorded during the 2011 Tohoku earthquake and the statistical properties and attenuation laws of IF parameters are studied. Also, the analysis results are compared with those of 2008 Wenchuan earthquake (Mw7.9) to represent some unique properties of such mega earthquakes.

2. MODELLING OF INSTANANEOUS FREQUENCY

2.1. Instantaneous Frequency Model

As discussed above, non-stationary frequency content of ground motions has important influence on elastic-plastic response of structures under certain conditions. Since last two decades, considerable attention has been paid to the nonstationarity in amplitude, but not much significant progress has been achieved in nonstationary properties of frequency contents. Though several approaches are available to analyse such properties i.e. the evolutionary spectrum (Priestley 1965), physical spectrum (Mark 1970), time-frequency analysis based on Hilbert transform (Cohen 1989), zero-crossing rate (Li 1999 and 2008), time varying autoregressive moving average model (Conte 1990 and Dong 2004) and the recently developed Hilbert-Huang transform (Huang 1998), their analysis results still need to be parameterized if they are used to study the statistical properties and attenuation laws of the nonstationarity in frequency contents. To circumvent above difficulties, instantaneous frequency (IF) is used in Dong (2010) to describe the nonstationary property in frequency contents. IF is a physically meaningful notation which can be conveniently adopted in synthesis and simulation of strong ground motions.

For a given earthquake ground motion $x(t)$, the IF can be derived from its analytic signal $z(t)$ which is given by

$$z(t) = x(t) + iy(t) = A(t)e^{i\theta(t)} \tag{2.1}$$

$$A(t) = \sqrt{x^2(t) + y^2(t)} \tag{2.2}$$

$$\theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right) \tag{2.3}$$

where $y(t)$ is the Hilbert transform of $x(t)$, i.e. $y(t) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$ and the PV indicates the Cauchy principal value of the singular integral. $A(t)$ is the instantaneous amplitude (envelope), and $\theta(t)$ is the phase function. With above representation, $x(t)$ can be expressed as $x(t) = \Re\left(A(t)e^{i\theta(t)}\right) = A(t)\cos\theta(t)$, where $\Re(\cdot)$ indicates the real part of a complex number.

Then for the analytic signal $z(t)$, the IF $f(t)$, with the stationary phase approximation, can be defined as the derive of the phase function

$$f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \tag{2.4}$$

By surveying the changing rule of $f(t)$, an exponential decay model is proposed in Dong (2010) to represent the IF

$$f(t) = f_0 e^{-\gamma t} \tag{2.5}$$

where f_0 is initial frequency and γ is the decay coefficient. Accordingly, the cumulative phase function $\theta(t)$ is given by

$$\theta(t) = \frac{f_0}{\gamma}\left(1 - e^{-\gamma t}\right) \tag{2.6}$$

2.2. Parameter Estimation of IF Model

Based on above IF model (Eqns. 2.5 and 2.6), the optimization algorithm is used to estimate the corresponding model parameters (f_0 and γ). In this study, the downhill simplex method (Nelder 1965) is used.

In case that there are n parameters to be estimated, the downhill simplex method can be used to solve the minimization problem in n dimensions by maintaining at each iteration n+1 points that define a simplex. At each iteration, this simplex is updated by applying certain transformations to it so that it rolls downhill until it finds a minimum. Without any derivatives, this method only requires function evaluations, which makes it an elegant and robust method for function minimization.

In this paper, the weighted sum of square relative error between the actual and the estimated cumulative phase functions is used as the objective function to be minimized. Then, the downhill simplex method is used to solve the problem

$$\begin{aligned} &\text{Minimize} && J(\hat{f}_0, \hat{\gamma}) = \sum_{k=1}^{N} w(t_k)\left[1 - \hat{\theta}(t_k) / \theta(t_k)\right]^2 \\ &\text{subjected to} && \hat{f}_0 > 0 \\ &&& \hat{\gamma} > 0 \end{aligned} \tag{2.7}$$

where $J(\hat{f}_0, \hat{\gamma})$ is the objective function corresponding to the estimated parameters $\hat{f}_0$ and $\hat{\gamma}$, $w(t_k)$ is the weight function, t_k indicates the time $t = k\Delta t$ with Δt as the sampling time, N is the number of sampling points, $\theta(t_k)$ is the actual cumulate phase functions calculated according to Eqn. 2.3, and its estimate $\hat{\theta}(t_k)$ is calculated according to Eqn. 2.6 with parameters as $\hat{f}_0$ and $\hat{\gamma}$. Once the objective function is minimized, the optimum IF parameters are then obtained.

3. RESULTS AND DISCUSSION

3.1. Strong Motion Records

After this earthquake, the K-NET network of the National Research Institute for Earth Science and Disaster Prevention (NIED) in Japan released strong ground motion records of nearly 700 stations. These records provided a rare opportunity for researchers worldwide to study characteristics of such a mega earthquake from every aspect they are interested in.

Fig. 3.1 shows the spatial distribution (locations) of all the record sites of this earthquake. These record sites almost cover the whole territory of Japan well. In Fig. 3.2, histograms of fault distance and peak ground acceleration (PGA) of all the records are presented. For nearly 70% of records, the fault distance is below 400 km, and only for 9% of records the fault distance is below 100 km. The minimum fault distance is about 50 km. As for the PGA, there are about 70% of the records with PGA below 100 gal (cm/s^2), PGA of nearly 15% of the records is between 100 and 200 gal, and nearly for another 15% of the records the PGA is upon 200 gal.

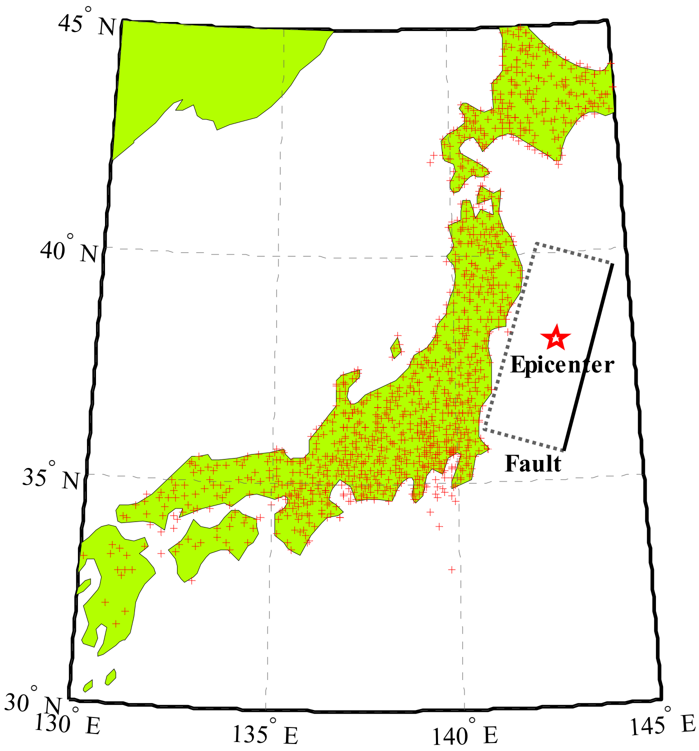


Figure 3.1. Spatial distribution of record sites

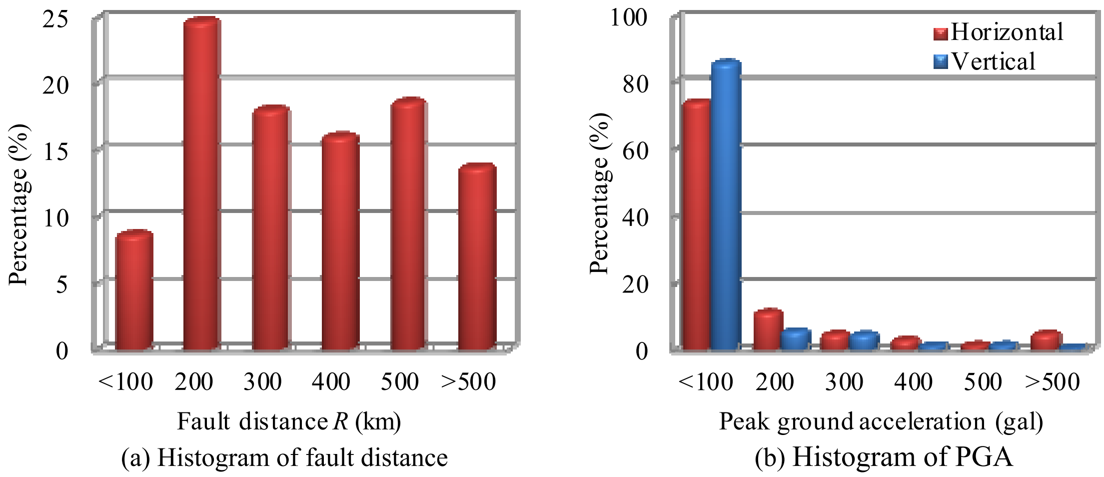
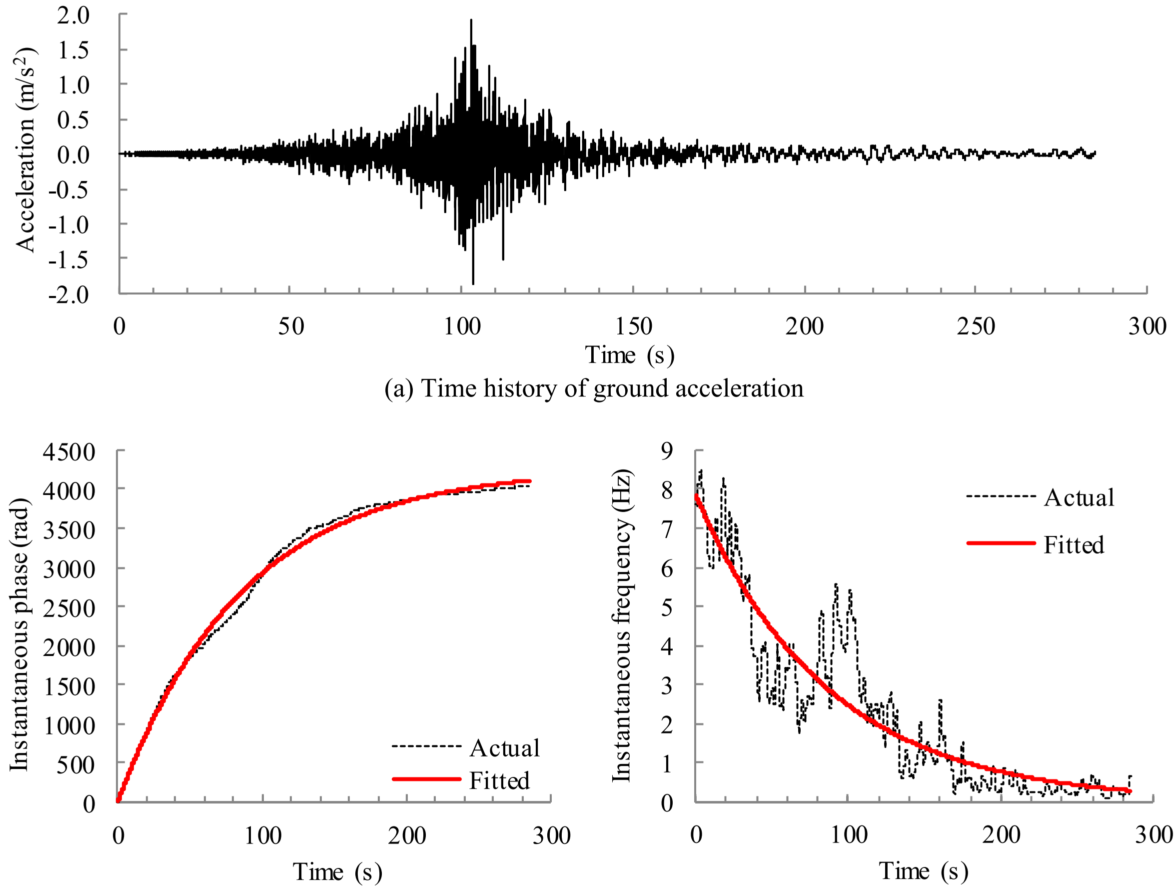


Figure 3.2. Histograms of fault distance and PGA

To ensure our analysis is physically meaningful for strong ground motions, records with fault distance upon certain limited value are not used in analysis. According to the prediction relation by Fukushima (2003), records with fault distance within 390 km are adopted in our analysis which nearly account for 65% of all the records released.

3.2. Modelling and Parameter Estimation of Instantaneous Frequency

In Fig. 3.3, the time history and corresponding IF estimation result of the North-South component of the ground acceleration recorded at the station TKY007 during this earthquake is shown. It clearly demonstrates the nonstationarity in frequency content of the actual ground acceleration. Before 180 s, there is a sharp decrease in IF from about 8 to 1 Hz, at the same time, the instantaneous phase increases from 0 to about 3600 rad quickly. After 180 s, the IF decreases from 1 to 0.3 Hz gradually and the instantaneous phase increases from 3600 to about 4000 rad accordingly.



(a) Time history of ground acceleration

(b) Estimation result of instantaneous phase

(c) Estimation result of instantaneous frequency

Figure 3.3. Histograms of fault distance and PGA

3.3. Correlation of IF Parameters

3.3.1. Two horizontal components

As has been mentioned above, 445 ground acceleration records with fault distance within 390 km are adopted here for analysis. Each record consists of two horizontal (North-South and East-West) components and one vertical component. Using linear and log-linear regression analysis, the correction of IF parameters between two horizontal components are surveyed and the corresponding results are given in Table 3.1. The superscript *NS* indicates the parameter corresponding to the north-south horizontal component of ground acceleration, the superscript *EW* indicates the one corresponding to the east-west horizontal component, and the superscript *H* indicates the geometric mean of the parameters of two horizontal components. The precision factor p is used to evaluate the correlation of two samples. For two samples X and Y, the precision factor p is given by $p = \sigma_{\Delta X}/\sigma_X + \sigma_{\Delta Y}/\sigma_Y$, where $\sigma_{\Delta X}$ and $\sigma_{\Delta Y}$ are standard deviations of regression errors between estimates and observations of X and Y, σ_X and σ_Y are standard deviations of X and Y. A smaller p

means a better correlation between X and Y. As can be seen, for f_0 the linear correlation is better than log-linear correlation, while for γ the log-linear correlation is better than linear correlation. In both linear and log-linear cases, the correlation of f_0 between different components is better than that of γ.

3.3.2. Vertical and two horizontal components

In the same manner, the correlation rules of IF parameters between vertical and two horizontal components are studied, and the results are shown in Table 3.2. The superscript V indicates the parameter corresponding to the vertical component of ground acceleration. For both f_0 and γ, the log-linear correlation between vertical and two horizontal components is better than the corresponding linear correlation. In both linear and log-linear cases, the correlation of f_0 between different components is better than that of γ.

Table 3.1. Correlation of IF Parameters Between Two Horizontal Components

Y	X	Linear regression $Y = c_1X + c_2$				Log-linear regression $\log Y = c_1\log X + c_2$			
		c_1	c_2	Precision Factor p	Rank	c_1	c_2	Precision Factor p	Rank
f_0^{NS}	f_0^{EW}	0.9849	0.0388	0.4986	9	0.9945	-0.0025	0.5016	10
f_0^{NS}	f_0^{H}	1.0087	-0.0449	0.2456	1	1.0131	-0.0106	0.2465	2
f_0^{EW}	f_0^{H}	0.9925	0.0570	0.2515	3	0.9869	0.0106	0.2530	4
γ^{NS}	γ^{EW}	0.8496	0.0008	0.8803	12	0.9056	-0.2239	0.7882	11
γ^{NS}	γ^{H}	0.9670	0.0002	0.4419	8	0.9887	-0.0287	0.3941	6
γ^{EW}	γ^{H}	1.0338	-0.0001	0.4392	7	1.0113	0.0287	0.3855	5

Table 3.2. Correlation of IF Parameters Between Vertical and Two Horizontal Components

Y	X	Linear regression $Y = c_1X + c_2$				Log-linear regression $\log Y = c_1\log X + c_2$			
		c_1	c_2	Precision Factor p	Rank	c_1	c_2	Precision Factor p	Rank
f_0^{V}	f_0^{NS}	1.1276	0.6880	1.3613	8	0.9840	0.0973	1.1197	2
	f_0^{EW}	1.1510	0.5496	1.3465	7	1.0025	0.0806	1.1458	3
	f_0^{H}	1.1582	0.5440	1.3278	6	1.0088	0.0799	1.1039	1
γ^{V}	γ^{NS}	0.8541	0.0020	1.6546	11	0.7852	-0.4158	1.2528	4
	γ^{EW}	0.7908	0.0023	1.6835	12	0.7297	-0.5485	1.4169	9
	γ^{H}	0.8534	0.0020	1.6320	10	0.7862	-0.4154	1.2753	5

3.4. Attenuation Rules of IF Parameters

Based on the IF parameters estimated, attenuation rules of IF parameters can be obtained accounting for the effects caused by source mechanism, propagation path and local site conditions. In this paper, the attenuation model is designated as $\log Y = c_1 \log(R+10) + c_2R + c_3T_g + c_4 + \varepsilon$, where Y indicates the IF parameters of various components, R is the fault distance, T_g is the characteristic period (turning point) of response spectrum which is used to take into account the effects of R and local site condition which can be obtained using the method described in Dong (2010), ε is the residual with the standard deviation as σ_ε, c_1, c_2, c_3 and c_4 are regression coefficients. By using regression analysis, the attenuation rules of IF parameters are obtained and the regression coefficients are given in Table 3.3. For comparison purpose, the regression coefficients of IF parameters corresponding to the ground motions recorded during the 2008 Wenchuan earthquake (Mw7.9) in China are also listed in Table 3.4. For both earthquakes, there is no much difference between the attenuation rules of two horizontal IF parameters, which indicates the identical nonstationarity in frequency contents of horizontal ground motions.

In Fig. 3.4, changes of f_0 with R and T_g are presented. It is clear that f_0 decreases as R and T_g increases. For vertical component, f_0 is larger than that for horizontal components in all cases, which implies that vertical ground motion component consists of much more high frequency contents than horizontal ground motion components. However, the difference of f_0 between horizontal and vertical components decreases with R and T_g increasing, which indicates that f_0 of vertical component decrease more quickly than that of horizontal components as R and T_g increases. In Fig. 3.5, changes of γ with R and T_g are presented. For all components, γ increases as T_g increases. When R is less than 200 km, there is an increasing tendency in γ and for vertical component γ increases much faster than that for horizontal components. When R is larger than 200 km, γ decreases gradually as R increases. It is also clear that R has much more influence on γ than T_g.

Table 3.3. Regression Coefficients of IF Parameters (2011 Tohoku Earthquake)

Component	Y	c_1	c_2	c_3	c_4	σ_ε
Horizontal	f_0^{NS}	0.3070	-0.0018	-0.1096	0.3660	0.2014
	f_0^{EW}	0.2112	-0.0015	-0.1070	0.5332	0.2381
	f_0^{H}	0.2591	-0.0017	-0.1083	0.4496	0.2253
	γ^{NS}	1.7166	-0.0041	0.2679	-5.6717	0.2285
	γ^{EW}	1.4224	-0.0034	0.2702	-5.1362	0.2277
	γ^{H}	1.5695	-0.0037	0.2690	-5.4039	0.2277
Vertical	f_0^{V}	-0.1220	-0.0013	-0.1582	1.3735	0.2130
	γ^{V}	1.9518	-0.0036	0.1561	-6.1073	0.2089

Table 3.4. Regression Coefficients of IF Parameters (2008 Wenchuan Earthquake)

Component	Y	c_1	c_2	c_3	c_4	σ_ε
Horizontal	f_0^{NS}	-0.0086	-0.0004	-0.4535	0.9585	0.0738
	f_0^{EW}	0.0385	-0.0006	-0.4285	0.8740	0.0949
	f_0^{H}	0.0149	-0.0005	-0.4410	0.9162	0.0746
	γ^{NS}	0.3206	-0.0002	0.7488	-3.7111	0.2249
	γ^{EW}	0.4574	-0.0007	0.7725	-3.9813	0.2273
	γ^{H}	0.3890	-0.0005	0.7606	-3.8462	0.2252
Vertical	f_0^{V}	-0.1937	0.0004	-0.5286	1.2988	0.0727
	γ^{V}	0.1371	0.0008	0.3313	-3.0877	0.1837

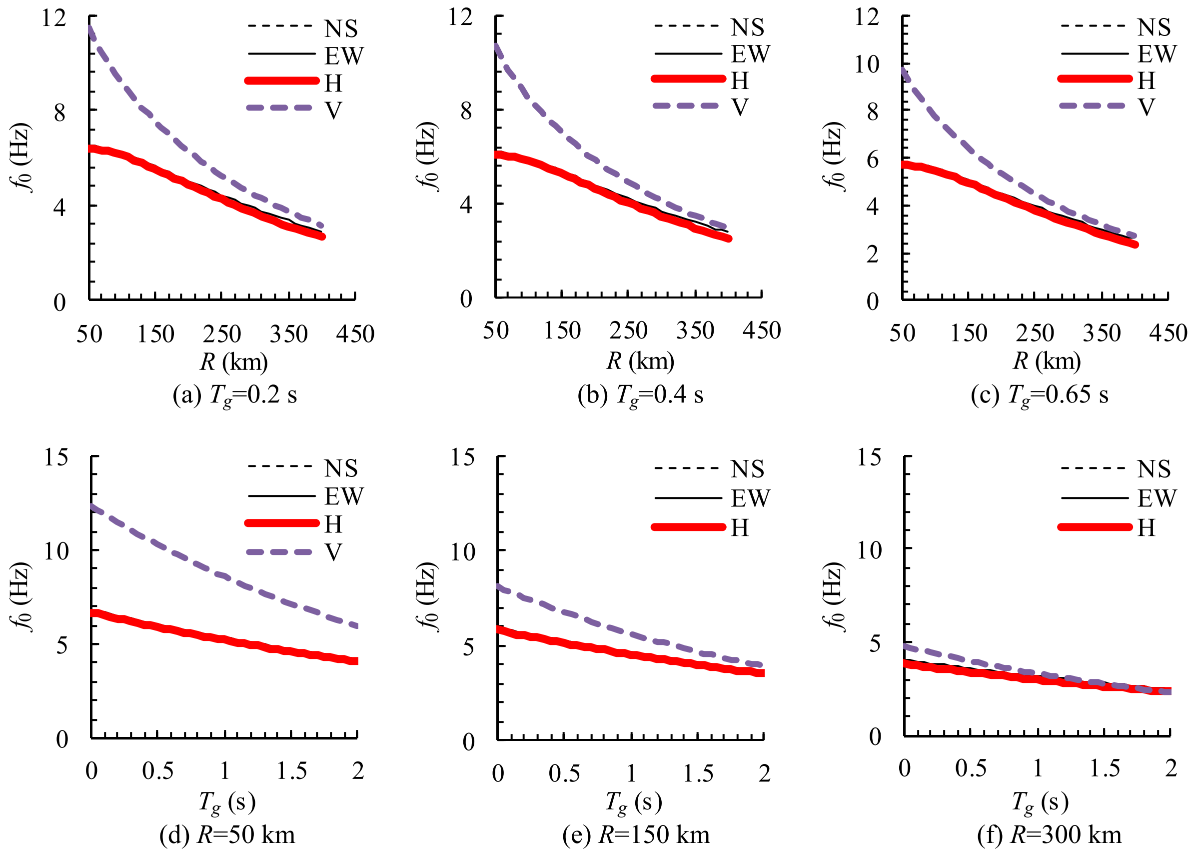
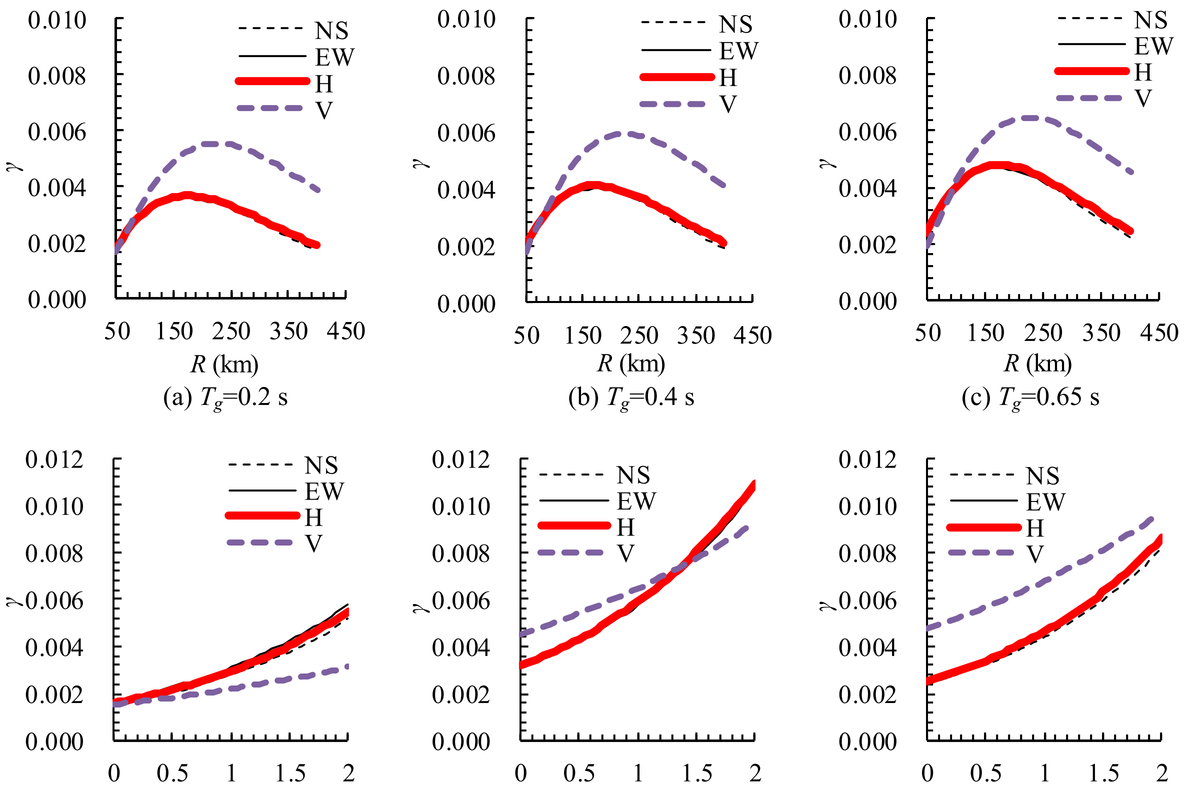


Figure 3.4. Changes of f_0 with R and T_g

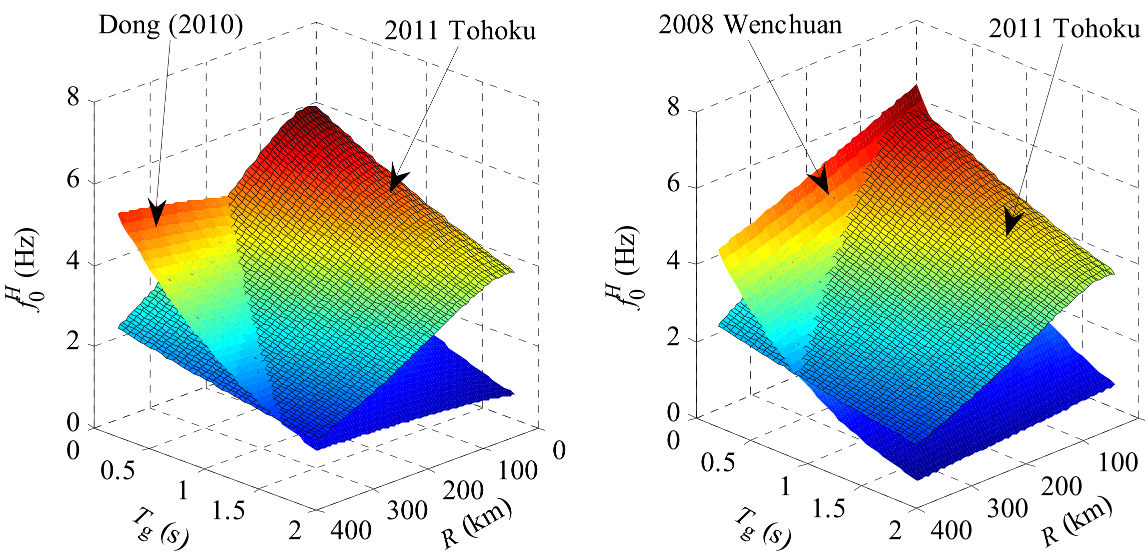


(a) T_g=0.2 s (b) T_g=0.4 s (c) T_g=0.65 s

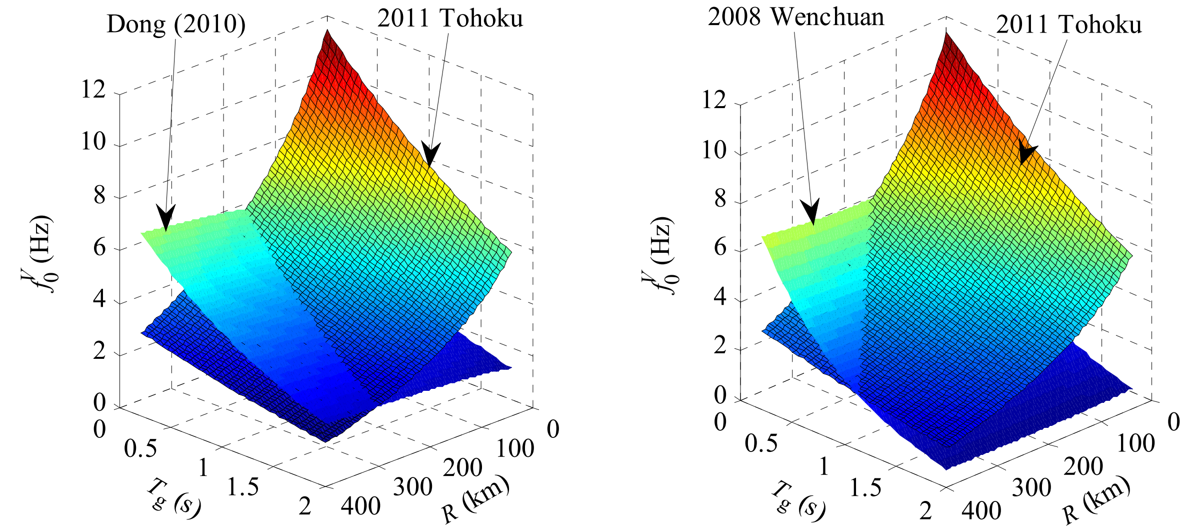
(d) R=50 km (e) R=150 km (f) R=300 km

Figure 3.5. Changes of γ with R and T_g

In Figs. 3.6 and 3.7, the attenuation relations of IF parameters are compared with the corresponding results in previous studies. Compared with the 2008 Wenchuan earthquake, ground motions of 2011 Tohoku earthquake consist of more high frequency contents and the high frequency contents decay slightly more slowly with time. By comparing the results with those by Dong (2010) based on strong ground motions recorded worldwide since 1930, it is clear ground motions of both 2011 Tohoku and 2008 Wenchuan earthquakes consist of much more high frequency contents and the high frequency contents decay significantly more slowly with time than the earthquakes of smaller magnitude.

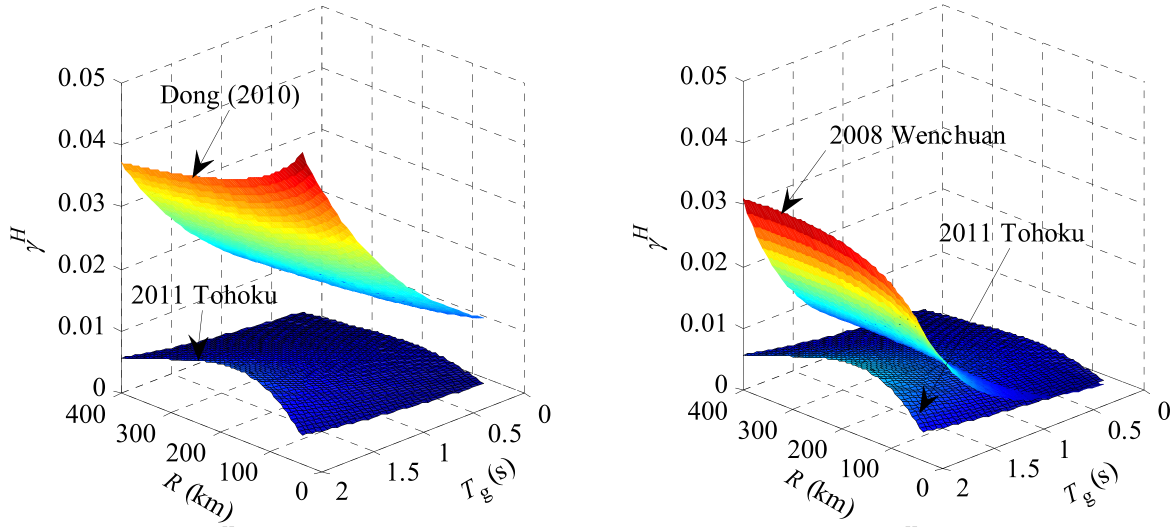


(a) Comparison of f_0^H with the result by Dong (2010) (b) Comparison of f_0^H during two earthquakes

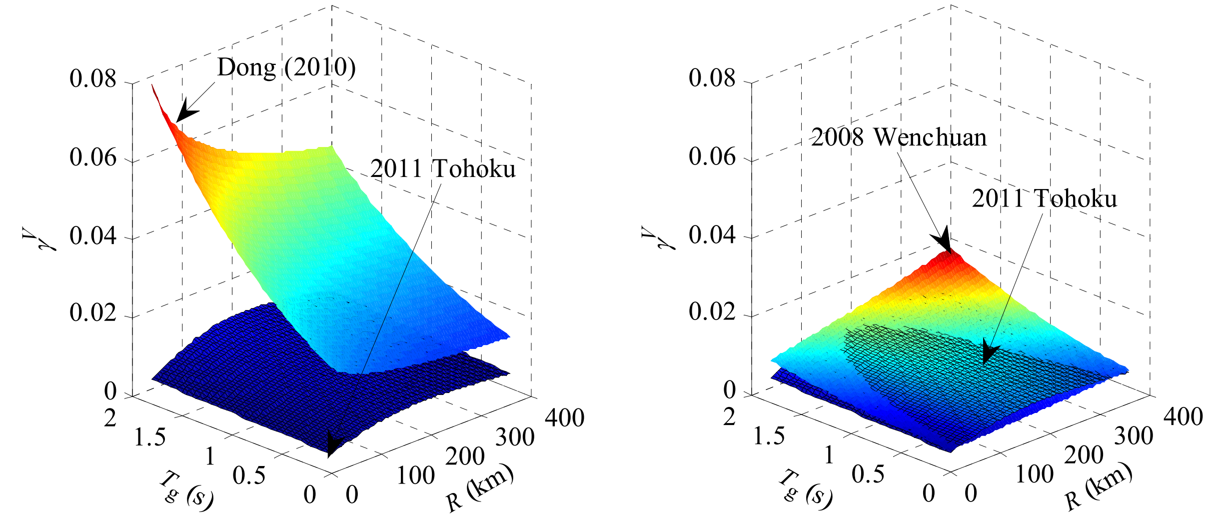


(c) Comparison of f_0^V with the result by Dong (2010) (d) Comparison of f_0^V during two earthquakes

Figure 3.6. Comparison of f_0 with the results of previous studies



(a) Comparison of γ^H with the result by Dong (2010) (b) Comparison of γ^H during two earthquakes



(c) Comparison of γ^H with the result by Dong (2010) (d) Comparison of γ^V during two earthquakes

Figure 3.7. Comparison of γ with the results of previous studies

4. CONCLUSIONS

In this paper, the nonstationarity in frequency content of strong ground motions from the 2011 Tohoku earthquake is investigated by using an exponential decay IF model. Correlation and attenuation relations of IF parameters are presented by using regression analysis. It is found that there is no much difference between the attenuation rules of two horizontal IF parameters which indicates the identical nonstationarity in frequency contents of horizontal ground motions. Vertical ground motion component consists of more high frequency contents than horizontal components and the high frequency contents decay more quickly with time increasing. Compared with ground motions from earthquakes of smaller magnitude, ground motions from mega earthquakes, such as 2011 Tohoku earthquake and 2008 Wenchuan earthquake, consist of much more high frequency contents and the high frequency contents decay significantly more slowly with time which is mainly due to the long rupture length and duration of such earthquakes.

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