Probabilistic site characterization based on Bayesian inversion of ambient vibration array recordings in SW British Columbia, Canada

15 WCEE
LISBOA 2012

S. Molnar, C.E. Ventura, W.D.L. Finn, & M. Taiebat
University of British Columbia, Vancouver, BC, Canada

S.E. Dosso
University of Victoria, Victoria, BC, Canada

J.F. Cassidy
Natural Resources Canada, Sidney, BC, Canada

SUMMARY:
One of the most important properties for characterizing earthquake site response is the shear-wave velocity (V_S) profile. The microtremor array method extracts phase velocity Rayleigh wave dispersion curves from recordings of ambient vibrations for inversion of V_S structure at a site. A variety of sediment sites with diverse amplification behaviour are chosen to evaluate the ability of this passive-source method to provide V_S profiles for site response characterization in SW British Columbia, Canada. A Bayesian inversion method, with evaluation of data errors and model parameterization, is applied here to produce the most-probable V_S profile together with quantitative uncertainty estimates. The resulting V_S profile probability distribution at each site generally demonstrates: (1) a well resolved V_S gradient within the upper sediment layer; and (2) poor resolution of the basement layer V_S, consistent with surface seismic methods in general. Probabilistic site response analyses are performed, in which common predictors of site amplification, including V_{SZ} (harmonic average V_S to a depth z) and linear 1D amplification spectra, are computed using a sample of V_S profiles drawn from the posterior probability density of the Bayesian inversion. Site amplification probability distributions are shown to be more informative than amplification estimated for a single best-fit V_S profile by characterizing the uncertainty and therefore level of confidence in the predictions. Amplification probability spectra are evaluated by comparison to empirical earthquake and microtremor spectral ratios, with generally good agreement. This result provides confidence that when the V_S profile probability distribution from Bayesian inversion of dispersion data is included in theoretical site amplification calculations the primary influence of site-specific structure is accounted for appropriately.

Keywords: site response, microtremor array method, Rayleigh wave dispersion, Bayesian inversion

1. INTRODUCTION

Characterization of earthquake ground motion amplification due to near-surface sediments, i.e. site response, is important for seismic hazard assessment. Earthquake ground motion recordings demonstrate large variability, both site-to-site variability for a given earthquake and earthquake-to-earthquake variability at a given site. The National Earthquake and Hazards Reduction Program (NEHRP) categorizes site conditions into six classes (A-F) based primarily on the harmonic average of shear-wave velocity (V_S) over the upper 30 m, V_{S30}, at a site. Overall, characterization of earthquake site response relies heavily on an assessment of V_S at a site.

Molnar et al. (2010) developed a Bayesian inversion scheme for quantitative uncertainty estimation from microtremor dispersion data. Probability distributions for common predictors of site amplification can then be computed based on a sample of V_S profiles drawn from the posterior probability density (PPD) of the microtremor inversion. Sites of contrasting site amplification behaviour in SW British Columbia, Canada, were chosen, collocated with invasive V_S measurement sites, to apply and evaluate the microtremor array method for site response characterization. In a companion 15WCEE paper, Molnar et al. (2012a) present the parameterization study and best-fit V_S profiles determined via Bayesian inversion of microtremor array dispersion data at two deep (> 100 m), and a shallow (< 30 m), sediment sites in SW British Columbia, and evaluate the inversion results with respect to collocated invasive V_S measurements. Overall, excellent agreement was obtained between the inversion results and the invasive methods over the depth interval for which the inversion results are well resolved: the average relative difference is 5% from surface to 120 m depth for Fraser River delta site 1, is 25% from to 60 m depth for Fraser River delta site 2, and is 11% to 17 m depth for Victoria (Molnar et al. 2012b). Hence, the Bayesian inversion results appear reliable and are applicable for characterization of site response.

Molnar et al. (2012b) examined uncertainty in predicted linear site amplification and corresponding site response characterization due to the uncertainty in V_S structure as quantified by Bayesian inversion of microtremor array dispersion data for Fraser River delta site 1 and the Victoria site. The methodology and results for these two sites, as well as Fraser River delta site 2, are presented here. As a first step, the probability distribution of V_{SZ}, the harmonic average V_S to an arbitrary depth z, is computed. Profiles of V_{SZ} (with uncertainties) are presented here as a means of choosing the most appropriate averaging depth z for a particular site. The probability distribution of V_{S30} is examined further, as this is the site amplification predictor used in current building codes and ground motion prediction equations. V_{S30} is also examined in terms of probability of NEHRP site class. As a second step, uncertainties for site amplification spectra are calculated using the full V_S profile rather than an average property of the soil column. Amplification probability spectra for vertically propagating transverse shear (SH) waves are computed here based on full wave effects. SH-wave amplification distributions represent the most complete site response measure considered here, and are compared with empirical earthquake and microtremor spectral ratios for evaluation of the microtremor array method to provide V_S profiles with sufficient accuracy for site characterization.

2. BAYESIAN INVERSION

A Bayesian formulation represents the solution of the inverse problem in terms of the posterior probability density (PPD) of the geophysical model parameters, a multi-dimensional probability distribution including both data and prior information. The distribution is typically interpreted by computing properties defining parameter estimates, uncertainties, and inter-relationships, such as the maximum *a posteriori* (MAP) model and one-dimensional marginal probability distributions. The most-probable V_S model was computed numerically using an adaptive hybrid optimization algorithm. To compute parameter uncertainty distributions, an efficient implementation of Metropolis-Hastings sampling in principal component space was used to provide unbiased sampling from the PPD. Bayesian inversion essentially maps data uncertainty distributions into parameter uncertainty distributions (modified by the prior). The Bayesian formulation used here is outlined explicitly in Molnar et al. (2010). The most appropriate model parameterization was determined using the Bayesian information criterion which indicates the simplest model consistent with the resolving power of the dispersion data (Molnar et al. 2010; 2012a,b). Wide prior bounds were set on all parameters, encompassing physically realistic values, to allow the data (not the prior) to primarily determine the solution. The marginal probability V_S profile for each site is obtained by converting each model in the PPD sample into a profile, and then integrating the probability of all profiles over a velocity-depth grid. The total number of models in the Markov chains generated for the Fraser River delta and Victoria sites were between 100,000, and 600,000.

2.1. Application to SW British Columbia

Three locations in SW British Columbia are chosen for application and evaluation of the microtremor array method: two deep (> 100 m) sediment sites on the Fraser River delta, and one shallow (< 30 m) sediment site in Victoria. The Fraser River delta is composed of up to 300 m of Holocene deltaic sands and silts from the Fraser River overlying up to 500 m of over-consolidated Pleistocene glacial material, resulting in up to 800 m of material over Tertiary sedimentary bedrock. In contrast, the local geology of Victoria generally exhibits a strong near-surface impedance contrast with < 30 m of marine clayey silt deposited atop of over-consolidated glacial material and/or hard bedrock.

At Fraser River delta site 1, downhole V_S measurements generally increase with depth from 125 m/s near the surface to ~500 m/s at the Holocene-Pleistocene boundary at 235 m, with high (> 600 m/s) velocities within the underlying Pleistocene material. The marginal profile in Fig. 1 (left panel) shows decreasing probability with depth, such that the distribution is scarcely visible on the plot below ~140 m. To more clearly illustrate the relative probability distribution at each depth, the probability is normalized independently at each grid depth (right panel). Fig.1 shows a well-determined power-law gradient that transitions to a ~500-1000 m/s half-space between 110-160 m depth.

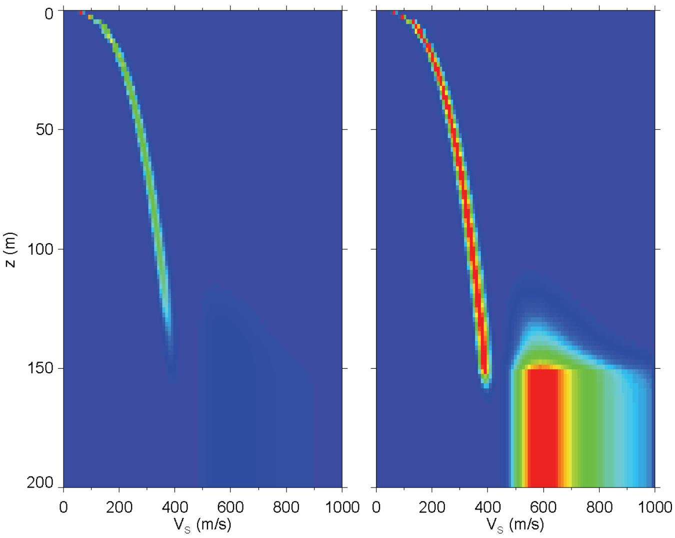

Figure 1. Marginal probability V_S profile for Fraser River delta site 1 fully normalized (left panel) and normalized at each depth (right panel).

The upper 60 m at Fraser River delta site 2 is composed of Holocene deltaic sands and silts, which overlies either over-consolidated Pleistocene glacial material or Tertiary sedimentary bedrock at an unknown depth. Invasive V_S measurements (downhole and eight SCPTs) show V_S increases from ~90 m/s near surface to ~330 m/s at 58 m depth, with significant velocity reversals in the 25-35 m depth range. The resulting marginal probability V_S profile shows a well-determined power-law gradient that transitions to a high-velocity (> 550 m/s) half-space after 180 m depth. The most-probable V_S profile shows a power-law gradient to 260 m depth where V_S increases significantly from ~500 m/s to ~1900 m/s in the uniform half-space.

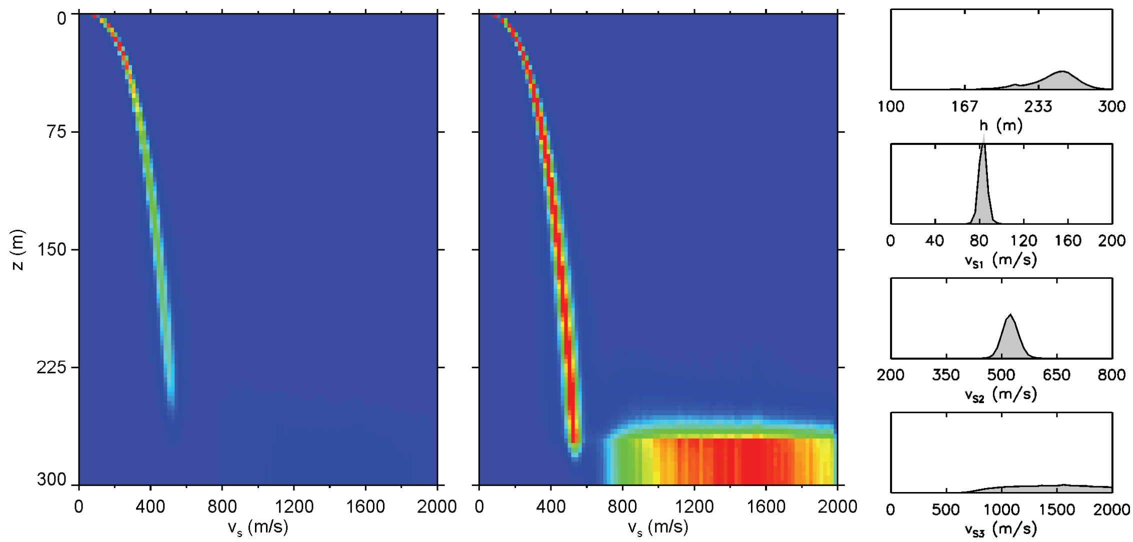

Figure 2. Marginal probability V_S profile for Fraser River delta site 2 fully normalized (left panel) and normalized at each depth (middle panel). The right panel shows 1D marginal distributions of the total thickness of the powerlaw layer (*h*), velocity at the top (V_{S1}) and bottom (V_{S2}) of the powerlaw gradient layer and in the

half-space (V_{S3}); limits of each plot correspond to the prior distribution limits (minimum limit for *h* is set to 2 m).

At Victoria, ~17 m of low-velocity marine silt overlies either over-consolidated glacial material and/or hard bedrock. Fig. 3 presents the marginal probability V_S profile for the Victoria site which shows a well resolved linear gradient of the upper layer and a significant increase in V_S at about 15-18 m depth, with the half-space V_S poorly resolved.

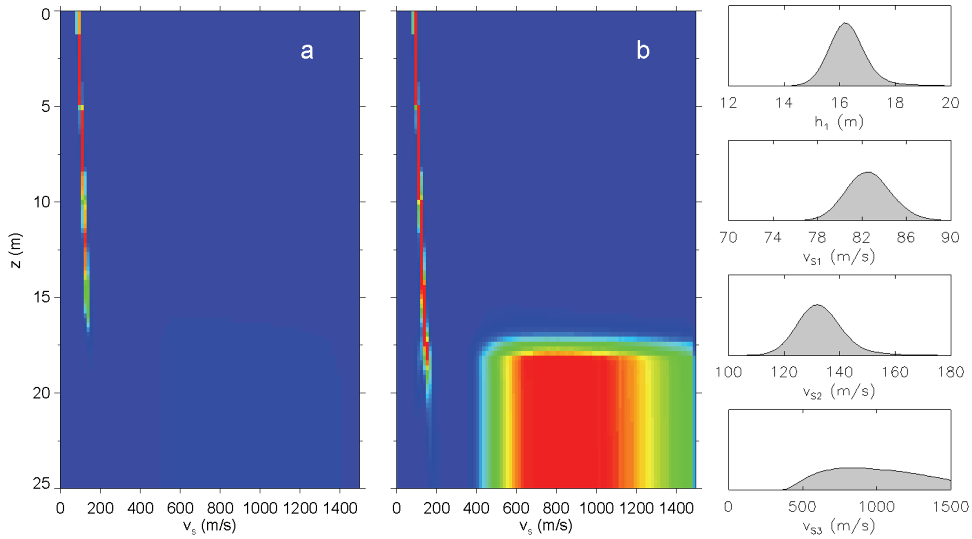

Figure 3. Marginal probability V_S profile for Victoria site fully normalized (left panel) and normalized at each depth (middle panel). The right panel shows 1D marginal distributions of the total thickness of the linear gradient layer (*h*), velocity at the top (V_{S1}) and bottom (V_{S2}) of the linear layer and in the half-space (V_{S3}).

3. SITE RESPONSE PROBABILITY ANALYSIS

In this section, uncertainty estimates of 1D, linear site amplification are calculated using the V_S profile probability distribution derived from Bayesian inversion and are examined for application to site response characterization. The probability distribution of V_{S30} is examined in terms of probability of NEHRP site class with respect to hard ground conditions. Uncertainty of the amplification spectrum is calculated in terms of vertical propagation of transverse shear (SH) waves. The most complete predictor is the SH-wave amplification spectra probability distribution, which is then compared to empirical (microtremor) amplification ratios. The most-probable V_S profile presented in Molnar et al. (2012a) is also tracked through into predictions of V_{S30} and amplification spectra representative of a single, albeit best-fit, estimate of V_S structure.

3.1. Uncertainty of V_{S30}

In general, surface-wave methods cannot resolve fine structure; hence, uncertainty of an average V_S estimate (e.g. V_{S10}, V_{S30}, etc.) is a good initial choice for assessing the ability of the microtremor array method to characterize site amplification. The average V_S from surface to a particular depth *z* is termed here as V_{SZ} and calculated as

$$V_{SZ} = \frac{z}{\int_0^z \frac{dz}{V_S(z)}}, \tag{2.1}$$

where $V_S(z)$ is the V_S profile. Statistical measures and probability distributions for V_{SZ} can be computed by applying Eqn. 2.1 to a sample of V_S models drawn from the PPD.

Figure 4a shows V_{SZ} for Fraser River delta site 1. Two standard deviation uncertainties are included to best describe the full distribution. The mean V_{S30} estimate ± two standard deviations for the delta site is

164 ± 43 m/s which spans the boundary between NEHRP site classes D and E. The V_{S30} estimate computed for the most-probable V_S profile is 157 m/s corresponding to class E, but without any indication of uncertainty. As a more meaningful analysis, the V_{S30} probability distribution shown in Fig. 4b indicates the probability the delta site corresponds to site class E (95%), rather than class D (5%).

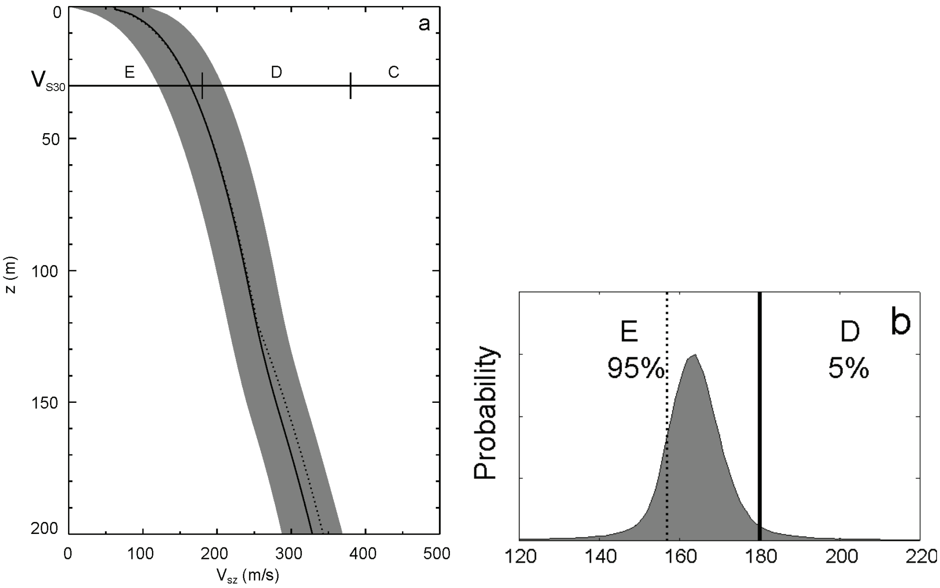

Figure 4. (a) Probability distribution for depth-averaged shear-wave velocity V_{SZ} for Fraser River delta site 1. Solid line represents the mean and shaded region corresponds to mean ± two standard deviations. NEHRP site class boundaries, based on V_{S30}, are indicated. MAP model V_{SZ} profile shown by dotted line. (b) Probability distributions of V_{S30}. MAP model V_{S30} estimate shown by dotted line.

Figure 5a shows V_{SZ} for Fraser River delta site 2. The mean V_{S30} estimate ± two standard deviations for the delta site is 182 ± 23 m/s which spans the boundary between NEHRP site classes D and E. The V_{S30} estimate computed for the most-probable V_S profile is 194 m/s corresponding to class E. The V_{S30} probability distribution shown in Fig. 5b indicates the probability the delta site corresponds to site class D (62%), more probable than class E (38%).

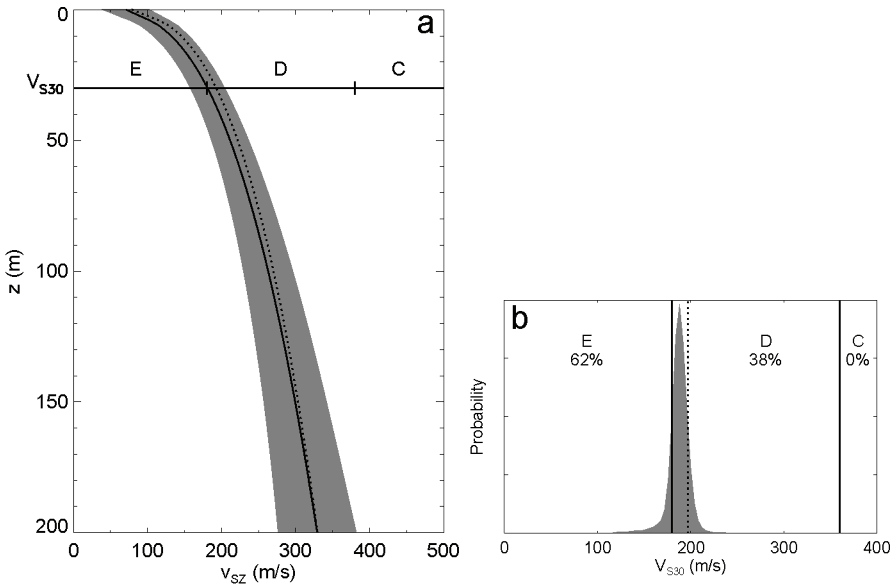

Figure 5. (a) V_{SZ} probability distribution and (b) probability distributions of V_{S30} for Fraser River delta site 2. MAP model estimate shown by dotted line in both panels. Details as in Fig. 4.

For the Victoria site, Fig. 6a shows that the uncertainty of V_{SZ} is ≤ 15 m/s in the upper 16 m due to the well-determined upper low-velocity layer (Fig. 3), then increases with depth due to the poorly constrained high-velocity half space. Fig. 6b shows the mean V_{S30} estimate ± two standard deviations is 182 ± 31 m/s, at the boundary between NEHRP site classes D and E. The V_{S30} probability distribution in Fig. 6b indicates 58% probability that the Victoria site corresponds to site class D, slightly more probable than class E (42%). Overall, the low predicted amplification based on V_{S30} is in disagreement with known high amplification effects from earthquakes at the Victoria site (Molnar et al. 2012b); V_{S30} fails as an amplification predictor for this site. The low-velocity soil layer to about 17 m depth causes significant amplification; however, this effect is not properly represented by V_{S30} which averages into the high-velocity basement. By examining the V_{SZ} probability distribution, V_{S15} ~100 m/s, is a more appropriate response indicator for this site.

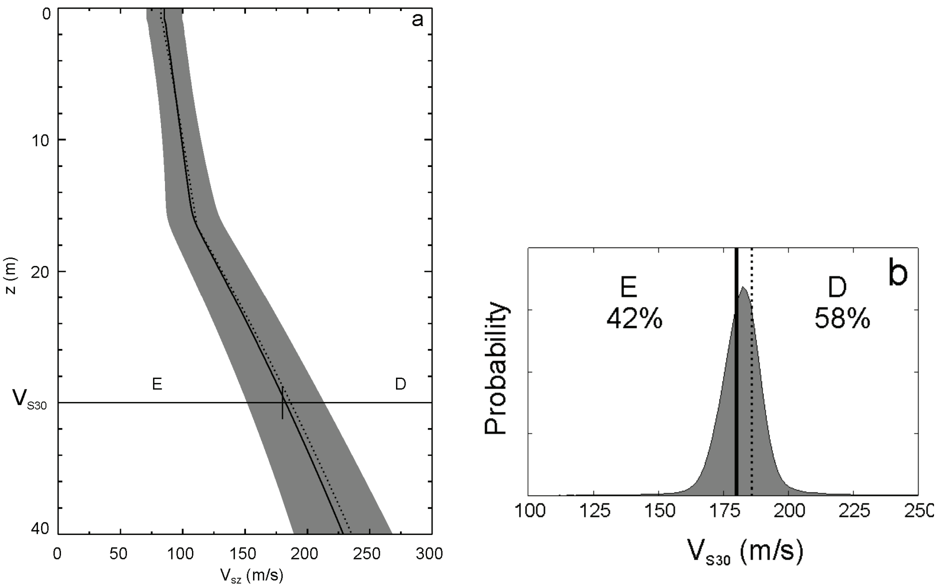

Figure 6. (a) V_{SZ} probability distribution and (b) probability distributions of V_{S30} for Fraser River delta site 2. MAP model estimate shown by dotted line in both panels Details as in Fig. 4.

3.2. Uncertainty of amplification spectra

A better indicator of earthquake site response is the calculation of frequency-dependent amplification based on the entire V_S profile, rather than an average property of the soil column such as V_{S30}. Linear site amplification that accounts for resonance of vertically-propagating SH-waves is calculated using the NRATTLE routine included in the ground motion simulation package of Boore (2005). For input, a subsample of up to 20,000 V_S profiles from the PPD are drawn with a wide spacing to skip over locally correlated models. The assumption of vertical incidence is used in all calculations. Assumptions for density and Q_S values for each layer are set to a Gaussian distribution. For the Fraser River delta sites, density and Q_S for the powerlaw layer are set to a mean (one standard deviation) of 2000 (200) kg/m^3 and 20 (10; cut off at minimum of 2), respectively. The density of the half-space is taken to be a Gaussian with mean and standard deviation 2200 and 200 kg/m^3. For the Victoria site, density and Q_S of the upper silt layer is represented by a Gaussian distribution with mean (standard deviation) of 1850 (200; minimum cut off of 1650) kg/m^3 and 10 (10; minimum cut off of 2), respectively. Density of the underlying glacial material and/or bedrock layer is taken to be a Gaussian with mean and standard deviation of 2400 and 300 kg/m^3.

Marginal probability distributions of the resulting SH-wave amplification are obtained by integrating the probability of all calculated amplification spectra over an amplification-frequency grid (normalized independently at each frequency). For the Fraser River delta sites, Figure 7 shows multiple resonant peaks are resolved up to ~ 5 Hz and ~2 Hz for Fraser River delta site 1 and 2, respectively, with relatively low amplification (< 5). For delta site 1, peak frequencies of the first-three modes are well determined to within 0.4 Hz. The fundamental peak frequency is determined to be 0.6 ± 0.1 Hz with a large uncertainty in amplification of 2.1. For delta site 2, the first two resonant peaks are resolved with relatively low amplification (< 5); the fundamental peak frequency is resolved to be 0.5 Hz ± 0.1 Hz with an amplification factor of 3.7 ± 1.0 Hz. Variation in the fundamental peak frequency and amplification is related to variation in the depth and strength of the impedance ratio with the half-space for all 20,000 V_S models.

Figure 7 shows microtremor horizontal-to-vertical (H/V) spectral ratios calculated from the 3-component microtremor recordings at each delta site. For delta site 1, peaks occur at approximately 0.35, 1.0 and 4.0 Hz, with the largest amplification at 1.0 Hz. For delta site 2, peaks occur at ~0.18 and 0.7 Hz. The low fundamental peak frequencies (0.35 and 0.18 Hz at sites 1 and 2, respectively) are indicative of thick accumulations of low-velocity deltaic material. Earthquake recordings at four strong-motion instrument sites on the delta with similar thick sequences of material also demonstrate a consistent low fundamental frequency (~0.3 Hz) in Fig. 8 (left panel). Peaks in some of the earthquake ratios are observed at higher frequencies of approximately 1.0 and 1.4 Hz related to site-dependent first higher-order modes. The microtremor inversion results provide useful SH-wave amplification estimates in reasonable agreement with the empirical spectra.

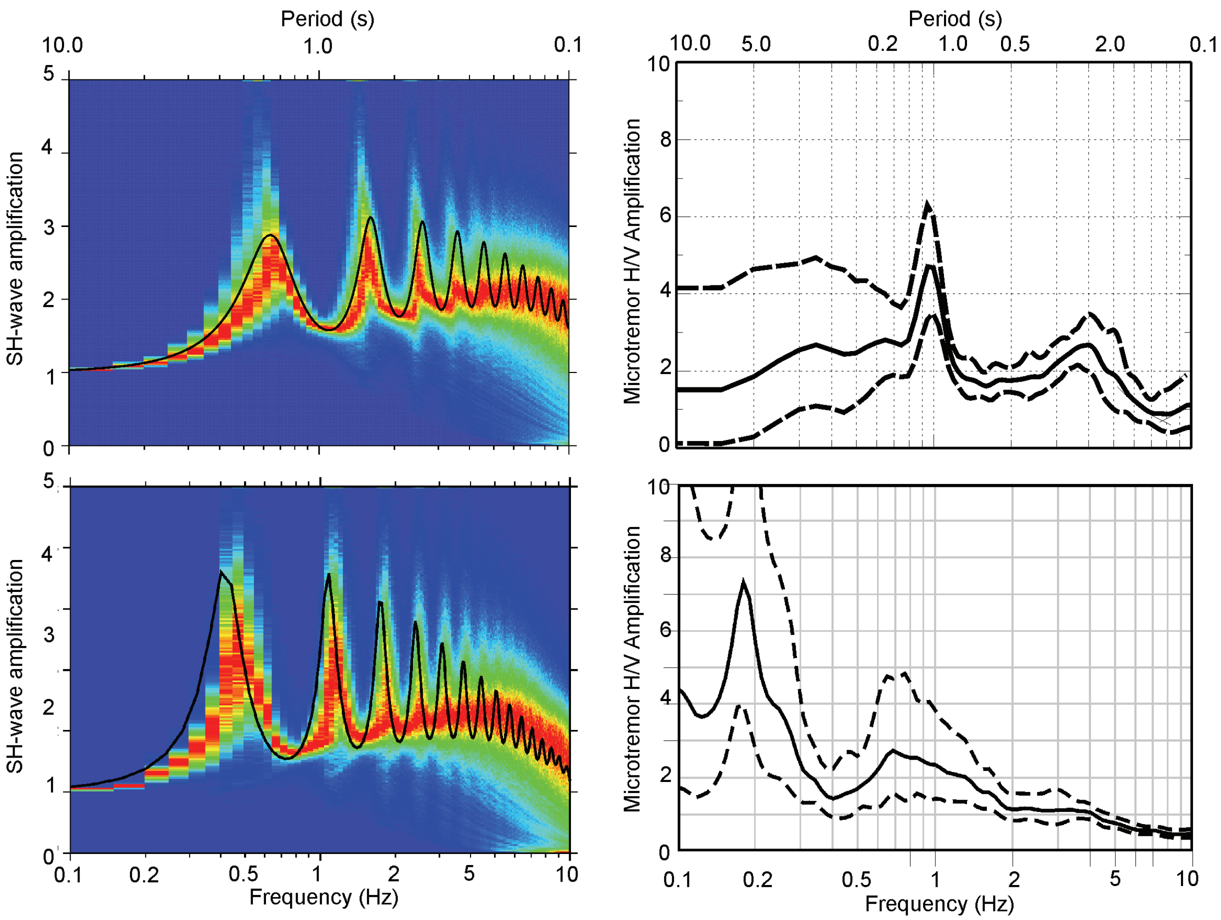

Figure 7. Left panels: Marginal probability amplification spectra for delta site 1 (top) and 2 (bottom) calculated from the sample of V_S profiles shown in Fig. 1 and 2, respectively. Black line corresponds to spectrum predicted for the most-probable V_S profile. Right panels: Average microtremor H/V spectral ratio (dashed lines denote ± two standard deviations) for delta site 1 (top) and 2 (bottom).

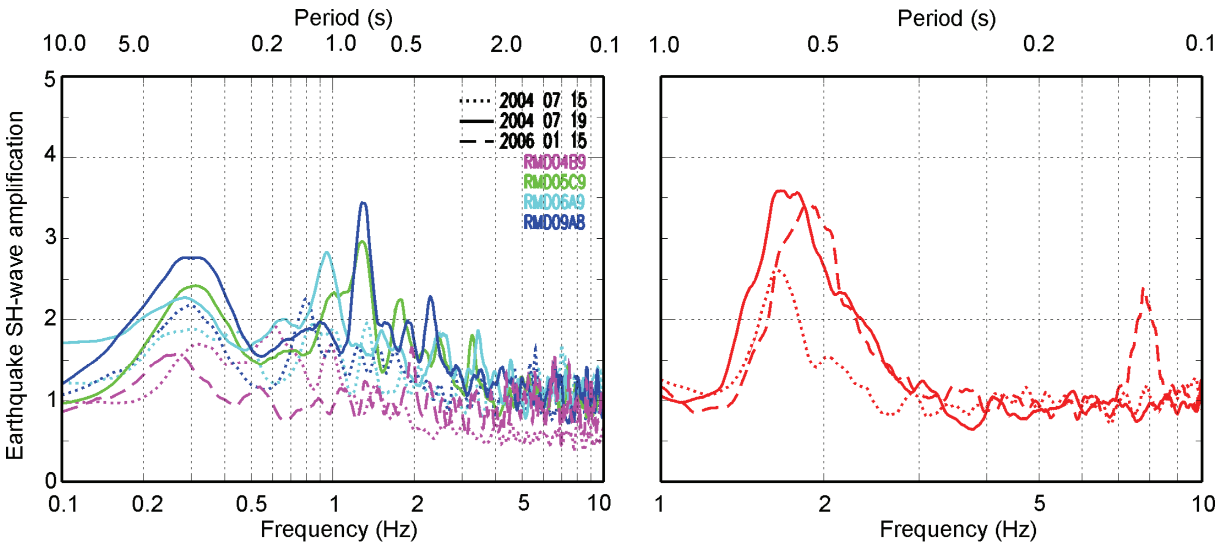

Figure 8. Earthquake SH/V ratios from three events recorded at four Geological Survey of Canada strong-motion stations on the delta (left panel; station names given in legend) and at the Victoria site (right panel).

For the Victoria site, the uncertainty distribution in SH-wave amplification displays three resonant modes (Fig. 9). The fundamental frequency is relatively well determined at 1.8 ± 0.3 Hz as the depth of the strong impedance contrast is well constrained between 15-18 m depth, whereas the peak amplification is poorly determined, 6.8 ± 5.0, due to the poorly constrained half-space velocity (500-1500 m/s). The prominent fundamental peak at 1.8 Hz is also clearly observed in both the microtremor H/V (Fig. 9) and earthquake SH/V ratios (Fig. 8; right panel). The high predicted SH-wave amplification at the fundamental frequency is in better agreement with the empirical amplification from the average microtremor response of 5.5 than from earthquake SH-waves of 2.5-3.5. The higher modes predicted at 5 and 8 Hz are generally not observed in the microtremor H/V and earthquake SH/V ratios. However, the SH/V ratio from a nearby high-frequency content earthquake shows amplification at 8 Hz as predicted. These discrepancies between predicted and empirical amplification for higher-order modes suggest strong attenuation occurs at the Victoria site.

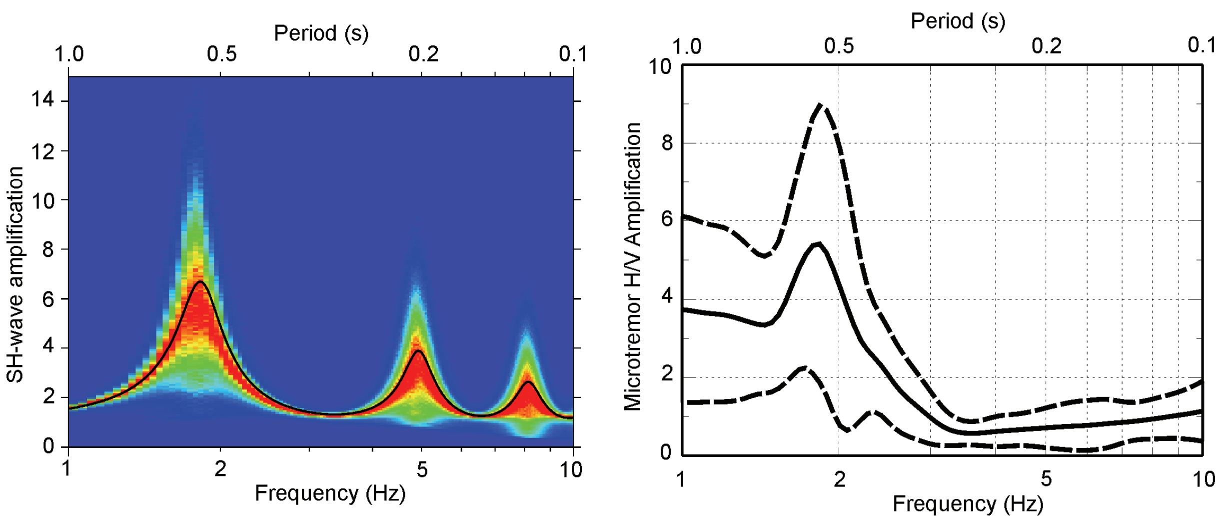

Figure 9. Left panel: SH-wave amplification probability distributions for the Victoria site calculated for the V_S probability distribution shown in Fig. 3. Right panel: Average microtremor H/V spectral ratio (dashed line s denote ± two standard deviations).

Finally, the SH-wave amplification spectrum for the most-probable V_S profile at each site generally agrees well with the mean of the amplification spectrum probability distribution. However, the spectral uncertainties provided by the Bayesian inversion provide a more robust and informative estimate of SH-wave amplification than simply the best-fit estimate.

4. CONCLUSIONS

This paper examines uncertainty in linear site amplification due to uncertainties in V_S structure as quantified via Bayesian inversion from microtremor array dispersion data at deep (> 100 m) sediment sites on the Fraser River delta and a shallow (< 30 m) sediment site in Victoria British Columbia, Canada. The Bayesian inversion draws a large sample of V_S models from the PPD to represent the V_S profile probability (uncertainty) distribution. To propagate the uncertainty distributions in V_S structure into uncertainty in predictions of site amplification, all V_S profile samples are used in the calculations of site amplification, thereby providing a sampled uncertainty distribution. Probability distributions are computed for common predictors of site amplification including V_{S30} and associated NEHRP site class, and SH-wave amplification spectra.

The Bayesian inversion determines a well resolved V_S profile to at least 110 m and 200 m depth at Fraser River delta sites 1 and 2, respectively for a power-law gradient parameterization. At the Victoria site, a layer with low V_S and a weak linear gradient is indicated to 15-18 m depth, above an abrupt increase to higher velocity material. In all three cases, the half-space velocity is not resolved as well as near-surface structure, consistent with previous observations that basement velocity is rarely retrieved with microtremor inversion (Cornou et al. 2006).

Computing the V_{SZ} probability distribution is shown to be more informative for site response characterization than a single V_{SZ} estimate determined from the most-probable (best-fit) V_S profile. The probability distribution of V_{S30} is examined in further detail as this is the site amplification predictor in current building codes and ground motion prediction equations. For Fraser River delta sites 1 and 2, the V_{S30} uncertainty distribution spans NEHRP site classes D and E with a 95% probability of class E, and 62% probability of class D, respectively. At Victoria, the V_{S30} distribution occurs at the D/E class boundary with 58% probability of class D, associated with lower amplification. This is in disagreement with known amplification effects of earthquakes and the strong near-surface impedance contrast at ~17 m depth. These results indicate that V_{S30} is not a useful indicator of site response for the Victoria site. However, the V_{SZ} probability profile computed from the microtremor inversion indicates that V_{S15} would provide a meaningful indicator.

Amplification spectra calculated using the entire V_S profile generally provide a better indicator of earthquake site response than an average V_{SZ} estimate. In this paper, uncertainty distributions for SH-wave amplification spectra based on seismic impedance variations and resonance are calculated from the sampled V_S profile probability distribution. The powerlaw gradients resolved for Fraser River delta sites result in relatively low amplification at multiple modes; peak frequencies are determined within 0.4 Hz and amplification within a factor of 2. At the Victoria site, the SH-wave amplification spectrum predicts the fundamental resonant frequency at 1.8 ± 0.3 Hz, with large uncertainties in amplification (factor of 5). The predicted fundamental peak frequency agrees well with that from empirical earthquake and microtremor spectral ratios. The inversion results provide useful SH-wave amplification estimates in reasonable agreement with the empirical spectra, indicating that the limited sensitivity depth of the microtremor data is not a serious constraint.

In summary, this paper illustrates the benefits of rigorous uncertainty estimation in site response characterization based on non-invasive seismic methods such as microtremor dispersion inversion. Propagating V_S profile probability distributions, estimated via Bayesian inversion, through to various measures of site amplification quantifies the uncertainties in key predictors such as NEHRP site class, and resonant frequencies and amplifications, indicating the level of confidence that is justified in these predictors.

AKCNOWLEDGEMENT
The authors would like to thank Karen Simon for field assistance. Financial support was provided by the National Sciences and Engineering Research Council of Canada, Geological Survey of Canada, and University of Victoria. This is ESS contribution number 2012####.

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