

Sensitivity of the Nonlinear Seismic Response of a Short RC Bridge to Numerical Modeling



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SUMMARY:

A numerical experiment that investigates the effect of nonlinear structural modeling on the influence of spatially variable seismic excitations on the response of reinforced concrete bridges is conducted. A short bridge is modeled in the nonlinear range using two approaches for the representation of the pier columns. One approach considers the formation of lumped plastic hinges and the other utilizes fiber elements. The effect of structural redundancy is also evaluated by making the structures stiffer or more flexible at their abutments. The models are subjected to spatially variable and uniform excitations. The results indicate that, by modeling the deformation of the columns with lumped plastic hinges, the structure becomes more flexible than when the columns are modeled with fiber elements. Furthermore, the degree of structural redundancy is important as, for the stiffer structures, the effect of spatially variable excitations is significant, but becomes negligible for the more flexible structures.

Keywords: spatially variable seismic ground motions; reinforced concrete bridges; numerical modeling

1. INTRODUCTION

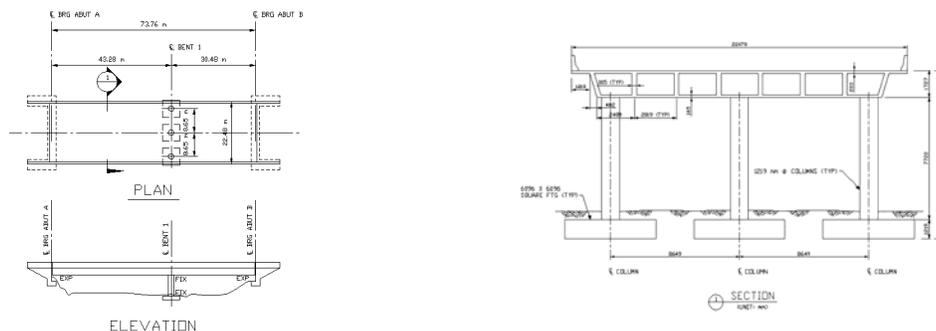
The effect of the spatial variation of seismic ground motions on the response of reinforced concrete (RC) bridges has been evaluated, essentially in its entirety, with nonlinear numerical studies. These studies utilized different modeling assumptions and different numerical tools for the simulation of the nonlinear behavior of the structures. Their majority considered lumped plastic hinges for the modeling of the inelastic behavior of the pier columns (e.g., Saxena et al., 2000; Lou and Zerva, 2005; Lupoi et al., 2005), and a few studies permitted the spread of plasticity along the length and across the section of the member (e.g., Tzanetos et al., 2000; Burdette and Elnashai, 2008). It is well-known, however, that the results of nonlinear analyses of structural systems can be highly dependent on the modeling assumptions (e.g., among others, Elnashai and McClure, 1996; Nielson and DesRoches, 2006).

This study conducts a numerical experiment to investigate the effect of nonlinear structural modeling assumptions on the influence of the spatial variation of seismic ground motions on the response of RC bridges. A typical, short, two-span bridge is modeled using two different numerical approaches for the representation of the pier columns, which have been known to carry the damage caused in bridges during earthquakes. One approach considers the formation of lumped plastic hinges and the other utilizes fiber elements. Furthermore, in order to analyze the effect of redundancy on the seismic response of the systems, the bridge models are made stiffer by artificially closing the gap between the deck and the seat-type abutments, or more flexible by permitting the gap to be significantly wide. The models are then subjected to spatially variable and uniform seismic excitations at their supports.

2. NUMERICAL MODELING OF THE BRIDGE

The bridge model in this study is the first (bridge no. 1) of the seven seismic design examples presented by the Federal Highway Administration (FHWA, 1996). The structure is a two-span, straight,

RC bridge. The plan and elevation of the bridge are illustrated in Fig. 1(a). The total length of the bridge is 73.76 m. Its super-structure is a 22.48 m-wide post-tensioned continuous box girder. Seat-type abutments are utilized for the bridge; space behind the end diaphragm can be prescribed to accommodate the longitudinal movement of the superstructure. The superstructure and the columns are connected with a cap beam. The intermediate bent consists of three columns that are fully connected with square spread footings underneath (Fig. 1(b)). The cross section of the columns is circular with a 1.2 m diameter and 22 #11 bars equally spaced around the perimeter of the column; #5 spirals are used with a spacing of 89 mm through the entire length of the columns.



(a) Plan and elevation of the bridge

(b) Bridge layout at section 1 of part (a)

Figure 1. Configuration of the first (bridge no. 1) of the seven seismic design examples presented by the Federal Highway Administration (1996)

The nonlinearities of the bridge models considered in this study include boundary nonlinearities, i.e., the contact between the two ends of the box girder and the abutments, and material nonlinearities, i.e., the inelastic behavior of the columns and the elastomeric bearings at each end of the box girder. The prestressed superstructure is modeled using 3-D elastic beam elements with flexural and torsional moments of inertia corresponding to the gross section properties, I_g and J_g , respectively, as recommended by ATC-32 (1996) and Caltrans (2001). Each abutment is represented by two elastic, fully plastic spring elements in parallel with initial gaps; the force-deformation relationship of the spring elements is determined according to Caltrans (2001). A bilinear model is adopted for the shear force-deformation relationship of the elastomeric bearings. The nonlinearity of the pier columns is emulated using two approaches: one considers the formation of lumped plastic hinges and the other utilizes fiber elements.

The lumped plastic hinge model (Fig. 2(a)) was created using the DRAIN-3DX software (Prakash et al., 1994). The potential plastic hinge zones are simulated with a rigid element and a nonlinear rotational spring, and placed at both ends of the pier columns to account for the expected double-curvature behavior. The determined plastic hinge length conforms to Caltrans (2001). The properties of the rotational spring elements are obtained from the moment-rotation analysis of the column section based on the moment-curvature relationship with the assumption of uniformly distributed plastic curvature along the plastic hinge zone. The code USC_RC (2012) was used to determine the moment-curvature curve (Fig. 2(b)). Mander's (1988) stress-strain model for confined concrete and the USC_RC (2012) steel model for the reinforcement were used for the moment-curvature analysis.

The object-oriented software framework OpenSees (Mazzoni et al., 2005) was utilized to model the columns with fiber elements. Fiber elements permit the spread of plasticity both along the member and across its cross section by explicitly taking into consideration the stress-strain relationship of different fibers such as reinforcing steel, confined concrete, unconfined concrete, etc. A stress-strain relationship with a parabolic ascending branch (Hognestad, 1951) and a linear softening branch is used for the unconfined concrete, and Mander's (1988) high-strain-rate model for the confined concrete. The reinforcing steel is simulated using a bilinear strain-hardening model with a hardening ratio of 1.6% (Caltrans, 2001). Sensitivity of different fiber arrangements for the cross section of the columns (Fig. 3(a)) was performed to determine their optimal configuration for the nonlinear dynamic analysis. The

moment-curvature analysis results (Fig. 3(b)) suggest that 89 fibers (FBR 089 in Fig. 3) are sufficient to capture the cross section response of the pier columns of the bridge. The comparison of the moment-curvature curves resulting from the fiber discretization in OpenSees (Fig. 3(b)) and that evaluated by USC_RC (Fig. 2(a)) indicate that the two models produce similar section properties.

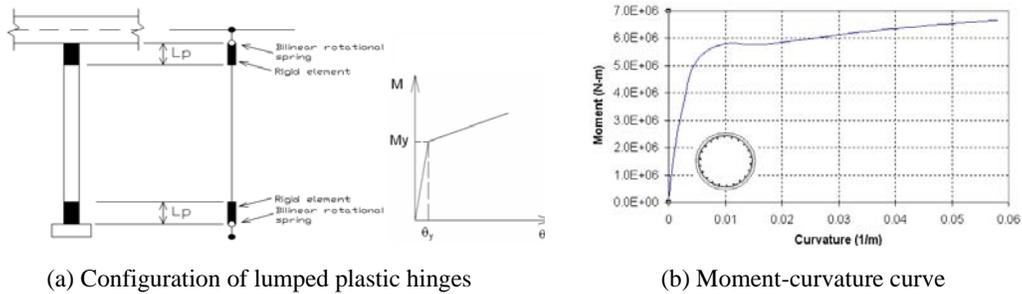


Figure 2. Lumped plastic hinge modeling of the columns

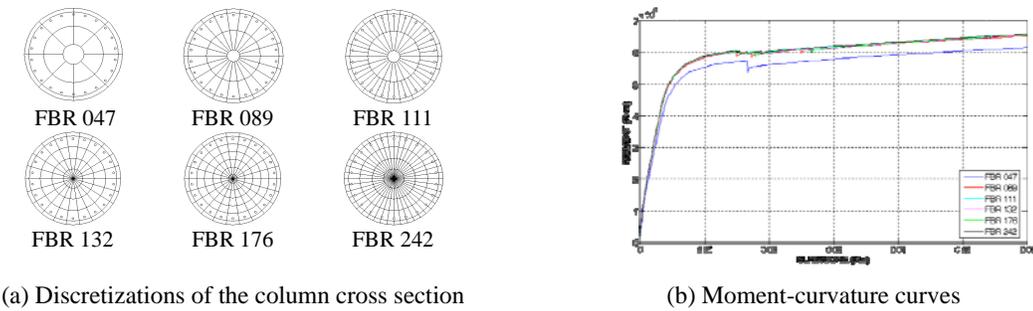


Figure 3. Fiber element modeling of the columns

3. DESCRIPTION OF THE SEISMIC EXCITATION

The seismic excitations utilized in this study were generated with the approach proposed by Hao et al. (1989), and processed with the methodology suggested by Liao and Zerva (2006). It was considered that the bridge supports were all located on “soft soil” conditions described by the corresponding UBC spectra. To ensure a nonlinear bridge response, the target peak acceleration was set at 0.5g. Furthermore, to observe more variability in the displacement time series at the short distances spanned by the analyzed bridge, the lagged coherency model of Harichandran and Vanmarcke (1986), which provides partial correlation of the motions at low frequencies, was adopted herein. An assumed fairly low apparent propagation of the motions (750 m/s) was also utilized.

Figure 4 presents the single set of simulated displacement time series that were applied in the longitudinal direction at the supports of the bridge models in this numerical experiment. In the figure, “TH1” denotes the displacement time history at abutment A, “TH2” the time history at bent 1, and “TH3” the time history at abutment B (Fig. 1(a)). It can be observed from the figure that the lagged coherency model of Harichandran and Vanmarcke (1986) leads to some variability in the displacement waveforms, and that the apparent propagation of the motions is almost not visible for the short separation distances between the bridge supports (Fig. 1(a)).

To compare the effect of spatially variable seismic excitations on the bridge response with that induced by uniform motions, two scenarios of uniform input excitations are also considered. The ground motions with the largest (TH3) and smallest (TH1) peak value in Fig. 4, corresponding, respectively, to the excitations at the right and left abutment, are selected as uniform ground motion input at all bridge supports. Because, presumably, these two scenarios will provide the highest and lowest seismic demand on the bridge, they are referred to hereafter as the worst-case and best-case scenario uniform

input motions, respectively. In all subsequent figures, “BEST” denotes response quantities induced by the best-case scenario uniform motions, “WORST” by the worst-case scenario uniform motions, and “SV” by the spatially variable input motions.

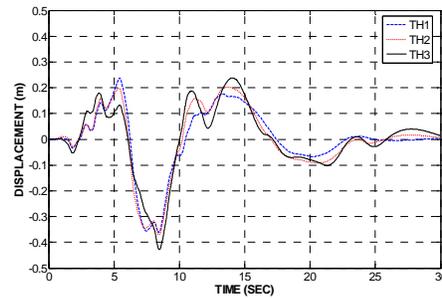


Figure 4. Simulated displacement time series at the bridge abutments and pier

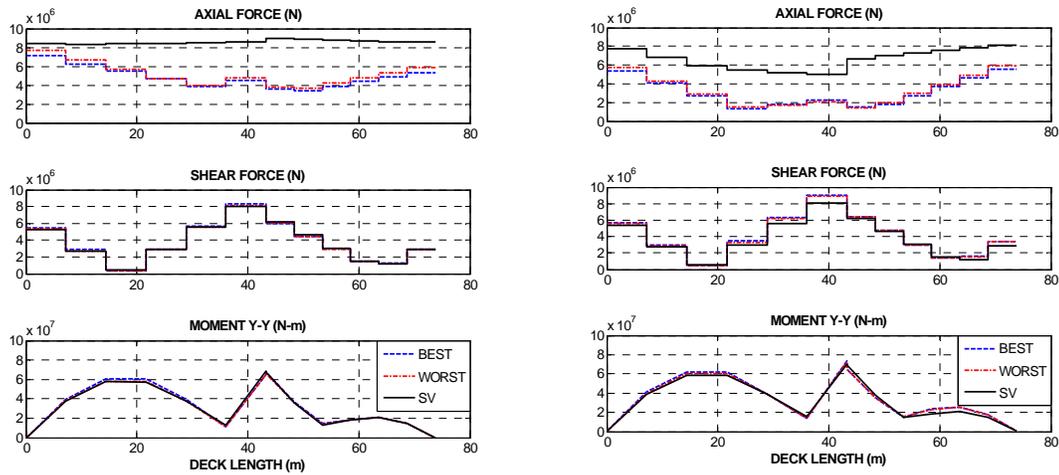
4. NUMERICAL RESULTS

The seismic response results of the bridges subjected to spatially variable and uniform excitations in their longitudinal direction are compared in this section. Two cases of boundary conditions at the abutments are also considered for both numerical models to investigate the effect of structural redundancy on the bridge response. In the first case, it is assumed that the initial gaps at the two abutments (Fig. 1(a)) are completely “closed”, and, in the second case, the initial gaps are “open”, i.e. a fairly significant gap size (15.2 cm) is utilized in the evaluation. The closed-gap case results in more constrained structures, whereas the open-gap case reflects more flexible structures, as the deck is permitted to move relative to the abutments. Rayleigh damping with a ratio of 5% of critical is considered in all evaluations.

4.1 Results of nonlinear analysis for a closed gap at the abutments

With a closed gap at the abutments, the excitation at the abutments fully guides the ends of the deck. Figure 5 presents the envelope functions of the absolute deck response, in terms of axial forces, shear forces and bending moments, for the lumped plastic hinge model using DRAIN-3DX in part (a) and the fiber element model using OpenSees in part (b). The shape of the bending moment envelope functions is very similar for both models, with the fiber element model in OpenSees providing slightly higher values. For the lumped plastic hinge model, the bending moments essentially coincide for all excitation cases, whereas, for the fiber element model, the bending moments caused by the uniform excitations along the spans are only slightly higher than the ones induced by the spatially variable excitations. Similarly, shear forces follow the same pattern for both models and all three excitation cases. The uniform excitations tend to produce a higher shear force response at the location of the pier, with the OpenSees results being higher than the DRAIN-3DX results. Very substantial differences are, however, observed for the axial force demand along the deck. The spatially variable excitations induce significantly higher axial forces in the deck than the uniform motions. Furthermore, contrary to the shear force and bending moment envelopes, which are similar for both numerical models, the envelopes of the axial forces differ significantly. For the uniform support excitations (best- and worst-case scenarios), the deck at the left abutment is more stressed in the DRAIN-3DX model (Fig. 5(a)) than the OpenSees model (Fig. 5(b)), and axial forces through the left span, at the location of the pier and, continuing, up to the midspan of the second span are higher for the DRAIN-3DX model (Fig. 5(a)) than the OpenSees model (Fig. 5(b)). Spatially variable excitations induce a very high axial force demand along the entire deck for the DRAIN-3DX model (Fig. 5(a)), whereas axial forces drop at the location of the pier for the OpenSees model (Fig. 5(b)). The significantly higher axial force values induced by the spatially variable excitations compared to those induced by the uniform motions in both models may be attributed to the fact that, for the uniform excitations, the structure moves “in-phase,” as the excitation at all supports is the same, whereas it moves “out-of-phase” for the spatially

variable excitations. Since the deck is fully constrained in this closed-gap analysis, it becomes more strained for the spatially variable excitations than the uniform motions. The high values of the axial forces at the location of the pier for the DRAIN-3DX model, however, are an effect of the column modeling, as will be shown subsequently.



(a) Deck absolute force demand envelopes in the lumped plastic hinge model using DRAIN-3DX

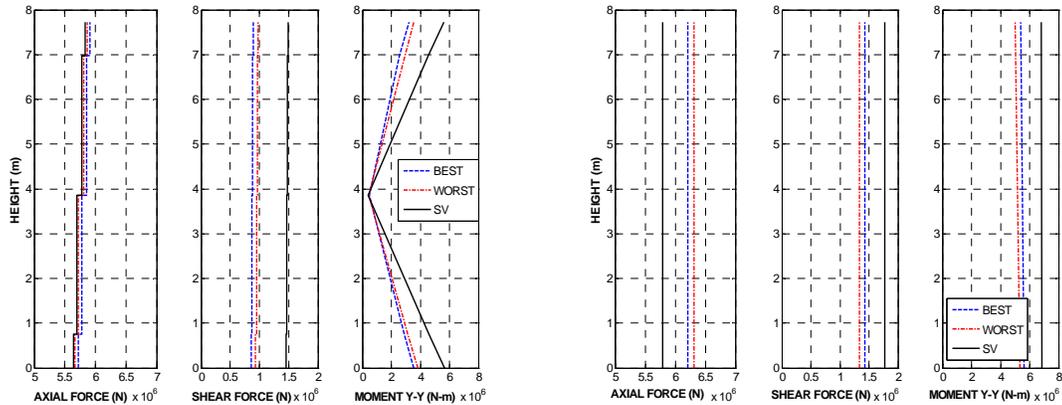
(b) Deck absolute force demand envelopes in the fiber element model using OpenSees

Figure 5. Deck force demand envelopes caused by the three input motion scenarios for the closed-gap analysis

Figure 6 presents the column absolute force demand envelopes, in terms of axial forces, shear forces and bending moments, for the lumped plastic hinge model in DRAIN-3DX (part (a)) and the fiber element model in OpenSees (part (b)). Higher values for all response quantities are obtained with the fiber element model in OpenSees (Fig. 6(b)) than the lumped plastic hinge model in DRAIN-3DX (Fig. 6(a)). For both nonlinear models, shear forces and bending moments are higher for the spatially variable than the uniform excitations, whereas, again for both models, uniform excitations induce the highest axial forces. For the lumped plastic hinge model (Fig. 6(a)), the worst-case scenario uniform excitations induce higher bending moments and shear forces than the best-case scenario, whereas the best-case scenario induces higher axial forces than the worst-case scenario. The opposite occurs for the forces and moments induced in the fiber element column (Fig. 6(b)) for the two uniform input excitation scenarios. Figures 5 and 6 indicate that the speculation that the lowest peak amplitude ground displacement time series will, presumably, induce the lowest response is obviously erroneous. In spite of this fact, the notation “best-” and “worst-” case scenario is maintained herein as a means to distinguish between the two analyzed cases of uniform excitations.

Substantial pounding at the abutments occurred in all cases, as expected, for this closed-gap bridge model. The DRAIN-3DX analysis resulted in pounding forces of 3.73×10^6 N, 4.0×10^6 N and 4.0×10^6 N for the best-, worst- and spatially variable ground motion scenarios, respectively, whereas the corresponding values in the OpenSees model were 3.06×10^6 N, 3.23×10^6 N and 4.0×10^6 N. Interestingly, the structure modeled both with the lumped plastic hinge model in DRAIN-3DX and the fiber element model in OpenSees responded in the linear range in terms of the behavior of its bearing elements and columns when subjected to uniform ground motions, but nonlinearly when the excitation was spatially variable. For illustration purposes, Fig. 7 presents the hysteretic response of a representative bearing element (located at the southern corner of abutment A in Fig. 1(a)) for the three input motion scenarios when the columns are modeled using fiber elements in OpenSees, and Fig. 8 the hysteretic response of a section at the top of a representative column modeled, again, in OpenSees. Results similar to those in Figs. 7 and 8 were obtained for all bearing and column responses of the model in OpenSees as well as the lumped plastic hinge model in DRAIN-3DX. It can be seen from Figs. 7 and 8 that the uniform excitation cases (labeled “BEST” and “WORST” on the left and middle

part, respectively, of the figures) induce a linear response, whereas the spatially variable excitations (labeled “SV” on the right part of the figures) induce a nonlinear response. For this structure, that is fully constrained at the abutments because the gap is closed, the uniform excitations, that cause all supports to move in phase, limit the deformation at the columns and bearings, whereas the spatially variable ground motions, that induce an “out-of-phase” component, strain the columns and bearings more significantly, thus leading to their nonlinear response. Furthermore, the differences in the response may also be attributed to the fact that nonuniform excitations induce a pseudo-static contribution to the response, which the uniform excitations do not, and, also, nonuniform motions excite the dynamic structural response in a different manner than uniform ground motions (Zerva, 2009).



(a) Column absolute force demand envelopes in the lumped plastic hinge model using DRAIN-3DX (b) Column absolute force demand envelopes in the fiber element model using OpenSees

Figure 6. Column force demand envelopes caused by the three input motion scenarios for the closed-gap analysis

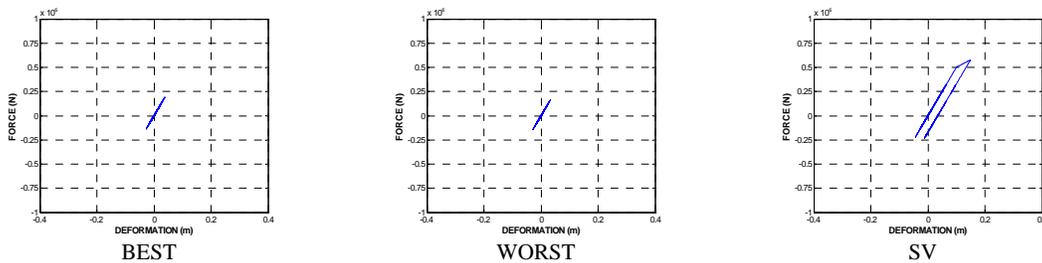


Figure 7. Hysteretic response of a representative bearing caused by the three input motion scenarios in the fiber element model of the bridge columns using OpenSees for the closed-gap analysis

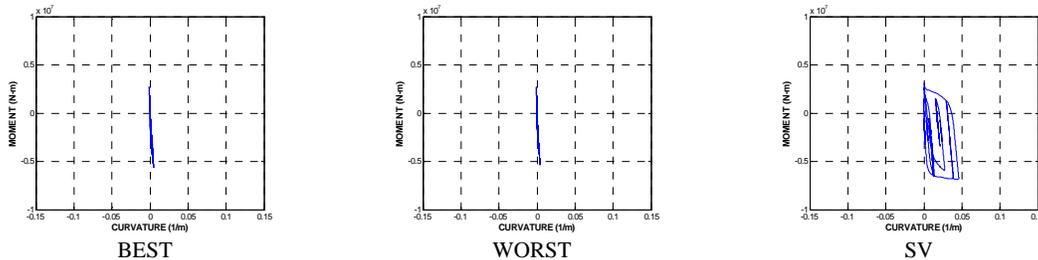


Figure 8. Hysteretic response of a section at the top of a representative column caused by the three input motion scenarios in the fiber element model of the bridge columns using OpenSees for the closed-gap analysis

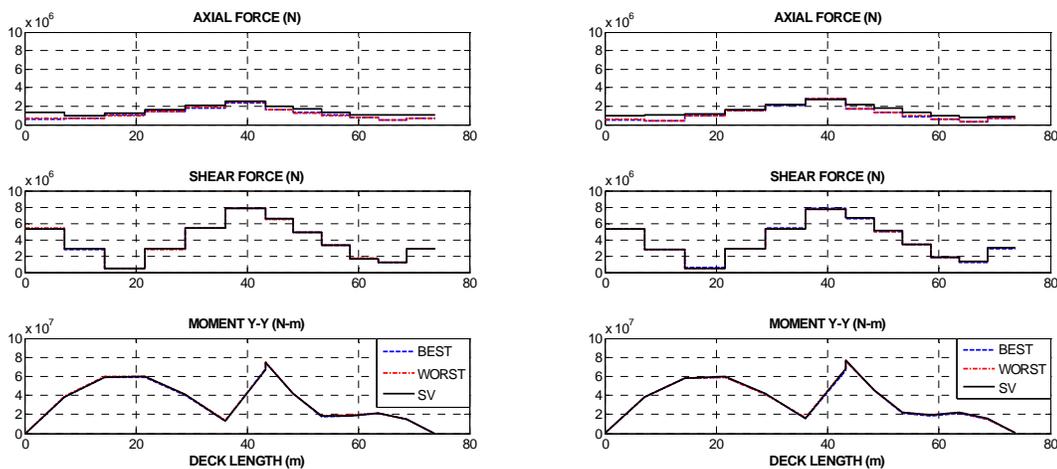
The major differences between the responses of the two nonlinear models are observed in the values of the peak absolute bending moments at the top and bottom of the columns on the left part of Figs. 6(a) and (b), and the axial force envelope distributions along the deck on the top part of Figs. 5(a) and (b).

For all input motion scenarios, the lumped plastic hinge model in DRAIN-3DX produces consistently lower bending moments (Fig. 6(a)) than the fiber element model in OpenSees (Fig. 6(b)). It appears then that, with the introduction of the lumped plastic hinges, the DRAIN-3DX model becomes more flexible than the OpenSees model. However, the stiffer deck cannot absorb the flexibility of the columns in the horizontal direction in the DRAIN-3DX model, especially since the deck is fully constrained at its ends, and, hence, larger axial forces result in the deck, especially at the location of the pier for the spatially variable excitation case (Fig. 5(a)). This observation may explain the differences in the axial force envelope distributions along the deck in the top part of Figs. 5(a) and (b) for the nonuniform excitation case.

4.2 Results of nonlinear analysis for an open gap at the abutments

A different picture altogether is obtained when the gap size is increased significantly from 0 to 15.2 cm. Figure 9 presents the deck absolute force demand envelopes for the lumped plastic hinge model in DRAIN-3DX in part (a) and for the fiber element model in OpenSees in part (b). Figures 10(a) and (b) present the corresponding column absolute force demand envelope functions for the two models.

The two models yield very similar results, and the different input excitations do not seem to have any significant effect on these more flexible structures. The axial forces along the deck in this case (Fig. 9) are dramatically reduced compared to those of the models with no allowable gap (Fig. 5). Furthermore, in the open-gap analysis, because the deck has the ability to move in the longitudinal direction, axial forces for the lumped plastic hinge model (Fig. 9(a)) become slightly lower than the axial forces in the fiber element model (Fig. 9(b)) at the location of the pier. No significant response differences are induced by the different ground motion scenarios in the columns modeled in DRAIN-3DX (Fig. 10(a)) and in OpenSees (Fig. 10(b)). Moments and shear forces, however, are still higher in the fiber element model (Fig. 10(b)) than the lumped plastic hinge model (Fig. 10(a)). For this more flexible structure with a wide gap opening, the uniform excitations do not induce pounding at the abutments for both models. For the OpenSees model, there is no pounding at the abutments for the spatially variable excitations as well, whereas the nonuniform motions still induce a small pounding force (0.28×10^6 N) in the DRAIN-3DX model, further confirming that the lumped plastic hinge model is more flexible than the fiber element model.



(a) Deck absolute force demand envelopes in the lumped plastic hinge model using DRAIN-3DX

(b) Deck absolute force demand envelopes in the fiber element model using OpenSees

Figure 9. Deck force demand envelopes caused by the three input motion scenarios for the open-gap analysis

Because the structure becomes now more flexible, the bearings and columns of both models with an open gap responded nonlinearly for all input excitation scenarios. Again, for illustration purposes, only

the response of the fiber element model in OpenSees is presented herein. Figure 11 presents the results of the hysteretic behavior of the same representative bearing as in Fig. 7, and Fig. 12 the hysteretic response at the top of the representative column of Fig. 8. Figures 11 and 12 clearly indicate that the models responded nonlinearly for all cases of input ground motions, with the nonlinear response induced by the spatially variable seismic excitations being slightly more pronounced. Similar behavior to that presented in Figs. 11 and 12 was also obtained for the remaining bearings and columns in the OpenSees model as well as for all bearings and columns in the DRAIN-3DX model, when an open gap was considered at the abutments.

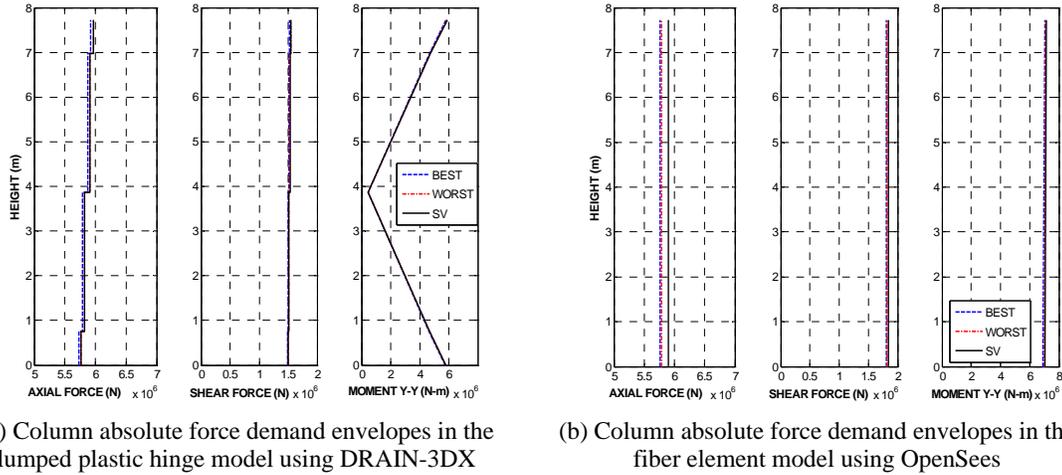


Figure 10. Column force demand envelopes caused by the three input motion scenarios for the open-gap analysis

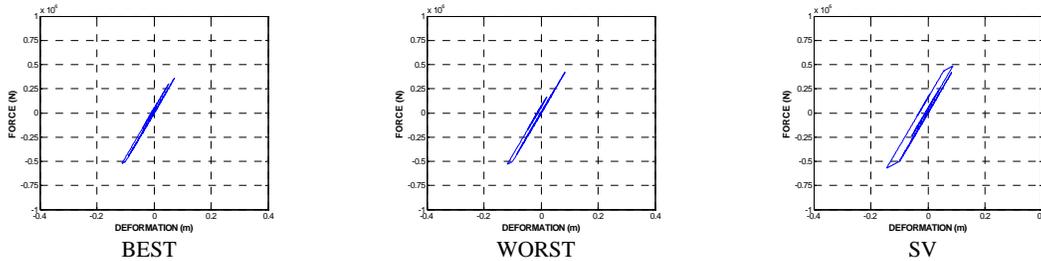


Figure 11. Hysteretic response of a representative bearing caused by the three input motion scenarios in the fiber element model of the bridge columns using OpenSees for the open-gap analysis

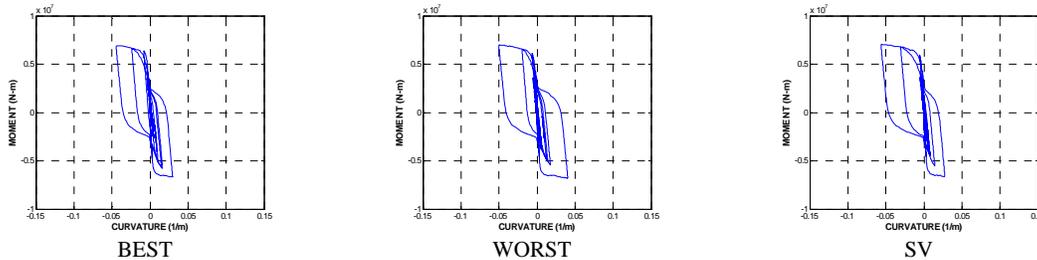


Figure 12. Hysteretic response of a section at the top of a representative column caused by the three input motion scenarios in the fiber element model of the bridge columns using OpenSees for the open-gap analysis

DISCUSSION AND CONCLUSIONS

This study undertook the examination of the effect of modeling assumptions on the influence of the spatial variation of the seismic ground motions on the response of RC bridges by means of a numerical experiment. For this purpose, a typical, short, two-span, RC bridge was selected for nonlinear analyses

that included both boundary and material nonlinearities. Two approaches were utilized for the modeling of the nonlinear behavior of the pier columns: The first approach introduced lumped plastic hinges at both ends of the columns to ensure the double-curvature effects at the supports and at the deck-column connections, and modeled the structure in DRAIN-3DX. The second approach utilized OpenSees and modeled the nonlinear behavior of the pier columns with fiber elements, which permit the spread of plasticity both along the member and across its cross section. The effect of the degree of structural redundancy on the response of the bridge was also investigated by controlling the size of the gap between the deck and the seat-type abutments: a stiffer structure was obtained by completely closing the gap, and a more flexible structure was obtained by considering a wide gap, so that the deck could move relatively freely in the longitudinal direction at its two ends. The systems were then subjected to a single set of spatially variable excitations and two sets of uniform motions at their supports in the longitudinal direction. It should be noted that the results of a single simulation with the excitation applied in a single direction do not suffice for the reliable estimation of the stochastic characteristics of the seismic response of a structure. However, the detailed analysis of the response characteristics of the system to these limited excitation cases, (and, hence, the use of the term “numerical experiment” herein), provide insight into the behavior of the system and the effect of the modeling assumptions on its response. The analyses led to the following conclusions:

The degree of indeterminacy of the structures plays a significant role in the influence of the spatial variation of the excitation on their response. For the nonlinear bridge models with the closed gap at the abutments and for both modeling approaches, spatially variable excitations induce a nonlinear response, whereas the same models subjected to uniform excitations respond linearly. This may be attributed to the fact that, for these stiff structures and for uniform excitations, the abutments guide the deck response and, because all supports move “in phase,” only a small dynamic response is induced in the structures. On the other hand, the “out-of-phase” motions at the supports cause a significant pseudo-static contribution that produces a nonlinear response. Pounding at the abutments, however, occurs for all excitation cases. For the nonlinear bridge models with the open gap between the deck and the abutments and for both modeling approaches, the effect of the spatial variation of the motions on the response is insignificant, as all excitation cases yield a similar response. For these more flexible structures, the dynamic contribution to the response becomes dominant, and the systems respond nonlinearly for all ground motion scenarios. Still, an additional small pseudo-static contribution yields, in general, slightly higher response values for the spatially variable excitation scenario.

Both the lumped plastic hinge model in DRAIN-3DX and the fiber element model in OpenSees predict the same trend in the seismic response of the bridge. However, some response quantities can vary significantly. Part of the differences may be attributed to the different material models that were employed in the two codes. When determining the properties of the rotational springs of the lumped plastic hinges in the DRAIN-3DX model, Mander’s (1988) concrete model and the USC_RC (2012) steel model were used, whereas, in the OpenSees fiber element model, the high-strain-rate Mander’s (1988) concrete model and a bilinearized steel model (Caltrans, 2001) were adopted. Because of the aforementioned differences in the material properties between the models, different damping properties of the systems were created using the two codes. The major difference, however, may be attributed to the fact that the use of lumped plastic hinges and fiber elements for the columns result in a different mechanical behavior of the models. For the lumped plastic hinge model, plastic deformation takes place only at a concentrated point, whereas fiber elements permit the spread of plasticity along the member and across its section. The use of lumped plastic hinges at the top and bottom of the pier columns, however, appears to make this model artificially more flexible than when the columns are modeled by fiber elements, causing large axial forces in the deck of the stiffer structure under the spatially variable excitation scenario, lower bending moments in the columns of both the stiffer and the more flexible structures subjected to all ground motion scenarios, and, even though the gap size for the more flexible structure is long, pounding still occurs for the lumped plastic hinge model under the spatially variable ground motion scenario. Considering that, in the lumped plastic hinge model, the behavior at the column ends is controlled by the user (i.e., through rotational springs and rigid elements of specified length), whereas, in the fiber element model the behavior of the column is, more or less, not controlled, the results of this numerical experiment suggest that the fiber element model

may be more appropriate for the evaluation of the effect of spatially variable ground motions on the seismic response of RC bridges.

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