New Direction Based (Fundamental) Periods of RC Frames Using Genetic Algorithms

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SUMMARY:

Expressions for estimating the fundamental period provided by Building Codes are generally given as a function of building height, building type (frame or shear wall), etc. Although these equations are widely used in practice, it has been indicated that they can be improved. A difference in the period of an RC frame structure is also noticed depending on whether the longitudinal or transverse direction of the structure is considered. We propose new expressions for fundamental periods of regular RC frames which take into account the direction of the structure considered, by performing nonlinear regression analysis using genetic algorithms on 600 different models of RC framed structures.

Keywords: fundamental period, RC frame, genetic algorithm, regular structures

1. INTRODUCTION

The determination of the natural period of vibration of a reinforced concrete structure is an essential procedure in earthquake design and assessment since it is the main property of the structure that determines the elastic demand and, indirectly, the required inelastic performance in static procedures. The fundamental period depends on the mass, stiffness and strength of the structure and is influenced by many factors, which include structure regularity, number of storeys and bays, infill panel properties, section dimensions, axial load level, reinforcement ratio and extent of concrete cracking. The fundamental period can be evaluated using simplified expressions found in codes, which are based on earthquake recordings in existing buildings, laboratory tests, numerical or analytical computations. These technical codes provide expressions which depend on basic parameters such as building height or number of storeys.

Building periods predicted by these expressions are widely used in practice although it has been pointed out by Amanat and Hoque (2006) and Verderame, Iervolino and Manfredi (2010) that there is scope for further improvement in these equations since the height alone is inadequate to explain period variability. It is also known that the period of a reinforced concrete (RC) frame structure differs depending on whether the longitudinal or transverse direction of the structure is considered.

Genetic algorithms are optimization techniques based on the concepts of natural selection, genetics and evolution. They have proven themselves as reliable computational search and optimization procedures for complex objectives involving large number of variables. They can effectively be implemented in nonlinear regression analysis.

In this paper, a brief review of simplified expressions for fundamental period estimation of RC moment resisting frames (MRF) is initially given. A description of the considered database of RC framed structure models, along with the periods calculated in both directions is then provided. These periods are compared with those obtained using building codes. An overview of genetic algorithms and their implementation in nonlinear regression analysis is given. With the aid of genetic algorithms, nonlinear regression analysis is performed using this database. As a result, new expressions are proposed for the period of regular RC frames which take into account the direction of the structure considered as well as the number of floors (or height).

2. EXISTING EXPRESSIONS FOR THE FUNDAMENTAL PERIOD OF RC FRAMES IN BUILDING CODES

The value of the fundamental period needs to be as accurate as possible in earthquake resistant designs with a special emphasis on designs which are based on either linear static (or lateral force) methods or performance level. Buildings are usually designed for seismic resistance using elastic analysis, but most will experience significant inelastic deformations under large earthquakes. Inelastic analysis should therefore be selected for moderate and major earthquakes since the structure's behaviour is in the inelastic range. Seismic design is preferably conducted using elastic analysis with seismic reduced forces due to several reasons: time and cost concerns, availability of elastic methods and lack of extensive inelastic analysis software (Hoseinzadeh, Edalatbehbahani and Labibzadeh, 2011). Strength reduction factor is usually used for reducing structure strength from elastic strength. Thus, building codes extract seismic loads of inelastic designs from a linear spectrum, which is dependent on the fundamental period of structure, and ground zone type. In other words, in current seismic code provisions, seismic forces estimation using design spectra requires either implicitly the use of empirical equations for the fundamental period determination or more specifically detailed dynamic analysis.

Since the predicted fundamental period is used to obtain the expected seismic load affecting the structure, a precise estimation of it is important for the safety of the applied procedure in the design steps and consequently in the future performance of the structure after it is constructed.

The fundamental period of vibration required for the simplified design of RC structures has been calculated for many years using a simplified formula relating the period to the height of the building. One of the first formulae of this type was presented over 30 years ago in ATC3-06 (ATC, 1978) and was of the form:

$$T = C_{c}H^{0.75}, (2.1)$$

where: H – height of the structure [m] and C_t – constant depending on the structure type. The coefficient C_t is calibrated in order to achieve the best fit to experimental data.

This particular form of Eqn. (2.1) was theoretically derived using Rayleigh's method with the assumptions that the equivalent static lateral forces are distributed linearly over the height of the structure, the seismic base shear is proportional to $1/T^{2/3}$ and the distribution of the stiffness with height produces a uniform interstory drift under the linearly distributed horizontal forces.

ATC3-06 proposed the value of 0.025 for the coefficient C_t for evaluating the period of vibration of RC MRF buildings. This was based on periods computed from motions recorded in 14 buildings during the San Fernando earthquake in 1971. In SEAOC-88 (SEAOC, 1998) C_t has a value of 0.030 (where H is measured in feet). The use of the form of period-height equation shown in Eqn. (2.1), along with the SEAOC-88 recommended 0.03 coefficient, has been adopted in many design codes since 1978, for example in UBC-97 (UBC, 1997), in SEAOC-96 (SEAOC, 1996), in NEHRP-94 (FEMA, 1994). In Eurocode 8 (CEN, 2004), this C_t coefficient has simply been transformed considering that the height is measured in metres, leading to $C_t = 0.075$:

$$T = 0.075H^{0.75}. (2.2)$$

Goel and Chopra (1997) collected data measured from eight Californian earthquakes, from 1971 (San Fernando earthquake) until 1994 (Northridge earthquake) and showed that Eqn. (2.1) generally underestimates the periods of vibration measured from 27 RC frames, especially those above sixteen storeys. Therefore, different formulas were proposed resulting from semiempirical analysis, with the best-fit plus 1 standard deviation recommended for displacement-based assessment, whilst the best-fit minus 1 standard deviation recommended for conservative force-based design (Chopra and Goel, 2000):

$$T_U = 0.067 H^{0.9}, (2.3)$$

$$T_L = 0.0466H^{0.9}, (2.4)$$

where *H* is the height of the structure [m]. The latter period-height formula has been included in ASCE 7-05 (2006).

The NEHRP-94 (FEMA, 1994), as well as National Building Code of Canada (NBCC, 2005) also recommends an alternative formula for reinforced and steel MRF buildings based only on the number of storeys, *N*:

$$T = 0.1N$$
. (2.5)

This simple formula is limited for buildings less than 12 storeys in height and with a minimum storey height of 10 ft.

In order to show the (in)accuracy of the given expressions for the fundamental periods of different RC frame structures a database of 600 different models of RC MRF structures was created.

3. DATABASE OF CALCULATED PERIODS OF RC FRAME STRUCTURE MODELS

Tha database consisted of 600 different RC frame structures, each with a rectangular plan shape and moderate number of storeys. The considered variable parameters of a given structure model were the longitudinal length (L_x) , transversal length (L_y) and the global height (H) excluding the foundation. The basic layout model was a 3D space frame model with the length of bay of 5.0m in both longitudinal and transversal directions. Interstorey height was constant and equal to 3.0m. All the models were generated by a modular combination of the basic model. The largest model was set to be ten basic models in length and height and three basic models in width. This structural configuration represents a lateral load resisting system consisting of moment-resisting RC frames in both the longitudinal and the transversal directions.

The self weight of the structural elements and a live load of $2kN/m^2$ were taken as the construction load in the models. The dimensions of cross sections of all elements of the structure were modeled in accordance with the necessary requirements given in EC8 (CEN 2004). The following material requirements of EC8 was considered: a concrete class lower than C20/25 shall not be used in primary seismic elements for ductility class DCH (high ductility class). Therefore, the concrete class C25/30 was used, implying that the cylindrical compressive characteristic strength of concrete $f_{c,cyl}$ is constant among the structures and equal to 25 N/mm². All cross sections in the beams had the same 25cm basis and a height of 45cm, which satisfy geometrical constrains of EC8 (the width of primary seismic beams shall not be less than 200 mm and requirement which take advantage of the favourable effect of column compression on the bond on the horizontal bars passing through the joint). By calculating the column forces using SAP2000, the dimensions of the columns were determined using the ductility criteria:

$$\frac{N}{A_c} \le 0.3 f_{cd} \,, \tag{3.1}$$

where f_{cd} is the design compression strength of the concrete [kN/cm²], N is axial force [kN] and A_c is the cross section of the column [cm²].

Analysis showed that gradually increasing the dimensions of the cross sections by 5cm for each storey did not produce large deviations from the required value of 0.3. Two datasets of modeled structures were created. The cross sections of the columns of the first dataset was increased from 25/25 for one storey to 70/70 cm for 10 storeys, while for the second dataset column, cross sections increased from 30/30 to 75/75 cm for 10 storeys. The first dataset is referred to herein as "DataSet1", while the second dataset is referred to as "DataSet2".

The building dimensions considered were as follows:

- longitudinal length: $L_x = [5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0, 40.0, 45.0, 50.0]$ m;
- transversal length: $L_y = [5.0, 10.0, 15.0]$ m;
- building height (*H*) was between $(3.0 \div 30.0)$ m corresponding to 1–10 storeys.

A sample of the model with three lengths of bays in transversal and four lengths of bays in

longitudinal directions and five storeys (labelled as 3-4-5) is displayed in Fig. 3.1.



Figure 3.1. A structure with 3 bays in transversal direction, 4 bays in longitudinal direction and 5 storeys (3-4-5) modeled in SAP2000.

For each model in the database, the elastic periods for both directions (longitudinal and transversal) were determined using SAP2000 using modal analysis. Since for a given 3D model two different periods are obtained, the fundamental period of the model is represented by the elastic period with the greater value. This elastic period corresponds to the direction with a lower stiffness, in other words to the shorter direction. This can be explained using the fact that the frame models are regular and therefore the stiffness of a given model is increased by the addition of equally spaced RC columns and beams.

4. COMPARISON OF PERIODS OF MODEL STRUCTURES WITH PERIODS OBTAINED USING BUILDING CODES

The results obtained for the periods in both directions of the RC frame systems in the database, "DataSet1" and "DataSet2", were compared to those obtained using the empirical expressions mentioned in Section 3. and are displayed in Figs. 4.1. and 4.2.

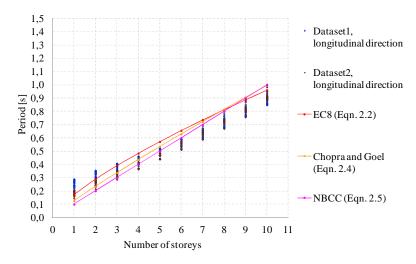


Figure 4.1. Calculated periods in longitudinal direction for all models.

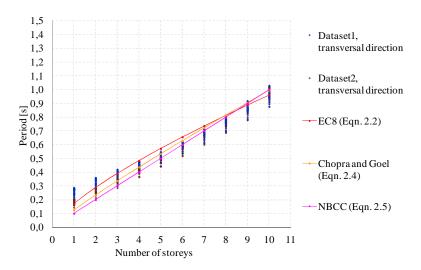


Figure 4.2. Calculated periods in transversal direction for all models.

Looking at Figs. 4.1 and 4.2, it can be noticed that, with the exception of the first two floors, the fundamental periods obtained using EC8 (Eqn. 2.2) are significantly greater than the obtained elastic period values in longitudinal and transversal directions, respectively. Structures with the same number of storeys have different values of elastic period. This can be explained using the fact that a change in the ratio of floor dimension parameters occurs without a change in the stiffness in the analyzed direction. The differences between the periods of the models and the corresponding periods obtained using building codes indicate that the expressions in building codes can further be improved.

Amanat and Hoque (2006) performed a sensitivity analysis of RC building with infills and concluded that the main parameters affecting the period are the height, the number and length of bays and the amount of infills. They also showed that the stiffness of RC members does not have a great influence. Verderame, Iervolino and Manfredi (2010) provided the periods in the two main directions using regression for sub-standard RC MRF buildings. They showed that height alone is inadequate to explain period variability and they included the plan area in their expressions for period evaluation.

Based on this, we pose several questions. Can we get more accurate expressions for the fundamental period if, in addition to the height, we take into consideration the length of the structure parallel to the considered direction? How does ratio between the lengths in the longitudinal and transversal directions affect the fundamental period? How does the floor plan area influence the fundamental period?

We considered six different expressions which, in addition to the height, take into consideration one of the following:

- the length parallel to the considered direction;
- the ratio between the lengths in the longitudinal and transversal directions;
- the floor plan area.

With the aid of genetic algorithms, nonlinear regression analysis was performed in order to determine the parameters of these expressions.

5. GENETIC ALGORITHM

Genetic algorithms (GA) have proven themselves as reliable computational search and optimization procedures for complex objectives involving large number of variables. In structural and earthquake engineering, genetic algorithms have been used in various problems (Naeim, Alimoradi and Pezeshk, 2004). Some examples include design optimization of nonlinear structures (Pezeshk, Camp and Chen, 1999), active structural control (Alimoradi, 2001) and performance-based design (Foley, Pezeshk and Alimoradi, 2003).

GAs are optimization techniques based on the concepts of natural selection, genetics and evolution. The variables are represented as genes on a chromosome. Each chromosome represents a possible solution in the search space. Like nature, GAs solve the problem of finding good chromosomes by

manipulating the material in the chromosomes blindly without any knowledge about the type of problem they are solving. The only information they are given is an evaluation (or fitness) of each chromosome they produce.

GAs feature a group of candidate solutions (population) in the search space. The initial population is usually produced randomly. Chromosomes with better fitness are found through natural selection and the genetic operators, mutation and recombination. Natural selection ensures that chromosomes with the best fitness will propagate in future populations. Using the recombination operator (also referred to as the crossover operator), the GA combines genes from two parent chromosomes to form two new chromosomes (children) that have a high probability of having better fitness than their parents. Mutation is a necessary mechanism to ensure diversity in the population, thus mutation allows new areas of the response surface to be explored. Given a problem, one must determine a way or method of encoding the solutions of the problem into the form of chromosomes and, secondly, define an evaluation function that returns a measurement of the cost value (fitness) of any chromosome in the context of the problem. A GA then consists of the following steps (Lin and Lee, 1996):

- 1. Initialize a population of chromosomes.
- 2. Evaluate each chromosome in the population.
- 3. Create new chromosomes by using GA operators.
- 4. Delete unsuitable chromosomes of the population to make room for the new members
- 5. Evaluate the new chromosomes and insert them into the population.
- 6. If the stopping criterion is satisfied, then stop and return the best chromosome, otherwise, go to step 3.

Due to their nature, the advantages to using GAs are many: they require no knowledge or gradient information about the response surface, discontinuities present on the response surface have little effect on overall optimization performance, they are resistant to becoming trapped in local optima, they perform very well for large-scale optimization problems and can be employed for a wide variety of optimization problems. However they do have some disadvantages, and these include having trouble finding the exact global optimum and requiring a large number of fitness function evaluations or iterations. This is more obvious in situations when the dimensionality of the problem is large (Nyarko, 2001).

Mathematical models are often used in applied research (physics, biology, etc.). By using experimental data, the parameters of the mathematical models can be determined. This is often referred to as the parameter identification problem (Nyarko and Scitovski, 2004). If the data are modelled by a function which is a nonlinear combination of the model parameters and depends on one or more independent variables, then it is also referred to as nonlinear regression.

Even though certain nonlinear problems can be transformed to linear regression problems, there are several advantages to performing nonlinear regression directly:

- Caution is needed in transforming nonlinear problems to linear problems since the influence of the data values on the dependent variable changes.
- The minimization of the sum of the squared residual values is based on the true nonlinear value rather than the linearized form.

Standard nonlinear regression methods, on the other hand, require an initial estimate of the parameters to be determined and the choice of good initial values is crucial. There is no standard procedure for getting initial estimates. One of the most obvious methods is to use prior information. We therefore implement GA for nonlinear regression analysis since initial values need not be defined.

Let us assume that the mathematical model of the period is defined as a function f

$$T = f\left(\mathbf{x}, \mathbf{p}\right),\tag{5.1}$$

where $\mathbf{x} = (x_1, ...x_k)^T$ is the vector of k independent variables and $\mathbf{p} = (p_1, ...p_m)^T$ the vector of m unknown real parameters. We are also provided with the experimental data (\mathbf{x}_i, T'_i) , i = 1,...n where \mathbf{x}_i represents the values of the independent variables and, T'_i , the measured values of the period. Usually we have k << n and m << n. With the given measured values one has to estimate the optimal parameter vector, \mathbf{p}^* for Eqn. (5.1) such that

$$F(\mathbf{p}^*) = \min_{\mathbf{p} \in \mathbb{R}^m} F(\mathbf{p}), \quad F(\mathbf{p}) = \frac{1}{n} \sum_{i=1}^n [f(\mathbf{x}_i, \mathbf{p}) - T_i']^2.$$
 (5.2)

The parameters which need to be determined and optimized are encoded into chromosomes using the floating point implementation. In the GA, the new values $f(\mathbf{x_i}, \mathbf{p})$ for each possible solution of \mathbf{p} are determined after each iteration. The fitness of each chromosome is then determined using Eqn. 5.2. The GA then consists of the following steps (Lin and Lee, 1996)

- 1. Initialize a population of chromosomes (possible solutions of **p**).
- 2. Find the values $f(\mathbf{x}_i, \mathbf{p})$ for each chromosome, \mathbf{p} , in the population.
- 3. Evaluate the fitness of each chromosome, **p**, in the population using Eqn. 5.2.
- 4. Create new chromosomes by using GA operators.
- 5. Delete unsuitable chromosomes of the population to make room for the new members.
- 6. Find the new values $f(\mathbf{x}_i, \mathbf{p})$ for each new chromosome, \mathbf{p} , in the population.
- 7. Evaluate the fitness of each new chromosome, **p**, in the population using Eqn. 5.2 and insert them into the population.
- 8. If the stopping criterion is satisfied, then stop and return the best chromosome, otherwise, go to step 4.

6. NEW EMPIRICAL EXPRESSIONS FOR THE PERIOD OF RC FRAMES USING GENETIC ALGORITHM

All expressions in the previously mentioned building codes depend on either the height or number of storeys. In order to determine more accurate expressions for the elastic period we considered seven basic expressions which, in addition to the number of floors, take into consideration each of the following: the number of bays parallel to the considered direction; the ratio between the number of bays in the longitudinal and transversal directions; the product between the number of bays in the longitudinal and transversal directions. The expressions considered are:

$$T = C_1 N^{C_2}, (6.1)$$

$$T = C_1 N^{C_2} \cdot B^{C_3} \,, \tag{6.2}$$

$$T = C_1 N^{C_2} + C_2 B^{C_4} \,, \tag{6.3}$$

$$T = C_1 N^{C_2} \cdot \left(\frac{B_x}{B_y}\right)^{kC_3},\tag{6.4}$$

$$T = C_1 N^{C_2} + C_3 \left(\frac{B_x}{B_y}\right)^{kC_4}, \tag{6.5}$$

$$T = C_1 N^{C_2} \cdot (B_x B_y)^{C_3}, \tag{6.6}$$

$$T = C_1 N^{C_2} + C_3 (B_x B_y)^{C_4}, (6.7)$$

where N is the number of storeys, B is the number of bays of the building parallel to the considered direction, B_x is the number of bays in longitudinal direction, B_y is the number of bays in transversal direction, k is a constant which has a value of 1 when the period in the longitudinal direction is to be determined and a value of -1 when the period in the transversal direction is to be determined and C_1 , C_2 , C_3 and C_4 are (unknown) parameters that need to be determined. The parameters of the expressions

are determined by performing nonlinear regression analysis using the models in the database described in section 3. Nonlinear regression analysis is performed with the aid of GA. A comparison of these expressions (Eqn. 6.1 to 6.7) is given in Table 6.1. and Table 6.2, by providing the mean squared error (MSE) of the regressions' residual.

Table 6.1 Comparison of expression errors for longitudinal period (T_x)

Expression		MSE			
	C_1	C_2	C ₃	C ₄	MSE
$T_{x} = C_{1}N^{C2}$	0.1557	0.7407	-	-	2.05·10 ⁻³
$T_x = C_1 N^{C2}$ $T_x = C_1 N^{C_2} \cdot B_x^{C_3}$	0.1605	0.7416	-0.0213	-	1.98·10 ⁻³
$T_x = C_1 N^{C_2} + C_3 B_x^{C_4}$	0.0369	1.2794	0.20104	-0.02784	1.02·10 ⁻³
$T_x = C_1 N^{C_2} \cdot \left(\frac{B_x}{B_y}\right)^{C_3}$	0.1605	0.7417	-0.0362	-	1.74·10 ⁻³
$T_x = C_1 N^{C_2} + C_3 \left(\frac{B_x}{B_y}\right)^{C_4}$	0.0368	1.2805	0.2077	-0.0824	8.49·10 ⁻⁴
$T_x = C_1 N^{C_2} \cdot (B_x B_y)^{C_3}$	0.1536	0.7404	0.0068	-	2.04·10 ⁻³
$T_x = C_1 N^{C_2} + C_3 (B_x B_y)^{C_4}$	0.0370	1.2790	0.1734	0.0496	9.76·10 ⁻⁴

Table 6.2 Comparison of expression errors for transversal period (T_y)

Expression	Parameters				MCE
	C_1	C_2	C_3	C ₄	MSE
$T_{y} = C_{1}N^{C2}$	0.1547	0.7751	-	-	$2.12 \cdot 10^{-3}$
$T_{y} = C_1 N^{C_2} \cdot B_{y}^{C_3}$	0.1574	0.7760	-0.0325	-	$2.04 \cdot 10^{-3}$
$T_{y} = C_{1}N^{C_{2}} + C_{3}B_{y}^{C_{4}}$	0.0409	1.2750	0.1946	-0.0425	1.05·10 ⁻³
$T_{y} = C_{1}N^{C_{2}} \cdot \left(\frac{B_{y}}{B_{x}}\right)^{C_{3}}$	0.1488	0.7755	-0.0410	-	1.68·10 ⁻³
$T_{y} = C_{1}N^{C_{2}} + C_{3} \left(\frac{B_{y}}{B_{x}}\right)^{C_{4}}$	0.0407	1.2773	0.1707	-0.1145	7.35·10 ⁻⁴
$T_{y} = C_{1}N^{C_{2}} \cdot (B_{x}B_{y})^{C_{3}}$	0.14807	0.7742	0.0214	-	1.99·10 ⁻³
$T_{y} = C_{1}N^{C_{2}} + C_{3}(B_{x}B_{y})^{C_{4}}$	0.0408	1.2762	0.1566	0.0904	8.61·10 ⁻⁴

Since the above expressions are expressed in terms of the number of bays and number of floors, equivalent length and height expressions can be obtained by substituting for the bay length of 5.0m and floor height of 3.0m.

Looking at both tables, it can be concluded that taking into account another parameter gives better results than when only the height is considered. The equations 6.5., 6.7. and 6.3. give the best results, with equation 6.5 being the expression with the least MSE irrespective of the considered direction. It can also be noted that each of these three expressions consists of the *sum* of two sub-expressions: one being the *standard* height expression and the other an expression of the parameter considered (the length parallel to the considered direction, the ratio between the lengths in the longitudinal and transversal directions or the floor plan area). It is also interesting to note that the period of the structure in a given direction depends on the reciprocal value of the length of the structure in the considered direction (Eqns. 6.2. to 6.5.)

The best equation (Eqn. 6.5) is compared to the expressions of building codes EC8 (Eqn. 2.2) and NBCC (Eqn. 2.5) for some chosen models from the database mentioned in Section 3. Figures 6.1 and 6.2 show the periods obtained in both directions for sample models having bay ratio of 1/4 and 3/10

respectively. It can be noticed that the new expression estimates the period better than that obtained using EC8.

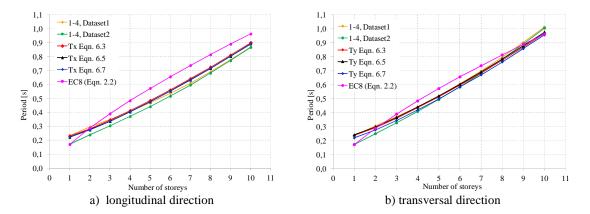


Figure 6.1. Calculated periods for RC frame models with 1 bay in transversal direction and 4 bays in longitudinal direction

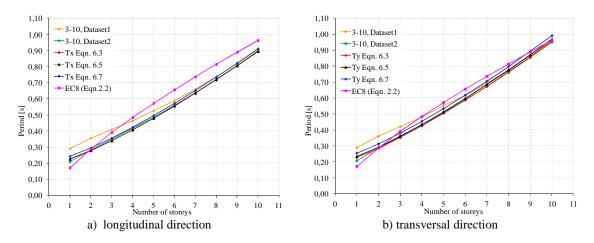


Figure 6.2. Calculated periods for RC frame models with 3 bays in transversal direction and 10 bays in longitudinal direction.

7. CONCLUSION

Several expressions for the evaluation of fundamental period given by building codes are analyzed. Using a database of 600 RC frames models, the differences between the periods of the models and the corresponding periods obtained using building codes indicate that the expressions in building codes can be improved. New direction based elastic period expressions are obtained using this database by performing nonlinear regression analysis implementing genetic algorithms. These direction based elastic period expressions are given in terms of the building height and one of the following: the length parallel to the considered direction, the ratio between the lengths in the longitudinal and transversal directions or the floor plan area. Since for a given structure two elastic period values are obtained (one for each direction), the fundamental period of the structure is represented by the elastic period with the greater value. Results indicate that these new expressions, generally, give a better estimation of the fundamental period of RC frame structures. The best results are obtained using expressions given in terms of building height and the ratio between the lengths in the longitudinal and transversal directions.

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