Bias and Variability in Site Response: Analysis of Residuals at 100 KiK-net Stations

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SUMMARY:
Using the Kiban-Kyoshin network (KiK-net) downhole array data in Japan, we analyze the accuracy (bias) and variability (precision) resulting from common site response modeling assumptions, and identify critical parameters that significantly contribute to the uncertainty in site response analyses. We perform linear and equivalent-linear site response analyses at 100 KiK-net sites using 3720 ground motions ranging from weak to strong in amplitude. We find that the maximum shear strain in the soil profile, the observed peak ground acceleration at the ground surface, and the predominant spectral period of the surface ground motion, are the best predictors of where the evaluated models become inaccurate and/or imprecise. The peak shear strains (γ_{max}) beyond which linear analyses become inaccurate in predicting surface pseudo-spectral accelerations (PSA) are a function of vibration period, and are between γ_{max} = 0.01% and 0.2% for periods less than 0.5 s. Equivalent-linear analyses become inaccurate at peak strains of approximately γ_{max} = 0.4% over this range of periods.

Keywords: Earthquake ground motion, equivalent linear, nonlinear, downhole arrays, uncertainty

1. INTRODUCTION

For many engineering design projects, a site-specific analysis of ground-motion amplification is necessary to quantify the seismic hazard. Site response analyses are used to estimate the ground motion at the surface, as a function of the properties of the soil profile and the "bedrock" ground motion at the base of the soil profile (Kramer, 1996). As observed in many historic and recent earthquakes, the softer materials near the free surface influence damage patterns over short distances (Borcherdt, 1970; Boatwright et al., 1991; Hanks and Brady, 1991; Bakir et al., 2002; Hough et al., 2011; Bradley and Cubrinovski, 2011). Accurate modeling of nonlinear soil behavior is an important component of seismic hazard assessment because the effects of the nonlinearity are most significant at near-source locations during large-magnitude events, the conditions where the most severe damage typically occurs. Although nonlinear site response is an important factor that affects the total seismic hazard in a region, the area affected by nonlinear soil behavior for a given earthquake is generally small, thus limiting the number of observations of nonlinear soil behavior for validation of constitutive models.

Vertical seismometer arrays represent a unique interaction between observed and predicted ground motions, and are especially helpful for validating and comparing site response models. However, most site response studies focus on few strong-motion recordings at well-documented vertical arrays, such as the Large-Scale Seismic Test (LSST) site in Lotung, Taiwan (e.g., Elgamal *et al*., 1995; Borja *et al*., 1999). In this study, we take advantage of the extensive database of ground motions recorded by the Kiban-Kyoshin network (KiK-net) of vertical seismometer arrays in Japan (Aoi et al., 2001; Okada et al., 2004). The 2011/3/11 M_w 9.0 Tohoku earthquake in Japan, as well as other recent events in Japan, such as the 2003/9/26 M_w 8.0 Tokachi-Oki, 2003/5/26 M_w 7.0 Miyagi-Oki, and 2008/7/24 M_w

6.8 Iwate events, have substantially increased the number of ground-motion records with large surface accelerations at these strong motion stations. In particular, the Tohoku earthquake of March 2011 added 525 records to the KiK-net database, with 43 records exceeding 0.3g. The reader is referred to Kaklamanos et al. (2012) for more details about the sites and ground motions used in our analyses.

Many studies have demonstrated the limitations of the underlying assumptions of typical constitutive soil models, even for relatively small-strain records (e.g., Baise et al., 2003; Boore, 2004; Stewart and Kwok, 2008; Thompson et al., 2012). Site response analyses are burdened with significant uncertainties in the input ("rock") motion, soil parameters, constitutive model parameters, constitutive model assumptions, and other general assumptions. In this paper, we use surface-downhole seismic arrays to eliminate the uncertainty of the input motion (in cases where the 1D wave propagation assumption provides negligible error), and this allows us to focus on the constitutive model of the soil.

We analyze 100 KiK-net stations that have recorded at least one strong ground motion with peak ground acceleration (PGA) greater than 0.3g at the ground surface. Using the 3720 ground-motion records at these sites, we perform linear and equivalent-linear site response analyses, and we identify critical parameters that most greatly contribute to uncertainty in site response analyses. We focus on linear and equivalent-linear site response analyses because the purpose of this paper is to identify trends in model performance of widely used site response models using a large database. Our goal is to sample a broad range of site conditions and strong ground-motion amplitudes to make general conclusions on how site response models perform, and the conditions under which standard site response models become inaccurate and imprecise.

Recently, Bradley (2011) developed a general mathematical framework for the validation of site response analyses using downhole seismic arrays. We apply a modification of this approach to the previously described ground motion database. First, we calculate the site response residuals for the ground-motion records, using both linear and equivalent-linear site response models. Using the model residuals, we then use mixed effects regression to determine which critical parameters control the magnitude of the residuals in terms of both inter- and intra-site variability. Inter-site variability quantifies the scatter in the residuals between sites, and intra-site variability quantifies the scatter in the residuals within a single site. We group the critical parameters into source, path, site, and ground-motion categories, and we identify which parameters have the greatest influence on the accuracy of the site response model. In these analyses, we quantify both the biases and the standard deviations (sigma's) predicted by linear and equivalent-linear site response models for predicting peak surface pseudo-acceleration spectral ordinates (PSA), as a function of spectral period (T).

2. METHODS

2.1. Site response analyses

The most common assumptions for performing a one-dimensional linear site response analysis are: (1) the medium is assumed to consist of laterally-constant layers overlying a non-attenuating halfspace; (2) wavefronts are assumed to be planar; (3) damping is assumed to be independent of frequency and strain; and (4) only the SH-wave (the horizontally-polarized component of the S wave) is modeled. We refer to these collective assumptions as the linear SH1D site response model. In the linear SH1D formulation, ground response is assumed to be viscoelastic; strain- and frequency-independent damping is allowed. Consistent with linear-elastic theory, the viscoelastic formulation uses the small-strain, linear-elastic shear modulus $G_{max} = \rho V_S^2$, where V_S is the S-wave velocity and ρ is the density of the material. The linear SH1D site response transfer function is computed by using the Thomson-Haskell matrix method (Haskell, 1953; Thomson, 1950), and the necessary input parameters are V_S, ρ, and the S-wave intrinsic quality factor (Q).

The equivalent-linear site response formulation, as implemented, for example, by the program SHAKE (Schnabel et al., 1972; Idriss and Sun, 1992; Ordóñez, 2010), is an approximation of nonlinear soil behavior that is frequently employed in earthquake engineering practice. In addition to the basic soil properties required by the linear SH1D model (ρ and V_S), SHAKE requires strain-dependent modulus-reduction and damping curves. Here, we use the Zhang et al. (2005) relationships for the equivalent-linear site response calculations in SHAKE.

2.2. Site response model residuals

To quantify the observed and predicted surface ground motions, we compute the 5%-damped pseudo-acceleration response spectra (PSA) from the acceleration time series. To quantify the goodness-of-fit of the site response models, we compare the response spectra of the observed surface ground motion, $\mathrm{PSA}_{\mathrm{obs}}(T)$, to the response spectra of the predicted surface ground motion using the site response model, $\mathrm{PSA}_{\mathrm{pred}}(T)$. We compute the residual ($\mathrm{PSA}_{\mathrm{resid}}(T)$) between the observed and predicted PSA values in natural logarithmic space as

$$\mathrm{PSA}_{\mathrm{resid}}(T) = \ln[\mathrm{PSA}_{\mathrm{obs}}(T)] - \ln[\mathrm{PSA}_{\mathrm{pred}}(T)]. \tag{2.1}$$

Negative residuals indicate overpredictions while positive residuals indicate underpredictions. Residuals for PGA ($\mathrm{PGA}_{\mathrm{resid}}$) can be computed using Eqn. 2.1 in the case of $T = 0$.

2.3. Site response validation framework

By implementing a modified version of the framework outlined in Bradley (2011), we identify critical parameters for site response by plotting model residuals of $\mathrm{PGA}_{\mathrm{resid}}$ and $\mathrm{PSA}_{\mathrm{resid}}(T)$ versus critical parameters of interest. The most useful parameters are those upon which the residuals display a clear dependence. Identification of these critical parameters can help inform us when or where a particular site response model breaks down (e.g., by illustrating bias due to nonlinear soil behavior). In the current work, we apply a portion of the Bradley (2011) approach (analysis of site response residuals using mixed effects regression, and identification of critical parameters), but we do not undertake a formal investigation of the sources of site response model uncertainty.

In order to obtain statistically significant inferences about site response it is necessary to consider multiple sites and observations. We use mixed effects regression (Pinheiro et al., 2008) to account for the dependence between multiple observations at a single site. To simplify the notation, we let $y_{i,j}$ denote $\mathrm{PSA}_{\mathrm{resid}}(T)$ for the ith site and the jth ground-motion. To account for the site dependence, we model the residuals as

$$y_{i,j} = a + \eta_{S_i} + \epsilon_{i,j}, \tag{2.2}$$

where a is the population mean of $\mathrm{PSA}_{\mathrm{resid}}(T)$ (i.e., the "fixed effect"), which represents the average bias in the site response model across all sites and ground motions; η_{S_i} is the inter-site residual (i.e., the "between-site residual"), which gives the deviation from the population mean of the mean residual for the ith site; and $\epsilon_{i,j}$ is the intra-site residual (or the "within-site residual"), which represents the deviation for ground-motion observation j at site i from the mean residual at site i. In other words, the intra-site residual $\epsilon_{i,j}$ is the residual after accounting for the site residual η_{S_i}. With the fixed effect a included as a term in the model, we assume that η_{S_i} and $\epsilon_{i,j}$ are each normally distributed random variables with mean zero and variances τ_S^2 and σ^2, respectively. We refer to τ_S as the inter-site standard deviation, which refers to the site-to-site variability, and σ as the intra-site standard deviation, which refers to the variability within a single site. The use of the representation of $y_{i,j}$ in Eqn. 2.2 with multiple prediction-observation pairs represents a linear mixed effects model (Lindstrom and Bates, 1990). A schematic of the residuals and parameters in the linear mixed effects regression model is presented in Fig. 1. The figure illustrates the different types of residuals, and how the interand intra- site residuals are extracted from the total residuals; we include it here as a reference to help follow the presentation and discussion of the results.

The three unknown parameters obtained from linear mixed-effects regression in Eqn 2.2 are: (i) a, the fixed effect; (ii) τ_S^2, the variance of η_{S_i}; and (iii) σ^2, the variance of $\epsilon_{i,j}$. The mean and variance of $y = \mathrm{PSA_{resid}}(T)$ obtained from regression are given by $\mu_Y = a$ and $\sigma_Y^2 = \tau_S^2 + \sigma^2$, respectively. Hence, the mean and variance can be used to examine the bias and precision of the site response model.

3. RESULTS AND DISCUSSION

3.1. Quantification of Site Response Model Uncertainty

To summarize the results over all periods and for both the linear and equivalent-linear site response formulations, we present plots of the site response model bias and variability versus spectral period (T). In Fig. 2, we plot the parameters of the linear mixed effects regression model versus T following Eqn. 2.2: (a) fixed effect, a; (b) total standard deviation, σ_Y; (c) intra-site standard deviation, σ; and (d) inter-site standard deviation, τ_S. Approximate 95% confidence intervals of these parameters are also plotted, to represent the uncertainty in the estimates.

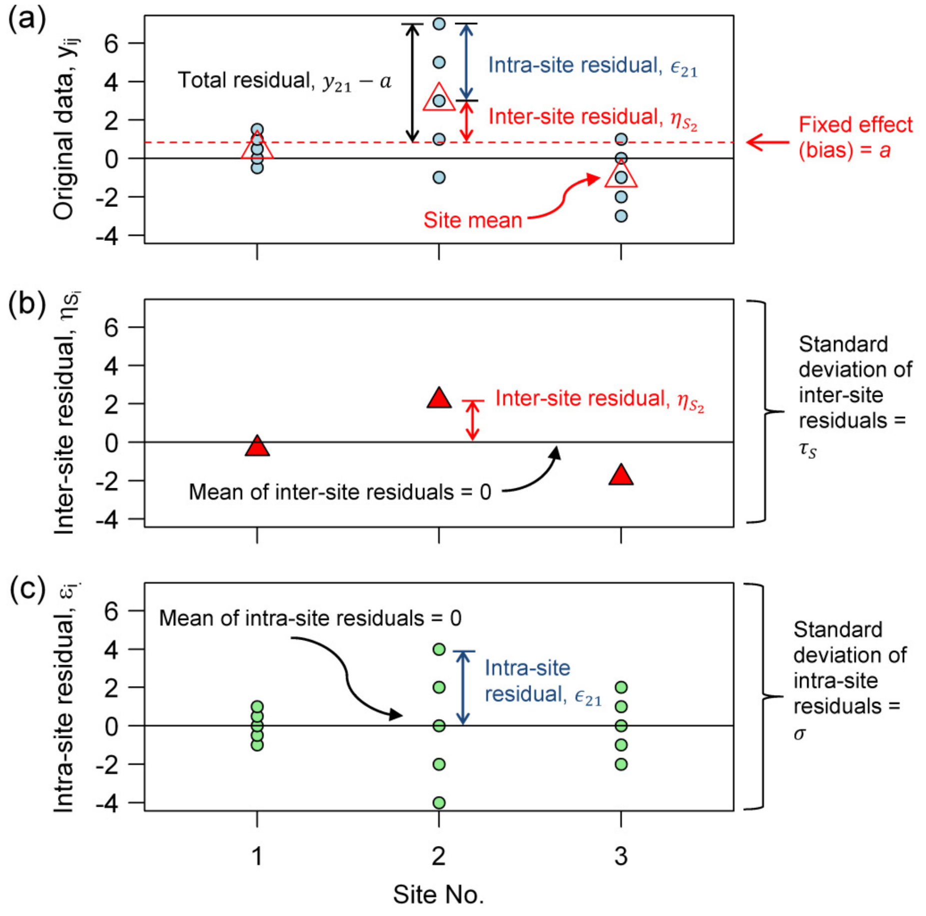

Figure 1. Illustration of the residuals and parameters in the linear mixed effects regression model: (a) original data ($y_{i,j}$), (b) inter-site residuals (η_{S_i}), and (c) intra-site residuals ($\epsilon_{i,j}$). The fixed effect (a), inter-site standard deviation (τ_S), and intra-site standard deviation (σ), are also illustrated.

Fig. 2(a) shows that both site response models generally have positive bias (underprediction of ground motions), except in the range of 0.5 to 2 s, where the bias is slightly negative. The bias of the equivalent-linear model is smaller than the linear at all periods, and the difference becomes more pronounced at shorter periods (T < 0.08 s), where the strain-dependent damping formulation of SHAKE has the strongest effect, and hence where nonlinear effects are most pronounced. Keep in mind that the bias reported here is persistent across all sites, ground motions, and intensity levels. We will investigate the intensity-dependence of the residuals later, but it is worth pausing to understand this persistent bias term. Noting the differences between the linear and equivalent-linear fixed effects in Fig. 2(a), the modest improvement of the equivalent-linear method comes from the fact that it solves for the strain-compatible equivalent damping for each layer, which also varies for each motion. However, the equivalent-linear method still tends to overdamp the ground motions at short periods. The equivalent-linear approach iterates to the secant shear modulus (G_{sec}) and damping based on the peak shear strain, and then uses this for all of the response history. The tangent shear modulus (G_{tan}) of soil during small-strain unloading/reloading is much greater than the value of G_{sec} determined from the peak strain. Hence, the equivalent linear approach will not amplify the small-strain response to the extent it should. As high-frequency response only produces small strains, equivalent-linear predictions of high-frequency ground motions are often too small. To adjust for this bias, several equivalent-linear formulations with frequency-dependent damping have been developed (e.g., Sugito et al., 1994; Joyner and Boore, 1998; Assimaki and Kausel, 2002; Park and Hashash, 2008), but these methods have yet to find their way into routine engineering practice. The remaining bias in the equivalent-linear method may be improved by the use of a fully nonlinear site response analysis in the time domain.

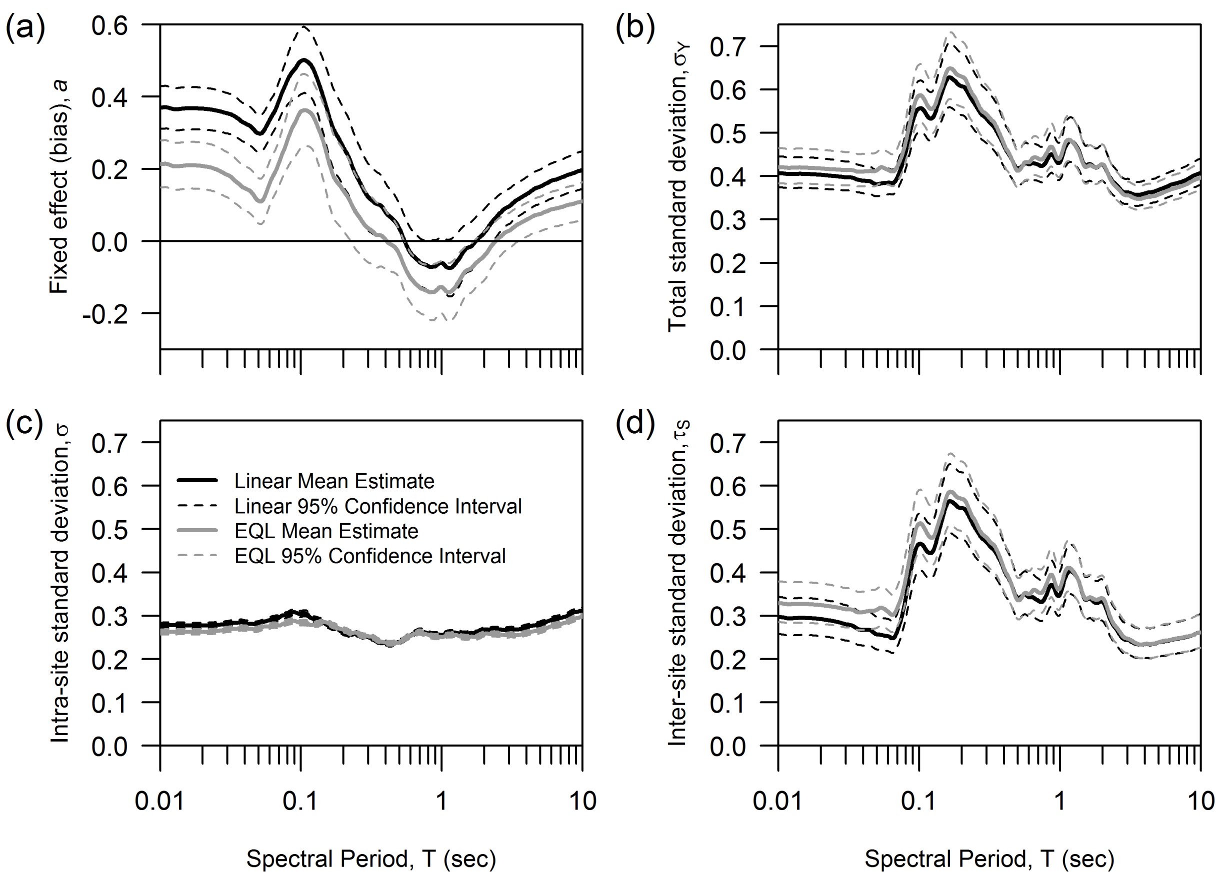

Figure 2. Period dependence of the parameters of the linear mixed effects regression model of Eqn. 2.2: (a) fixed effect, a; (b) total standard deviation, σ_Y; (c) intra-site standard deviation, σ; and (d) inter-site standard deviation, τ_S.

Figs. 2(b) through 2(d) provide an estimate of the variability of the residuals for the linear and equivalent-linear models. It can be seen that the differences in the inter-site, intra-site, and total standard deviations of the linear and equivalent linear analysis methods are statistically insignificant. The inter- and intra-site standard deviations, σ and τ_S, are similar in magnitude to each other at short and long periods (with values in the vicinity of 0.25 to 0.35 natural log units); however, within the range of approximately $0.07 < T < 0.5$ s, the inter-site standard deviations are elevated. In reviewing Fig. 2, it is significant that the linear and equivalent-linear residuals only differ when it comes to the fixed effect (Fig. 2a), and that the intra-site standard deviation (Fig. 2c) is not strongly dependent on period.

3.2. Identification of Critical Parameters

To identify critical parameters which affect the accuracy and/or precision in site response analysis, the dependence of PGA_{resid} and $PSA_{resid}(T)$ versus various source, path, site, and ground-motion parameters was examined. As detailed further in Kaklamanos et al. (2012), site parameters were plotted against the 100 intra-site residuals (η_{S_i}), and the source, path, and ground-motion parameters were plotted against the 3720 inter-site residuals ($\epsilon_{i,j}$). The critical parameters with the strongest trends are the maximum shear strain in the soil profile (γ_{max}), observed PGA at the ground surface (PGA_{obs}), and predominant spectral period of the observed ground motion (T_p). The trends in the residuals versus the observed strains and accelerations are valuable because we can use them to quantify the levels of strain, PGA, and T_p at which the predictive capabilities of the site response analyses begin to deteriorate.

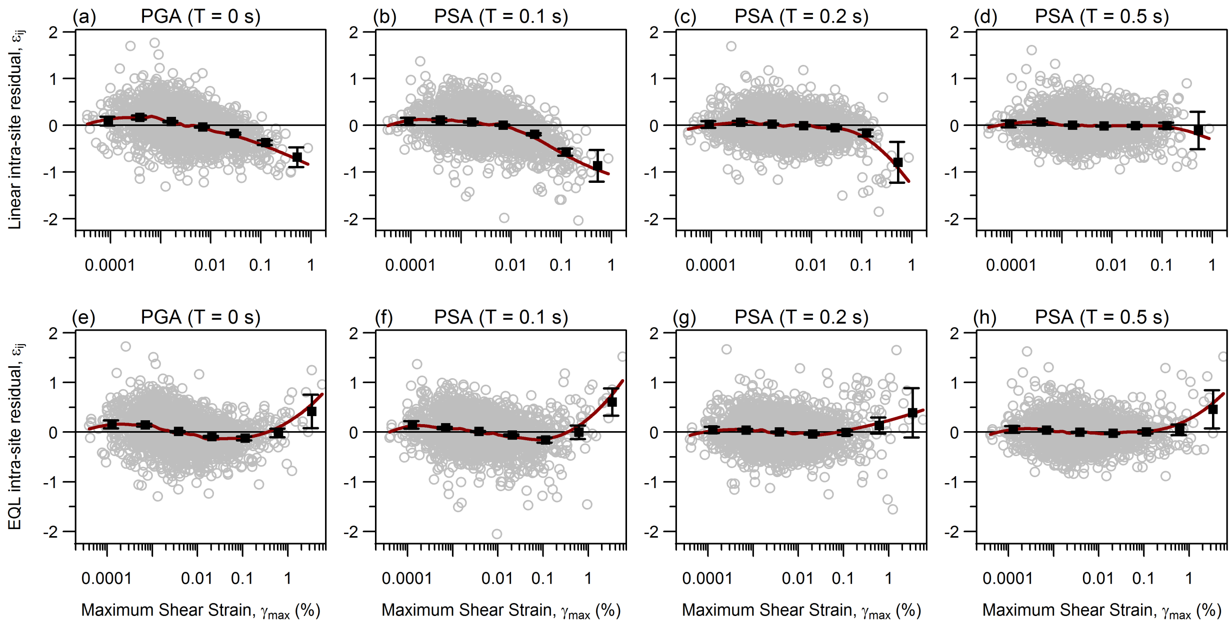

Figure 3. Plots of the intra-site residuals ($\epsilon_{i,j}$) versus the maximum calculated shear strain in the soil profile (γ_{max}), for the linear SH1D site response model (a-d; first row), and the equivalent-linear SHAKE site response model (e-h; second row), with four columns corresponding to PSA at different spectral periods. For each plot, we also display an estimate of the trendline, and the binned means and error bars (representing +/- two standard errors).

In Fig. 3, we display plots of $\epsilon_{i,j}$ versus the maximum shear strain in the soil profile (γ_{max}), for the linear (SH1D) and equivalent-linear (SHAKE) site response models. The plot contains four columns corresponding to PSA at different spectral periods (0, 0.1, 0.2, and 0.5 s); at longer periods, there were little to no trends in the residuals. In Fig. 3, noticeable trends in the residuals are observed, especially at short spectral periods. In the first row of Fig. 3, the linear SH1D model residuals slope downward at high strains. The downward-sloping pattern in the residuals indicates that the linear SH1D site response model increasingly overpredicts PSA as γ_{max} increases. This negative trend in the residuals occurs as a result of the fact that linear site response does not capture the general deamplification of high-frequency surface ground motion due to shear stiffness reduction and energy dissipation that occurs in soil deforming in a nonlinear fashion. For longer periods (T = 0.5 s and greater), there is no obvious trend in the residuals using the linear SH1D analysis. We expect a decreased effect of nonlinearity at longer periods, because longer-period seismic waves sample a deeper (and stiffer) portion of the profile, and therefore longer-period waves are not as greatly affected by the shallow soft layers that typically experience the greatest nonlinear effects. Also, the modulus-reduction and damping curves for equivalent-linear analyses are dependent on confining pressure, and therefore G/G_{max} increases and ξ decreases with depth; for our linear analyses, however, G/G_{max} = 1 and ξ is constant with depth. The range of spectral periods for which we do not see a trend with maximum shear strain (periods 0.5 sec and greater) provides a useful quantitative estimate of the conditions under which the linear assumption holds for site response analysis. Specifically, the residual trends from the linear analyses begin to deviate from zero in the range 0.01% < γ_{max} < 0.2% for periods below 0.5 s.

In the second row of Fig. 3, the equivalent-linear SHAKE model residuals show a somewhat different trend compared to the linear SH1D model residuals. All the plots display an upward slope at large strains—opposite the downward slope seen in the linear SH1D model residuals. The upward slope in the SHAKE residuals occurs only for large strains, slightly larger than the levels of strain at which the downward slope occurs in the SH1D residuals. The upward slope indicates that SHAKE is underpredicting the level of ground motion at large strains, especially for the shorter spectral periods; there is little bias at large strains for the longer periods. Unlike the SH1D model, SHAKE uses a decreased shear modulus (G) and increased damping ratio (ξ) to account for nonlinear soil behavior. Because the iterative adjustment of G and ξ is based on peak strain (which is correlated with long-period ground motion), the short-period ground motion predictions are not as accurate. As seen in Fig. 3, the equivalent-linear iteration algorithm results in an underprediction of ground motion at large strains, where the calculated value of G is likely to be much less than G_{max}. In this manner, SHAKE is overpredicting the amount of nonlinearity that actually occurs, potentially resulting in unconservative estimates of ground motion.

3.3. Onset of Nonlinearity

The trends in the eight panels of Fig. 3, which are summarized succinctly in Fig. 4, show the usefulness of γ_{max} as a critical parameter and provide a useful quantitative estimate of the conditions under which the linear and equivalent-linear formulations hold. The linear SH1D site response model begins to overpredict ground motions at shear strains between 0.01% and 0.1%. At spectral periods beyond 0.5 s, the linear model does not display any trends, indicating that, for the sites and ground motions considered, the effects of nonlinearity are not apparent in the residuals at these periods. For most ground motions, there are no statistically significant differences between the predictive capabilities of the linear and equivalent-linear site response models. However, at shear strains greater than those mentioned above, and up to $\gamma_{max} \approx$ 0.1% to 0.4%, the equivalent-linear site response formulation improves the accuracy of site response predictions, because it is able to account for the reduced shear strength and increased damping associated with higher shear strains. However, at strains beyond 0.4%, the equivalent-linear site response formulation results in an underprediction of ground motion. For short spectral periods ($T \leq$ 0.5 s) and shear strains greater than 0.4%, linear and equivalent-linear site response models both have significant biases. At these levels, nonlinear time-domain site response models are needed to more accurately capture the soil behavior.

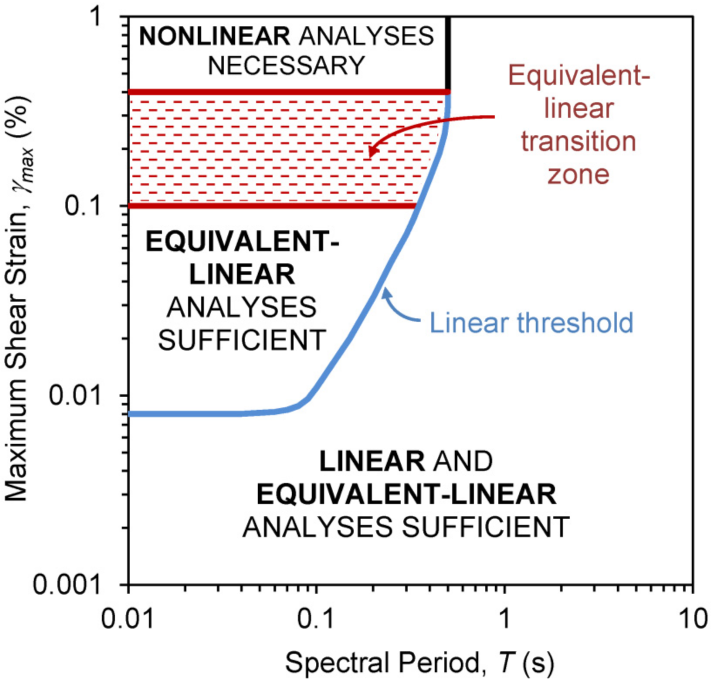

Figure 4. Illustration of the period dependence of the thresholds at which the linear and equivalent-linear models begin to exhibit bias, using γ_{max} as a critical parameter. The equivalent-linear thresholds are dashed because they are not as well-constrained as the linear thresholds. The approximate ranges of applicability of linear, equivalent-linear and nonlinear site response analyses are labeled.

The reader is referred to Kaklamanos et al. (2012) for a more detailed discussion of the results for the other critical parameters (PGA_{obs} and T_p). At predominant periods of approximately 0.2 to 0.3 s, and at values of PGA_{obs} in the range of 0.1 to 0.3g, the linear site response model begins to exhibit bias (overprediction). For the equivalent linear analyses, the intra-site residual trends do not noticeably deviate from zero (unlike for γ_{max}), indicating that PGA_{obs} and T_p are less useful as critical parameters than is γ_{max}.

4. CONCLUSIONS

To gain insights into the reliability and accuracy of site response models, we have evaluated site response residuals for both linear and equivalent linear site response formulations at 100 KiK-net sites that have recorded 3720 ground motions. To constrain the limitations of standard site response models, we analyze model residuals in terms of both inter-site and intra-site variability, employing the techniques of linear mixed effects regression after Bradley (2011). Our primary findings are:

1. Both linear and equivalent-linear site response methods generally have positive bias (underprediction of ground motions), except in the range of 0.5 to 2 s, where the bias is slightly negative.
2. The bias of the equivalent-linear model is smaller than the linear at all periods, and the difference becomes more pronounced at shorter periods ($T < 0.08$ s).
3. There are no significant differences in the standard deviations between the linear and equivalent-linear site response models.
4. The most influential parameters for characterizing site response uncertainty are the maximum shear strain in the soil profile (γ_{max}), followed by the observed peak ground acceleration at the ground surface (PGA_{obs}) and the predominant spectral period of the surface ground motion (T_p).
5. In terms of γ_{max} (summarized in Fig. 4), the linear site response analysis begins to break down (by overpredicting the ground motions) at strains in the range of 0.01% to 0.1%. At shear strains

greater than these values, and less than $\gamma_{max} \approx 0.1\%$ to 0.4%, the equivalent-linear site response formulation improves the accuracy of site response predictions. At shear strains beyond 0.4%, the equivalent-linear site response formulation results in an underprediction of ground motion; at these levels, nonlinear time-domain site response are needed to capture soil behavior.

6. We find that, for the sites and ground motions considered, site response residuals at spectral periods greater than 0.5 s, regardless of PGA or γ_{max}, do not systematically display noticeable effects of nonlinear soil behavior.

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