Flexure-Shear Coupling Fiber Model for the Nonlinear Analysis of Rectangular Hollow Section R/C Piers

N. Li, Z.X. Li & L.L. Xie
School of Civil Engineering, Tianjin University, China

N. Li, Z.X. Li & L.L. Xie
Key Laboratory of Coast Civil Structure Safety, Ministry of Education, China

L.L. Xie
Institute of Engineering Mechanics, China Earthquake Administration, China

SUMMARY:
Reinforced concrete (R/C) bridge piers may undergo strong nonlinear deformation when subjecting to severe earthquakes. The pier may perform flexure-shear coupling responses, especially for the thin wall of the hollow section pier. Many flexure-shear coupling damage phenomenon of R/C hollow section piers were observed, while there is few simulation model accounting for the flexure-shear coupling effect is proposed. In this paper, a Timoshenko beam element with section constructed by fibres, which are treated as cyclic soften membrane material model, is used in the determination of the nonlinear characteristic of hollow section R/C piers. The model has been implemented and verified on finite element analysis platform, OpenSees. Cyclic pseudo static experiments were carried out on the scaled hollow section piers. The result deduced from the numerical model is compared with the experiment, and it shows that the model has a good agreement for the strength degradation and pinching effect. The fibre-based Timoshenko beam model provides sufficient accuracy and computational efficiency.

Keywords: flexure-shear coupling, fibre model, hollow section, R/C pier, CSMM

1. INTRODUCTION

There are many reinforced concrete (R/C) bridges failed catastrophically due to shear deficiencies of piers in past earthquakes. The failure and collapse phenomenon of R/C bridges have been observed in Chichi earthquake (1999), Wenchuan Earthquake (2008), *et al*. Since the effect of flexure-shear coupled response on the seismic capacity of bridges are not clarity, the response simulation of R/C bridges subjected to strong seismic excitation is still a challenging task(Pinto, Molina et al. 2003). In particular, the determination of shear strength and deformation response is still far from reaching a mature state of development(Ceresa, Petrini et al. 2009). On the other hand, the geometric nonlinear behaviour of piers in the damage and progressive collapse phenomenon is attracting more and more attentions in recent years.

Nowadays, static and dynamic nonlinear analysis are required for common engineering practice. For R/C bridge piers, fibre beam-column model were developed about 20 years ago(Ceresa, Petrini et al. 2009). One of its main advantages is that it representing the coupled axial forces and flexure moments responses well(Spacone, Filippou et al. 1996). It catches the responses of flexure-dominant members, which adopts the assumption of Euler-Bernoulli beam theory, such as slender and high piers. However, the model fails when shear deformation becomes important, i.e., the hollow section members, thin-wall members and other shear-dominate members. In order to consider the shear deformation in frame elements, many studies take the advantages of Timoshenko beam theory with fibre model(Reddy 1997; Papachristidis, Fragiadakis et al. 2010). In addition, recently the applicability of multi-axial fibre model for flexibility-based element(Stramandinoli and La Rovere 2012) and mixed multi-field formula(Saritas and Filippou 2009) are investigated.

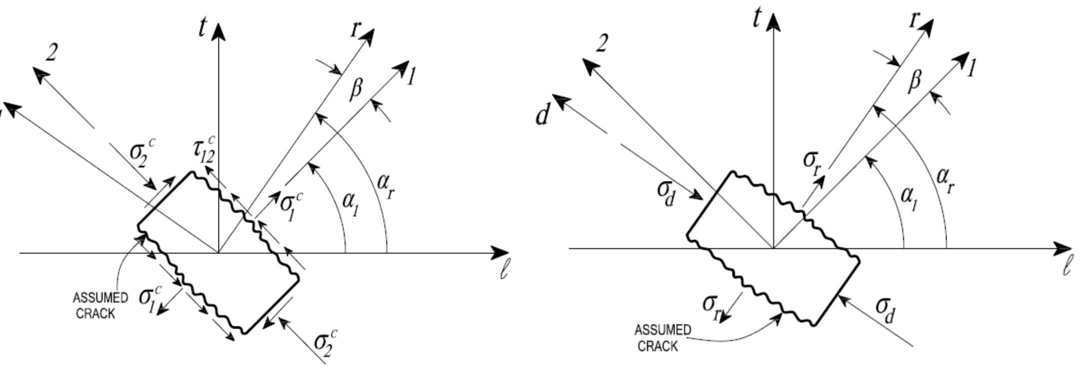
For bridge piers, the approach of this paper is a displacement-based Timoshenko formulation. The interpolation interdependent element (IIE) shape function is adopted in this model, which naturally avoiding the shear-locking phenomenon. Since the model interpolation function is interdependent with the section fibre behaviour in the nonlinear analysis, that an iterative procedure is included in the element state determination algorithm. To validate this model, the proposed Timoshenko element are compared with flexibility-based element formulation. The main advantage of flexibility-based formulation is that the distribution of internal forces along a frame element is exactly posed by the equilibrium equation of the bar in an auxiliary isostatic configuration, in which the rigid movements are removed from the element. The only errors obtained are exclusively due to the numerical integration. The deformation shape is integrated without assuming any shape function of the element(Mohr, Bairán et al. 2010). Another superiority of flexibility formulation is that the shear deformations can be automatically considered and numerical shear locking is eliminated. Here the results obtained from the flexibility-based element are used as reference value. Recently, the solution of flexibility-based geometrically nonlinear Timoshenko model is presented(Jafari, Vahdani et al. 2010).

The bi-axial reinforced concrete constitutive relationship need to be applied on the fibre-based section model in the equilibrium state determination of each fibre on section. Hsu *et al.*(Hsu and Zhu 2002; Mansour and Hsu 2005; Hsu 2010) proposed cyclic softened membrane model (CSMM) for shear dominate structural members. This model has been implemented into the finite element analysis platform, OpenSees(OpenSees 2012-4-30). Murat and Scott(Murat 2010) implemented the MCFT multi-axial material model into OpenSees platform, and the model is suitable for monotonous static simulations.

The main aim of this paper is to develop an R/C bridge pier numerical model with capacity of handling the flexure-shear coupling effect with adequate fibre-based section model. The main aspects of the model, including fibre section formulation and IIE Timoshenko beam element model, is presented. The new element is suitable for the nonlinear analysis of R/C bridge piers under shear, flexural and axial load combination. In addition, the new model is verified and compared with an experimental results obtained from scaled reinforced concrete pier tests subjecting psedu-static cyclic loading.

2. CSMM MODEL DESCRIPTION

CSMM model is implemented into OpenSees platform, and based on the rotation angle-soften truss model (RA-STM), further revised as fixed angle-soften truss model (FA-STM). The basic variables and parameters are defined in the following figures(Hsu and Mo 2010),



(a) Fixed angle model theory (b) Rotating angle model theory

Figure 2.1. The definition of rotational angle in FA-STM and RA-STM

β is the angle between rotating angle (incline angle α_r) and fixed angle (α_1). And the angle α_r is the angle between *d* direction in principle coordinates *d-r* of concrete and the *l* direction of reinforced steel. In RA-STM it is named as rotating angle. By introducing Hsu-Zhu ratio and experimental cyclic hysteretic rules of uniaxial steel and concrete materials, the CSMM could capture the membrane stress-strain behaviour of thin-walled members. The OpenSees platform is equipped with the calculation capacity for plane membrane member accounting shear deformation(Hsu and Zhu 2002; Mansour and Hsu 2005; Hsu 2010) by making utility of different uniaxial material constitutive relationships, such as SteelZ01 and ConcreteZ01 classes. By assuming that the status of RC plane stress element subjected complex load combination is orthotropic material and the response state could be represented by the modified uniaxial material model on principle directions. Each uniaxial material model is corresponding to one material hysteretic response on different principle direction in the plane member. Furthermore, in the OpenSees platform a bi-axial fibre model (BiaxialFiber2d class) is developed taking responsibility for the fibre stress-strain state determination in thin-walled members. The fibre section model is modified for the specific multi-axial behaviours instead of traditional uniaxial material fibre model in sectional response calculation, which is described in Section 3.2 in detail.

3. FIBER MODEL BASED TIMOSHENKO BEAM ELEMENT

3.1 Timoshenko Beam Element with IIE approach

The cubic shape functions of the Interpolation Interdependent Element (IIE)(Reddy 1997) are used in the derivation of the nature force vector and tangent stiffness matrix of Timoshenko element. The IIE functions are Hermitian polynomials with modification for including the flexure-shear coupled effects. The IIE shape function is based on the exact solution of the homogeneous form of the equilibrium equation for the Timoshenko deformation assumption. The axial displacement, *u*, the transverse displacement, *w*, and the local rotation, *θ*, are given by

$$\begin{cases} u = x\bar{u}/l_0 \\ w = \varphi_1\bar{\theta}_1 + \varphi_2\bar{\theta}_2 \\ \theta = \varphi_3\bar{\theta}_1 + \varphi_4\bar{\theta}_2 \end{cases} \tag{3.1}$$

where the deformation variables are illustrated in Fig. 3.1.

Figure 3.1. Elemental nodal forces and displacement excluding rigid body modes

And the shape functions are defined as

$$\begin{cases} \varphi_1 = \mu\xi\left(6\Omega(1-\xi) + (1-\xi)^2\right) l_0 \\ \varphi_2 = \mu\xi\left(6\Omega(\xi-1) - \xi + \xi^2\right) l_0 \\ \varphi_3 = \mu\left(1 + 12\Omega - 12\Omega\xi - 4\xi + 3\xi^2\right) \\ \varphi_4 = \mu\left(12\Omega\xi - 2\xi + 3\xi^2\right) \end{cases} \tag{3.2}$$

$$\mu = \frac{1}{1+12\Omega}, \quad \Omega = \frac{EI}{\kappa GAl_0}, \quad \xi = \frac{x}{l_0} \tag{3.3}$$

A, *I* is the sectional area and inertia moment of section, *κ* is the shear correction coefficient (for

rectangular cross-section fibres, κ=5/6). The interpolation functions are interrelated with the sectional material property, $EI/\kappa GA$. For nonlinear analysis, the shape function may be modified step by step following the nonlinear developing process, and must be determined iteratively in the element calculation procedure.
After the shape function is determined , each fibre strain can be derived from sectional deformations

$$\begin{cases} \varepsilon = \dfrac{\partial u}{\partial x} - \dfrac{\partial^2 w}{\partial x^2} y \\ \gamma = \dfrac{\partial w}{\partial x} - \theta \end{cases} \tag{3.4}$$

3.2 Fibre Section Model and Some Finite Element Implementation Consideration

For the beam-column element there is no compression stress between fibres ($\sigma_y=\sigma_z=0$) with satisfaction of the strain compatibility. The multi-axial material model for fibres should be condensed by introducing the physical conditions of stress and strain between fibres. This function is achieved by developing TimoshenkoSection2D (TimoshenkoSection3D for 3D issue) class based on OpenSees platform. For 3D material, the relationship between incremental stress and strain is $\Delta\sigma_{ij} = C_{ijkl}\Delta\varepsilon_{kl}$, where $i,j,k,l \in x,y,z$. For plane stress issue, the material constitute relationship is simplified as

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\tau_{xy} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{Bmatrix} \tag{3.5}$$

While the critical components of sectional strains are the axial strain and transverse shear strain on *y* direction. So the material modulus are associated with a 2×2 stiffness matrix and 2×1 stress and strain vectors for plane Timoshenko element. The stiffness matrix of each fibre should be

$$\mathbf{k} = \begin{bmatrix} \dfrac{\partial\sigma_x}{\partial\varepsilon_x} & \dfrac{\partial\sigma_x}{\partial\gamma_{xy}} \\ \dfrac{\partial\tau_{xy}}{\partial\varepsilon_x} & \dfrac{\partial\tau_{xy}}{\partial\gamma_{xy}} \end{bmatrix} \tag{3.6}$$

The transverse strain between fibres could be derived by satisfying that the transverse stress of each fibre is equal to 0,

$$\sigma_y = 0 \Rightarrow \varepsilon_y = \left(-\frac{\varepsilon_x k_{21} + \gamma_{xy} k_{23}}{k_{22}} \right) \tag{3.7}$$

Finally, the constitutive relationship of each fibre in incremental form is

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\tau_{xy} \end{Bmatrix} = \begin{bmatrix} k_{11} - \dfrac{k_{12}k_{21}}{k_{22}} & k_{13} - \dfrac{k_{12}k_{23}}{k_{22}} \\ k_{31} - \dfrac{k_{32}k_{21}}{k_{22}} & k_{33} - \dfrac{k_{32}k_{23}}{k_{22}} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\gamma_{xy} \end{Bmatrix} \tag{3.8}$$

The implementation of finite element analysis procedure is illustrated in Fig. 3.2. It is worth noting that, besides the new proposed Timoshenko element, the TimoshenkoSection2D class is also applicable to all kinds of beam column elements in OpenSees platform, including flexibility-based element and displacement-based element.

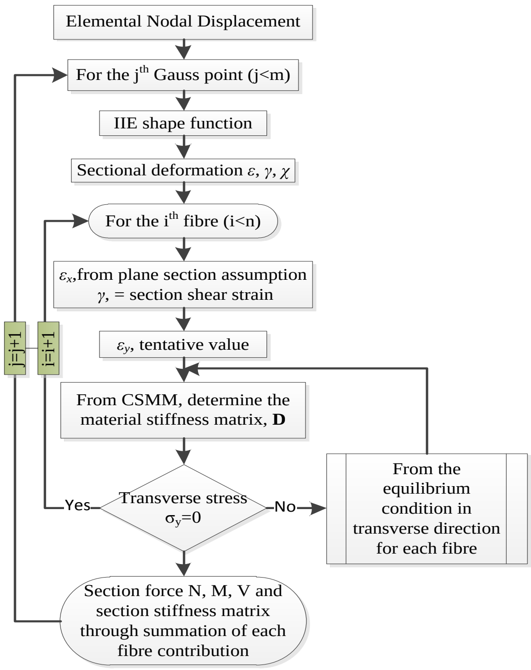


Figure 3.2. Fiber model based beam column element calculation procedure

It's worth mentioning that the conversion of fourth/second order tensors to matrices/vectors was performed in the finite element platform, OpenSees. The 2nd order Cauchy stress tensor is implemented in OpenSees as a 3×1 vector and 4th order material tangent tensor is implemented as a 3×3 matrix. However, the multiplication of two fourth/second order tensors cannot be directly converted to the multiplication of the corresponding matrices/vectors. For CSMM model, the constitutive law, $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$, cannot be directly converted in matrix form as (for plane issues),

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \neq \begin{bmatrix} C_{11,11} & C_{11,22} & C_{11,12} \\ C_{22,11} & C_{22,22} & C_{22,12} \\ C_{12,11} & C_{12,22} & C_{12,12} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} \tag{3.9}$$

Instead, it should be converted to

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} C_{11,11} & C_{11,22} & 2C_{11,12} \\ C_{22,11} & C_{22,22} & 2C_{22,12} \\ C_{12,11} & C_{12,22} & 2C_{12,12} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} \tag{3.10}$$

Currently, for beam elements existing in OpenSees platform, stresses and deformations can be determined through the uniaxial material fibre section model naturally. It means that the axial-flexure deformation does not interact with shear deformations. The biaxial material wrapper classes (BiaxialFiber2d) is implemented in OpenSees accounting for the multi-dimensional material behaviour in fibre model. Therefore, a new biaxial fibre section model that coupling sectional shear, axial and flexure deformation fields were proposed.

3.3 Numerical Verification

The stability of a right-angle frame is analysed with proposed IIE Timoshenko element and flexibility-based element. The problem definition is given in Fig. 3.3. and the analysis results are depict in Fig. 3.4.

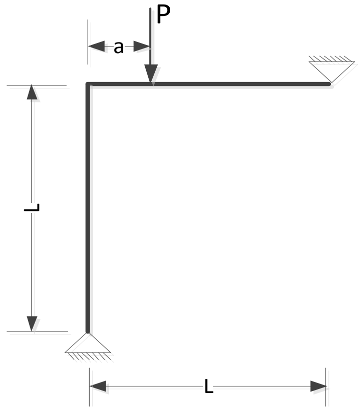


Figure 3.3. Post buckling analysis of hinged right-angle frame (*a*=*L*/5)

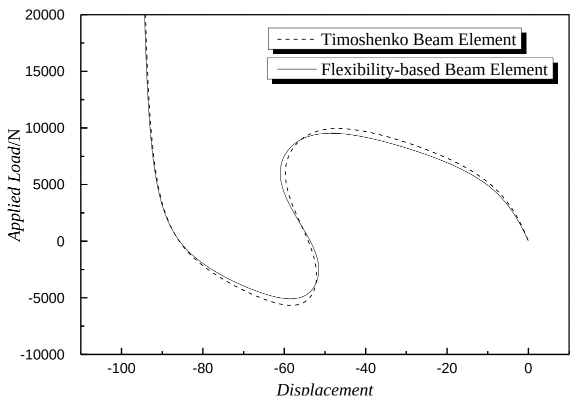


Figure 3.4. Analysis results (10 elements for 2 members)

Both the IIE Timoshenko element and flexibility-based element adopts the BiaxialFiber2d model in each integration point (with 3 integration points per element). Sectional parameter *A* is 6.0, *I* is 1.0, and the geometry transformation is corotational transformation. The multi-dimensional material model is J2 plastic model for 3D issue. The material elastic modulus is 27777.78 MPa, shear modulus is 9259.26 MPa, the initial yield stress is 50 MPa, final saturation yield stress is 65 MPa and the linear hardening parameter is 0. There are small error between the two numerical models as shown in Fig. 3.4, although it is acceptable. The numerical test also presented that the IIE Timoshenko element is capable for geometric nonlinear problems, and is independent with which geoTransf method was chosen. Further test and verification of the proposed IIE Timoshenko element is underway, and the shear-flexure coupled element for three dimensional issue is under developing.

4. MODEL TEST AND VERIFICATION

Table 4.1 One-way hysteretic test model size and parameters

No.	Height of pier	Height /Width ratio	Axial Load (MN)	Axial Compression ratio	Longitude Reinforcement		Transvers Reinforcement		
					Num. @ Dia. (mm)	ratio of reinforcement	Dia. (mm)	spacing	stirrup ratio
S1	1440	4	0.28	0.1	40@8	0.014	@6	40	0.035
S3	2880	8	0.28	0.1	40@8	0.014	@6	40	0.035
S8	3600	10	0.28	0.1	40@8	0.014	@6	40	0.035

The diameter of longitudinal rebar is 8 mm and transverse reinforcement is 6 mm with the design yielding strength f_y=300 MPa. The design compressive strength of concrete is f_c=19.1 MPa at 28 days. The yielding and ultimate strength stress measured by standard tensile tests on reinforcing steel coupons are 385 MPa and 498 MPa, respectively. The actual average compressive strength of concrete is 41.5 MPa, determined by 150 mm cubic concrete testing after 28-day curing process. The concrete cover is 25 mm, the cap and basement of pier is constructed and matched with the testing actuator and site. The geometry parameters are listed in Tab. 4.1, and the specimens size of horizontal section is illustrated in Fig. 4.2. The axial loading is constant and the test procedure follows the cyclic loading test method.

In order to compare the performance of proposed model and the model without coupling effect, the results concluded from the coupling model and non-coupling model are illustrated in the same figures (Fig. 4.3 ~ 4.5). For simulation model, the core concrete fibre is Concrete02, the cover concrete fibre is Concrete01, and the reinforcement is Steel02 for non-coupling model. Since the precision of pier element is not improved obviously when the pier is simulated using more than 3 elements with 4 integration points for each element, the piers are simulated by 3 elements with 4 Gauss-Lobatto integration points per element finally. The IIE Timoshenko model adopts BiaxialFiber2d class with 3 elements with 4 integration points, too. All the simulated results and test results are illustrated in the same figures (Fig. 4.3 ~ 4.5).

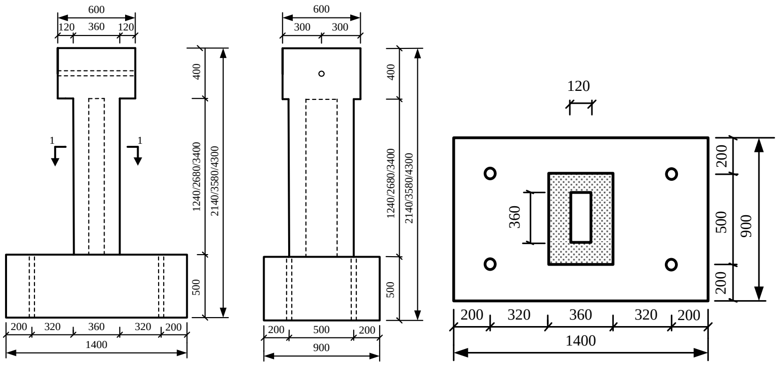


Figure 4.1. Model geometry parameter (unit: mm)

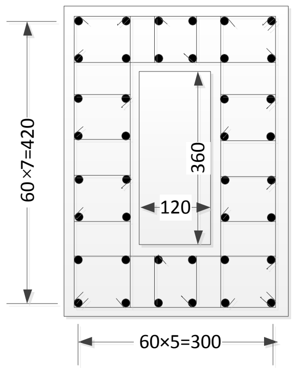


Figure 4.2. Steel reinforcement configuration (unit: mm)

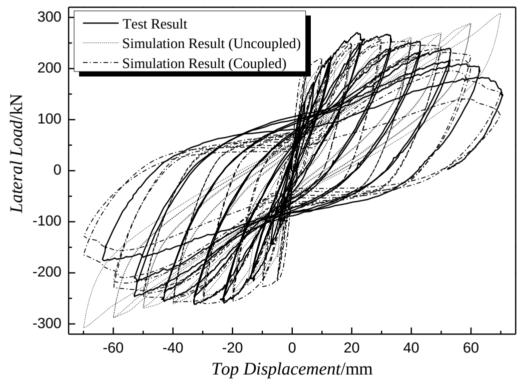


Figure 4.3. Comparison of numerical and test cyclic response of S1 column

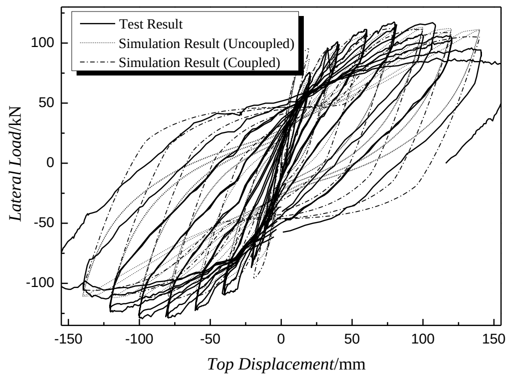


Figure 4.4. Comparison of numerical and test cyclic response of S3 column

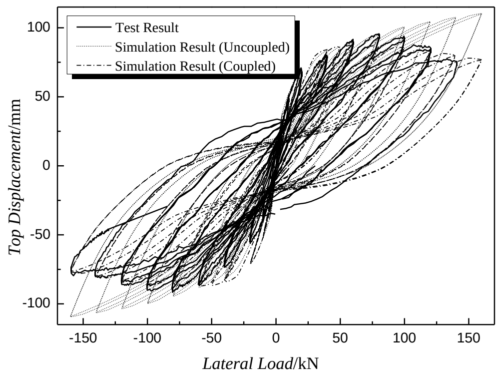


Figure 4.5. Comparison of numerical and test cyclic response of S8 column

CONCLUSION

Although there are many flexure-shear coupling damage phenomenon have been observed in recent earthquakes, few simulation model was proposed for R/C hollow section bridge piers mainly impacted by the flexure-shear coupling effects.

The paper presents a new analysis model for nonlinear analysis of shear-flexure dominated R/C members, like bridge piers with hollow section. The mathematical theory for this approach is multi-dimensional fibre-based section model, which is achieved by developing TimoshenkoSection2D class, and IIE Timoshenko formulation, which is implemented by developing Timoshenko2D element class in OpenSess platform. For hollow section R/C components, bi-axial constitutive relationships adopted for the fibre model is CSMM model. The calculation procedure is illustrated on the element level (Timosheko2d class), section level (TimoshenkoSection2d class) and fibre level (BiaxialFibre class). Cyclic pushover experiment carried on scaled hollow section piers. The result deduced from the numerical model is compared with the experiment results, and it shows good agreements for the strength degradation and pinching effect. This fibre-based section model provides sufficient accuracy and computational efficiency. And further researches will focus on the flexure-shear induced damage and collapse for bridge structures.

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