Nonlinear hysteresis model taking into account S-shaped hysteresis loop and its standard parameters

Y. Nogami
JR East Consultants company, Japan

Y. Murono
Railway Technical Research Institute, Japan

H. Morikawa
Tokyo Institute of Technology, Japan

SUMMARY:
Various stress-strain models have been proposed for a dynamic response analysis of ground. Most of these models, however, cannot represent "S-shaped" hysteresis loop of stress-strain relationship, which appears at large level of strain, because their hysteresis curves are formulated on the basis of the Masing's rule. To solve this kind of problem, we propose a new model, which is called GHE-S model. For the GHE-S model, we introduce a modified Masing's rule. In the modified Masing's rule, a similarity function, which is a quadratic function, is introduced. The proposed model is defined by eight parameters. These parameters should be determined through results of cyclic loading tests. However, it is not always carried out for all the samples of soil layers, because the cyclic loading test is costly. Therefore, we also provide standard values of eight parameters for the proposed model.

Keywords: S-shaped hysteresis loop, nonlinear stress-strain model, modified Masing's rule

1. GENERAL INSTRUCTIONS

It is very important for the earthquake engineering to evaluate seismic behavior at surface of ground. A dynamic response analysis is often used to estimate the response of sediments. For dynamic response analyses, not only accurate information of ground properties but also constitutive model of stress-strain relationship for soil is necessary. Various stress-strain models have been proposed for a dynamic response analysis of ground. Classical models include Hardin-Drnevich model [1], Ramberg-Osgood model [2] and so on. They are often used for dynamic response analysis, because classical models are very simple, in which only one or two parameters are required, and can be applied readily to various problems. However, these models cannot satisfy any results of stress-strain relationship obtained from cyclic loading tests perfectly. On the other hands, to solve such kind of problem, improved models have been proposed by Ishihara et al. [3], Nishimura and Murono [4] and so on. For example, Ishihara's model [3] can satisfy any results of hysteresis damping obtained from cyclic loading tests by modification of unloading stiffness to fit the hysteresis damping.

Most of these models, however, cannot represent "S-shaped" hysteresis loop of stress-strain relationship, which appears at large level of strain, because their hysteresis curves are formulated on the basis of Masing's rule. To solve this kind of problem, we have proposed a new model, which is called GHE-S model [5]. For GHE-S model, we have introduced a modified Masing's rule instead of conventional Masing's rule. GHE-S is defined by eight parameters. These parameters should be determined through results of cyclic loading tests. However, it is not always carried out for all the samples of soil layers, because the cyclic loading test is costly. Therefore, we also provide standard values of eight parameters for the proposed model.

2. STRESS-STRAIN RELATIONSHIP OF SOILS

Figs. 2.1 and 2.2 show hysteresis loops of stress-strain relationship, which is obtained from cyclic torsional shear test for a soil. A maximum strain is small in Fig. 2.1 and is large in Fig. 2.2. Shear stiffness, G, is defined by secant modulus at reversal points as shown in Fig. 2.3. Initial shear stiffness, $G_{\max}$, is shear modulus at infinitesimal strain; $\gamma = 10^{-6}$ to 10^{-5}. Hysteresis damping, h, is defined as

$$h = \frac{1}{4\pi} \cdot \frac{A_L}{A_T} \quad , \tag{2.1}$$

where, A_L and A_T are the area of the hysteresis loop in Fig. 2.3 and the area of the triangular in Fig. 2.3, respectively.

A normalized shear stiffness ratio, $G/G_{\max}$, decreases with increasing the strain, γ, in Fig. 2.4, because of plastic properties of soils. The hysteresis loop at small level of strain is generally represented as a "spindle-shape" of Fig. 2.1. Therefore, a hysteresis damping, h, increases with the strain, γ, as shown in Fig. 2.4. On the other hand, the hysteresis curve shows an "S-shape" of Fig. 2.2 in a case where the strain exceeds approximately 1%. Thus, the hysteresis damping, h, decreases with the strain, γ, as red crosses in Fig. 2.4.

As described in Chapter.1, most of conventional stress-strain models cannot represent "S-shape", which appears at large level of the strain. We, therefore, have already proposed a new model, which is called GHE-S model [5]. However, we show the outline of GHE-S model in the following chapter again, because the concept of GHE-S model is very important in this paper.

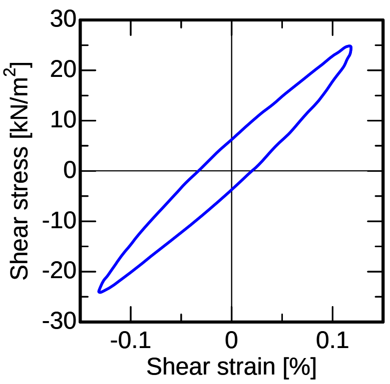


Figure 2.1 Hysteresis loop at small level of strain

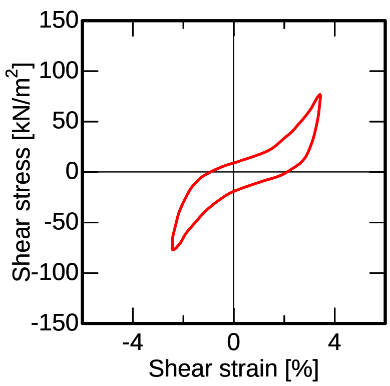


Figure 2.2 Hysteresis loop at large level of strain

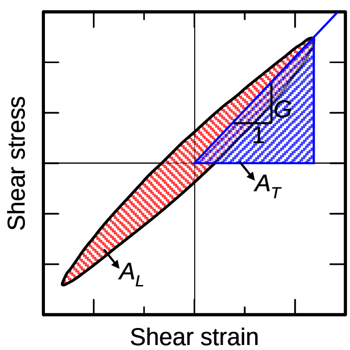


Figure 2.3 Definition of G, A_L and A_T

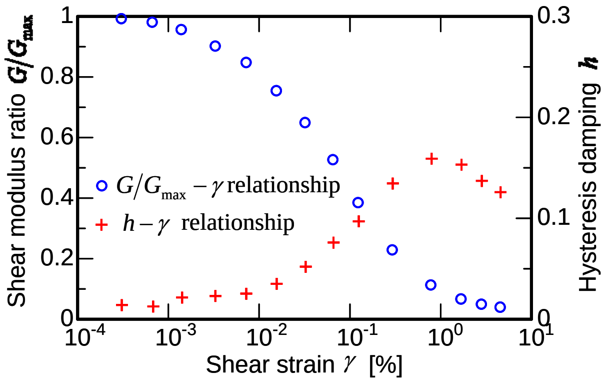


Figure 2.4 $G/G_{\max}$ - γ and h - γ relationship

3. GHE-S MODEL

3.1. Skeleton Curve

A skeleton curve of GHE-S model is defined by GHE (General hyperbolic equation) model [6]. GHE model is represented by the generalized hyperbolic equation:

$$\tau/\tau_f = \frac{\gamma/\gamma_r}{\dfrac{1}{C_1(\gamma/\gamma_r)} + \dfrac{\gamma/\gamma_r}{C_2(\gamma/\gamma_r)}} \quad , \tag{3.1}$$

where, τ stands for shear stress of soils, τ_f for shear strength of soils, γ for shear strain of soils, and reference shear strain of soils, γ_r, is defined as

$$\gamma_r = \frac{\tau_f}{G_{\max}} \quad . \tag{3.2}$$

Corrective coefficients, $C_1(\gamma/\gamma_r)$ and $C_2(\gamma/\gamma_r)$ are obtained from

$$C_1(\gamma/\gamma_r) = \frac{C_1(0)+C_1(\infty)}{2} + \frac{C_1(0)-C_1(\infty)}{2}\cdot\cos\left\{\frac{\pi}{\alpha/(\gamma/\gamma_r)+1}\right\} \quad , \tag{3.3}$$

$$C_2(\gamma/\gamma_r) = \frac{C_2(0)+C_2(\infty)}{2} + \frac{C_2(0)-C_2(\infty)}{2}\cdot\cos\left\{\frac{\pi}{\beta/(\gamma/\gamma_r)+1}\right\} \quad . \tag{3.4}$$

Stress-strain relationship of GHE model can be applied to various strain levels, namely, from infinitesimal strain to failure strain of soil. This means that GHE model can satisfy any results of stress-strain relationship obtained from cyclic loading tests. GHE model consists of six parameters, which are $C_1(0)$, $C_1(\infty)$, $C_2(0)$, $C_2(\infty)$, α and β. From Eqns.3.1 and 3.3, $C_1(0)=1.0$ is obtained as $\gamma \to 0$ and from Eqns.3.1 and 3.4, $C_2(\infty)=1.0$ is obtained as $\gamma \to \infty$. Four unknown parameters, therefore, should be determined. These parameters can be determined from results of cyclic loading tests.

3.2. Hysteresis Curve

3.2.1. Hysteresis rule

Conventional stress-strain models apply to Masing's rule for the hysteresis loop. In conventional Masing's rule, an enlarged skeleton curve is used to express the hysteresis loop in unloading and reloading processes. A procedure to generate a hysteresis loop is shown in Fig. 3.1. For example, when the unloading process occurs at a reversal point, $A(\gamma_a, \tau_a)$ of Fig. 3.1 on the skeleton curve, the hysteresis curve of blue lines is formed by enlarging the skeleton curve, $\tau = f(\gamma)$ of red line. The hysteresis curve depends on the homothetic ratio, λ as follows:

$$\frac{\tau-\tau_a}{\lambda} = f\left(\frac{\gamma-\gamma_a}{\lambda}\right) \quad . \tag{3.5}$$

A homothetic ratio, λ=2.0, is generally chosen as the hysteresis loop at any strain, because the hysteresis loop needs to be connected to the point B smoothly, which is the symmetric point of A with respect to the origin. As a result, only spindle-shaped hysteresis loop is drawn.

In order to connect the hysteresis loop smoothly from the point A to the point B in Fig. 3.1, λ should be equal to two only around the neighbourhood of reversal points, A and B. It is not necessary to be two between A and B. We, therefore, introduce a modified Masing's rule. The homothetic ratio, λ, is defined as a function of strain for the modified Masing's rule: similarity function $\lambda(\gamma)$, is introduced. S-shaped hysteresis loop can be realized by the similarity function, which depends on strain. We adopt a quadratic function, which passes through $\lambda(\gamma_a)=\lambda(-\gamma_a)$=2, as the similarity function in Fig. 3.2, because hysteresis curves of the modified Masing's rule with a quadratic function resembles those of cyclic loading tests for soil in their shapes. Practically, $\lambda_{\min}$ of Fig. 3.2 is determined from hysteresis damping of cyclic loading test.

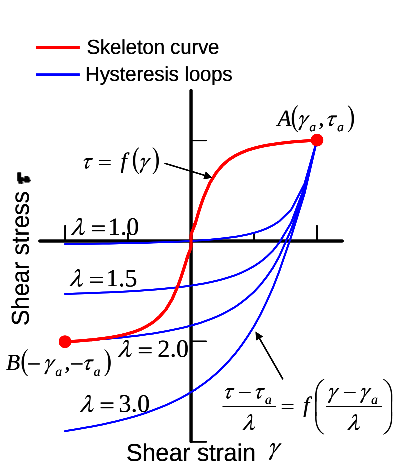


Figure 3.1 Hysteresis loops with some homothetic ratio

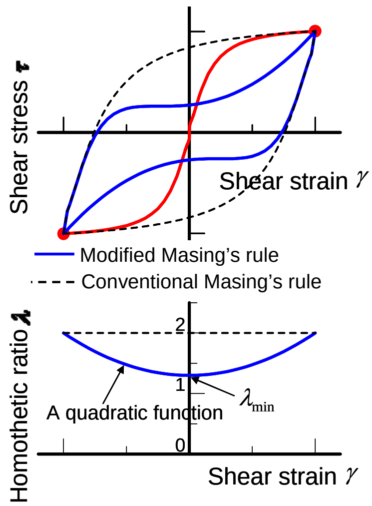


Figure 3.2 Concept of modified Masing's rule (Above: hysteresis loop, below: similarity function)

3.2.2. Relationship between hysteresis damping and strain

A relationship between hysteresis damping, h, and strain, γ, obtained from cyclic loading test is discrete data. One can use a linearly interpolated function of the discrete data, however we use a continuous function defined as

$$h = h_{\max}\left(1-\frac{G}{G_{\max}}\right)^{\beta_p} \quad . \tag{3.6}$$

The hysteresis damping h is expressed as a function of $G/G_{\max}$ with parameters of $h_{\max}$ and β_p. Eqn. 3.6 is not a function of γ explicitly, though h-γ relationship can be obtained by substituting Eqn. 3.1 and the relationship $\tau = G\cdot\gamma$ into Eqn. 3.6. The parameters of $h_{\max}$ and β_p can be determined by fitting to results of cyclic loading test.

3.2.3. Unloading stiffness

In general case, the unloading stiffness is constant and identified with initial shear stiffness, $G_{\max}$. Yoshida et al. [7], however, pointed out that the unloading stiffness shows non-linear characteristics. We, therefore, introduce Yoshida's formula for the unloading stiffness:

$$\frac{G_0}{G_{\max}} = \frac{1-G_{\min}/G_{\max}}{1+\gamma/\gamma_{r0}} + \frac{G_{\min}}{G_{\max}} \quad , \tag{3.7}$$

where, G_0 and $G_{\max}$ are unloading and initial shear stiffness, respectively, $G_{\min}/G_{\max}$ ratio of minimum shear stiffness, γ shear strain and γ_{r0} reference strain for unloading stiffness. Typical values of the parameters are provided by Yoshida et al. as listed in Table 3.1.

Table 3.1 Values in Eqn. 3.7

Soil	γ_{r0}	G_{min}/G_{max}
Sand (Dr[*1]=50%)	0.0006	0.18
Sand (Dr[*1]=80%)	0.0015	0.35
Clay	0.013	0.1

*1: Dr is relative density of soil.

3.3. Numerical Example

To confirm the proposed model, a numerical calculation is carried out. Input wave to the base is sinusoidal wave, in which the amplitude of acceleration increases gradually at a frequency of 2Hz. The calculated hysteresis loop of stress-strain relationship is shown in Fig. 3.3. It is observed that a spindle-shaped hysteresis curve can be drawn for small level of the strain in blue and that the shape of the hysteresis curve morphs into S-shape with the strain (e.g. green line).

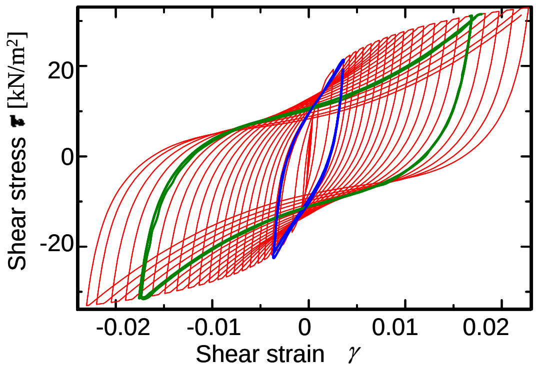


Figure 3.3 Hysteresis loop of a numerical calculation

4. STANDARD VALUES FOR THE PARAMETERS

The proposed model is defined by six parameters for G/G_{max} - γ/γ_r relationship and by two parameters for h - γ/γ_r relationship. These parameters should be determined through results of cyclic loading tests, such as cyclic torsional shear test. It is not always carried out for all the samples of soil layers, because the cyclic loading test is costly. We, therefore, provide standard values of eight parameters for the proposed model. In order to determine the standard values, we use results of cyclic loading tests, which were carried out with twenty samples under undrained condition. These tests are carried out under various conditions of confined pressure, type of soil, and so on.

4.1. Relationship between shear stiffness ratio and shear strain

Fig. 4.1 shows G/G_{max} - $\gamma/\gamma_{0.5}$ relationship obtained from 20 experiments, where $\gamma_{0.5}$ is the strain at $G/G_{max} = 0.5$ in G/G_{max} - γ relationship from cyclic loading test. It is seemed that G/G_{max} - $\gamma/\gamma_{0.5}$ relationship is distributed within a narrow range in spite of various conditions of confined pressure, type of soil, and so on. Kiyota et al. [8] pointed out that G/G_{max} - $\gamma/\gamma_{0.5}$ relationship is distributed within a narrow range regardless of confined pressure. From here onwards, we assume that G/G_{max} - $\gamma/\gamma_{0.5}$ relationship is independent of confined pressure and soil type.

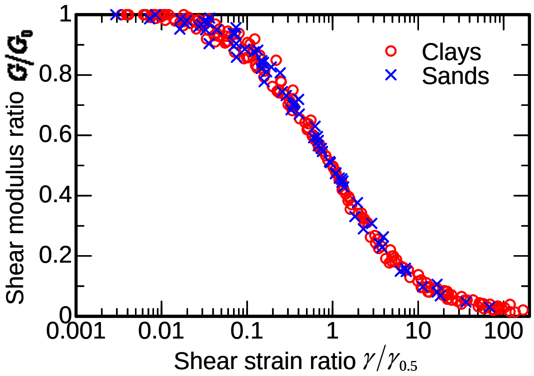


Figure 4.1 $G/G_{\max}$ - $\gamma/\gamma_{0.5}$ relationship obtained from cyclic loading tests

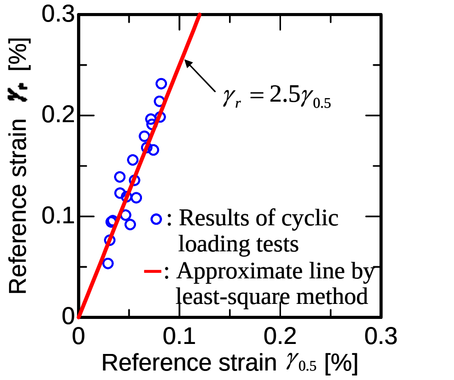


Figure 4.2 Relationship between γ_r and $\gamma_{0.5}$

From Eqn. 3.1, GHE-S model requires the reference strain, γ_r of Eqn. 3.2 in order to introduce the dependency of shear strength, τ_f. However, in a case where $\gamma_{0.5}$ is used as the reference strain instead of γ_r, an important properties, that is the dependency of shear strength, is definitely lost. To avoid this problem, we introduce the follows:

$$\gamma_r = 2.5\gamma_{0.5} \tag{4.1}$$

This equation is empirically determined using results of cyclic loading tests as shown in Fig. 4.2. Furthermore, in a case where the reference strain is γ_r, $C_2(\infty) = 1.0$ as $\gamma \to \infty$ from Eqns. 3.1 and 3.4. On the other hand, if the reference strain is $\gamma_{0.5}$, $C_2(\infty) = 2.5$ as $\gamma \to \infty$. There are two reasons why $\gamma_{0.5}$ is chosen to normalize the strain, γ. Firstly, variances of $G/G_{\max}$ - $\gamma/\gamma_{0.5}$ relationship are smaller than one of $G/G_{\max}$ - γ/γ_r relationship. This means that $\gamma_{0.5}$ can provide stable normalized strain. Secondly, we can apply the results by many researches to determine $\gamma_{0.5}$ without any experiments, though many experiments are necessary under various soil conditions to determine γ_r.

Twenty datasets of values for six parameters are obtained fitting $G/G_{\max}$ - $\gamma/\gamma_{0.5}$ relationship to results of twenty cyclic loading tests under the assumptions: small variances of $G/G_{\max}$ - $\gamma/\gamma_{0.5}$ relationship and Eqn. 4.1. The standard values of parameters are defined as the averaged values of datasets. Table 4.1 shows the obtained standard values. $G/G_{\max}$ - $\gamma/\gamma_{0.5}$ relationships are shown in Fig. 4.3, which includes results of twenty cyclic loading tests (red crosses), results by previous researches [9]-[15] (blue triangles) and "standard curve" determined from standard values (green line). It is observed that the standard curve agrees well with previous researches.

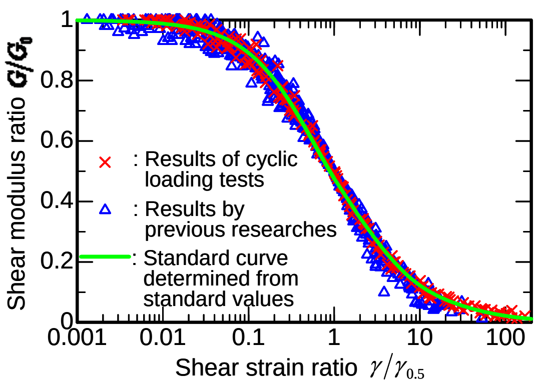


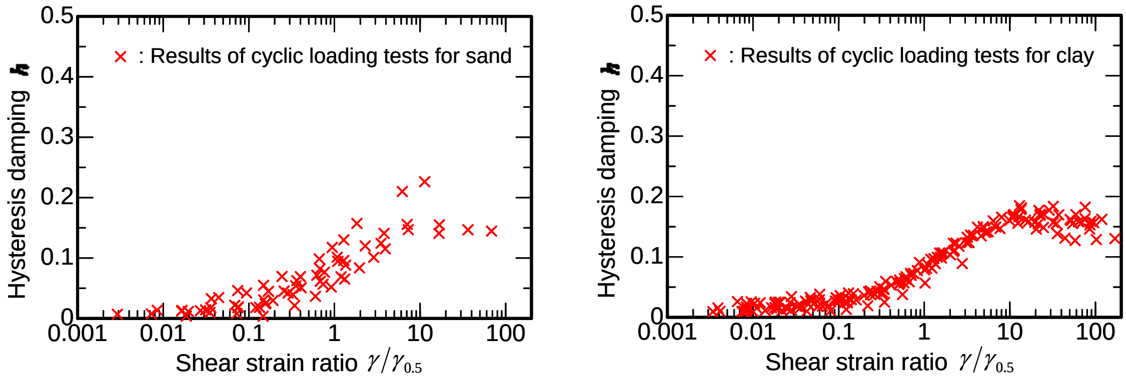
Figure 4.3 $G/G_{\max}$ - $\gamma/\gamma_{0.5}$ relationship with "standard curve"

Table 4.1 Standard values in Eqns. 3.3 and 3.4

$C_1(0)$	$C_1(\infty)$	$C_2(0)$	$C_2(\infty)$	α	β
1.000	0.170	0.830	2.500	2.860	3.229

4.2. Relationship between hysteresis damping and shear strain

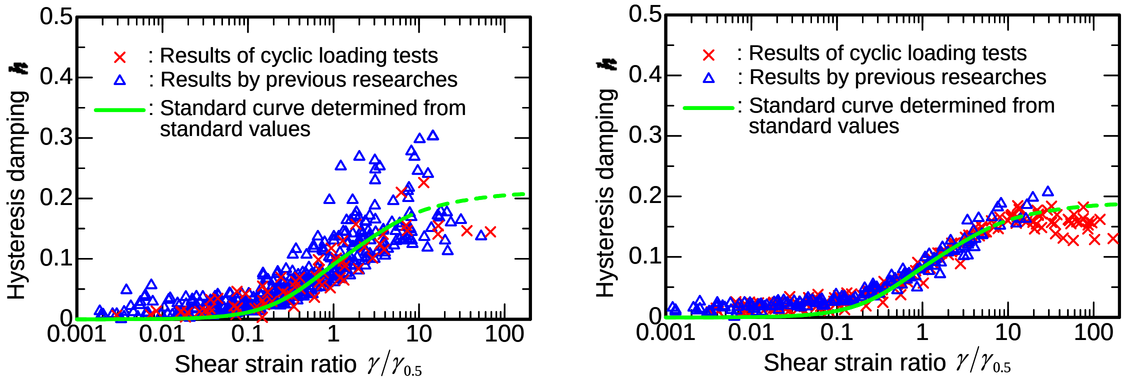
Standard values for h-γ relationship are determined in the same way as G/G_{max}-$\gamma/\gamma_{0.5}$ relationship. Figs. 4.4 (a) and (b) show h-$\gamma/\gamma_{0.5}$ relationship for sandy soils and clayey soils, respectively. In Fig. 4.4 (a), h-$\gamma/\gamma_{0.5}$ relationships show large variations, because hysteresis damping of sandy soil is affected by various conditions, such as excess pore water pressure, fraction content, confined pressure and so on. However, results of cyclic loading tests are not enough to consider various conditions.



(a) Sandy soils **(b)** Clayey soils

Figure 4.4 h-$\gamma/\gamma_{0.5}$ relationship obtained from cyclic loading tests

In general, hysteresis damping h of clayey soil is relatively smaller than that of sandy soil. We, therefore, divide soils into two types, that is, sand and clay to determine standard values for h-$\gamma/\gamma_{0.5}$ relationship. The standard values for h-$\gamma/\gamma_{0.5}$ relationship are determined from the averaged values of results by cyclic loading tests. The obtained values are listed in Table 4.2 and Fig. 4.5. The legends of Fig. 4.5 are the same as Fig. 4.3. From this figure, the standard curves also agree with the results by the previous researches.



(a) Sandy soils **(b)** Clayey soils

Figure 4.5 h-$\gamma/\gamma_{0.5}$ relationship with "standard curve"

Table 4.2 Standard values in Eqn. 3.6

Soil	h_{max}	β_p
Sand	0.21	1.31
Clay	0.19	1.29

5. CONCLUDING REMARKS

Various stress-strain models have been proposed for a dynamic response analysis of ground. Most of conventional models, however, cannot represent the stress-strain relationship with “S-shape”, which appears at large strain levels. We, therefore, have proposed a new model, which is called GHE-S model. GHE-S model consists of the skeleton curve expressed by GHE (general hyperbolic equation) model and the hysteresis curve expressed by a modified Masing’s rule. The proposed model has eight parameters. These parameters should be determined by results of cyclic loading tests. Cyclic loading tests, however, is not always conducted for all the soils at a target site. We also proposed standard parameters of GHE-S model in order to apply the model to a site, where there are no results of cyclic loading test.

REFERENCES

[1] Hardin, B.O. and Drnevich, V.P. (1972). Shear modulus and damping in soils : design equations and curves. *Journal of SMFD, Proc. ASCE* **Vol.98**, **SM7**, 667-692.

[2] Jennings, P.C. (1964). Periodic response of a general yielding structure. *Proc. ASCE* **EM2**, 131-163.

[3] Ishihara, K., Yoshida, N. and Tsujino, S. (1985). Modeling of stress-strain relations of soils in cyclic loading. *Fifth International Conference on Numerical Methods in Geomechanics* **Vol.1**, 373-380.

[4] Nishimura, A. and Murono, Y. (1999). Nonlinear hysteresis model using GHE model and modified Masing’s rule. *Proc. JSCE Earthquake Engineering Symposium* **Vol.25-1**, 309-312.

[5] Murono, Y. and Nogami, Y. (2006). Shear Stress-Shear Strain Relationship Taking into Account S-Shape Hysteresis Loop. *Symposium of 12th Japan Earthqake Engineering* **Vol. 12**, 494-497.

[6] Tatsuoka, F. and Shibuya, S. (1992). Deformation characteristics of soils and rocks form field and laboratory tests, Theme Lecture 1, *Proc. of Ninth Asian Regional Coference on Soil Mechanics and Foundation Engineering* **Vol. 2**, 101-170.

[7] Yoshida, N., Sawada, S., Takeshima, Y., Mikami, T and Sawada, S. (2003). Effect of hysteretic damping characteristics on earthquake response of ground. *JSCE journal of Earthquake Engineering* **Vol. 27:0158**, 1-8.

[8] Kiyota, Y., Hagiwara, T. and Tamura, H. (1995). Dynamic deformation characteristics of quartz sand #6. *Japan National Conference on Geotechnical Engineering* **Vol. 30: D-7**, 851-852.

[9] Tanaka, Y., Kudo, K., Kokusyo, T., Kato, M., Kato, A. and Nagasaki, K. (1985). Dynamic shear modulus and damping of gravelly soils. *Japan National Conference on Geotechnical Engineering* **Vol. 20: D-7**, 599-600.

[10] Hatanaka, M., and Suzuki, Y. (1987). Cyclic deformation and strength characteristics of an undisturbed gravel. *Japan National Conference on Geotechnical Engineering* **Vol. 22: D-7**, 611-612.

[11] Nishi, K., Kanatani, M., Touma, J., Nishi, T. and Kuroda, S. (1988). Dynamic shear modulus and damping of gravelly soil at low pressures. *Japan National Conference on Geotechnical Engineering* **Vol. 23: D-6**, 633-634.

[12] Yasuda, S., Nagase, H., Oda, S. and Kitsuji, K. (1993). Comparison of dynamic shear modulus and damping ratio between stage-test method and fresh-test method. Symposium of the *JGS* **I-1**, 127-132.

[13] Shamoto, Y. and Kusukame, T. (1993). The caracteristics of stress-strain relation of sands under cyclic loading on anisotoropic consolidation. *Japan National Conference on Geotechnical Engineering* **Vol. 28: D-7**, 923-924.

[14] Yamashita, S., Toki, S. and Shirasaki, K. (1994). Effect of membrane penetration on cyclic undrained triaxial deformation test on sands. *Japan National Conference on Geotechnical Engineering* **Vol. 29: D-7**, 767-770.

[15] Uchida, A., Hatanaka, M. and Tokimatsu, K. (2001). Modeling of cyclic deformation characteristics for soils. *Journal of Structural and Construction Engineering of AIJ* **Vol. 544**, 69-75.