Cumulative Damage Evaluation based on Energy Balance Equation

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SUMMARY:
This paper describes an evaluation method for cumulative damage by earthquakes based on energy balance equation. Industrial facilities such as power plants require high anti-seismic performance. Generally seismic design of mechanical structures in industrial facilities is based on static force in elastic region. However dynamic cyclic load may causes cumulative damage, and such damage induces fatigue failure on the structures. In addition, evaluation method of soundness after a huge earthquake is also required. Therefore quantitative evaluation of cumulative damage by earthquakes is required. Authors have dealt with this issue from the viewpoint of the energy balance equation. Since energy indicates cumulative information of motion, the energy balance equation is suitable to evaluate cumulative damage and a margin for failure. This paper consists of introduction of energy balance equation, investigation into relation between energy and fatigue failure, and an application of Miner's law to energy.

Keywords: Seismic evaluation, Energy balance equation, Fatigue failure, Cumulative damage

1. INTRODUCTION

Large earthquakes, such as the Niigataken Chuetsu-oki earthquake in 2007 and the great east Japan earthquake in 2011, attacked industrial facilities. Especially the great east Japan earthquake had long duration time and many aftershocks. From these disasters, some subjects regarding seismic design and evaluation of mechanical structures have been newly recognized. One of the subjects is rational evaluation of ultimate strength, and another one is evaluation of soundness after an earthquake. In general, seismic design of mechanical structures in industrial facilities is based on static force in elastic region, because of its easy calculability. However the static force is a momentary parameter, and it cannot consider the duration time and the number of aftershocks. Thus importance of seismic margin assessment (SMA) and damage indicating parameter (DIP) that can consider these subjects has discussed lately.

Authors focused on an energy balance equation [1]-[4] for above-mentioned subjects. The energy balance equation is one of valid methods for structural calculation, and it is easily yielded from the equation of motion. Therefore the energy balance equation is suitable for evaluation and monitoring of dynamic response. In addition, since energy indicates cumulative information of motion, the energy balance equation is suitable to evaluate cumulative damage such as fatigue.

This paper describes cumulative damage evaluation based on the energy balance equation. At first, energy balance equations of both SDOF and MDOF are introduced. Next relationship between energy and fatigue failure is investigated by forced vibration experiments and theory based on fatigue failure field. Finally Miner's law is adapted to energy in order to evaluate seismic damage against various waves.

2. ENERGY BALANCE EQUATION

2.1. Energy Balance Equation for Single-Degree-of-Freedom model

The energy balance equation is simply derived from the equation of motion as follows. Therefore the energy balance equation is able to explain vibration characteristics dynamically. Equation (1) shows the equation of motion of single-degree-of-freedom model.

$$m\ddot{x} + c\dot{x} + F(x) = -m\ddot{z}_H \tag{1}$$

Where x is the relative displacement from ground, m is the mass, c is the damping coefficient, $F(x)$ is the restoring force, and $\ddot{z}_H$ is the horizontal acceleration of ground motion. Multiplying Eqn. (1) by displacement increment dx (= $\dot{x}dt$) leads to the work of the time dt, as shown in Eqn. (2).

$$m\ddot{x}\dot{x}dt + c\dot{x}^2 dt + F(x)\dot{x}dt = -m\ddot{z}_H\dot{x}dt \tag{2}$$

Finally the energy balance equation is yielded by the time integral of Eqn. (2), as shown in Eqn. (3).

$$\begin{gathered} m\int_0^t \ddot{x}\dot{x}dt + c\int_0^t \dot{x}^2 dt + \int_0^t F(x)\dot{x}dt = -m\int_0^t \ddot{z}_H\dot{x}dt \\ W_k + W_d + W_e + W_p = E \end{gathered} \tag{3}$$

Where,

$$\begin{aligned} W_k &= m\int_0^t \ddot{x}\dot{x}dt \\ W_d &= c\int_0^t \dot{x}^2 dt \\ W_e + W_p &= \int_0^t F(x)\dot{x}dt \\ E &= -m\int_0^t \ddot{z}_H\dot{x}dt \end{aligned} \tag{4}$$

In Eqns. (3) and (4), W_k is the kinetic energy, W_d is the dissipation energy by viscous damping, W_e is the elastic strain energy, W_p is the cumulative plastic energy and E is the input energy. The energy balance equation, Eq. (3), shows a sum of the work till the time t although the equation of motion shows the momentary condition in a time t. As shown Eqns. (3) and (4), energy is expressed by integral, so the energy balance is adequate to investigate the influence of the cumulative load because it includes the cumulative information.

The behavior of the energy is shown in Fig. 1. In the energy balance equation, only dissipation energy by viscous damping W_d and cumulative plastic energy W_p are cumulated. Kinetic energy W_k and elastic strain energy W_e converge with convergence of response. Therefore the sum of the dissipation energy W_d and the cumulative plastic energy W_p is equal to the input energy E after vibration of system finished. Additionally input energy E is easy to calculate because it consists of parameters easy to measure. Therefore input energy E is important energy in the energy balance equation, so input energy E is specially focused on in this study.

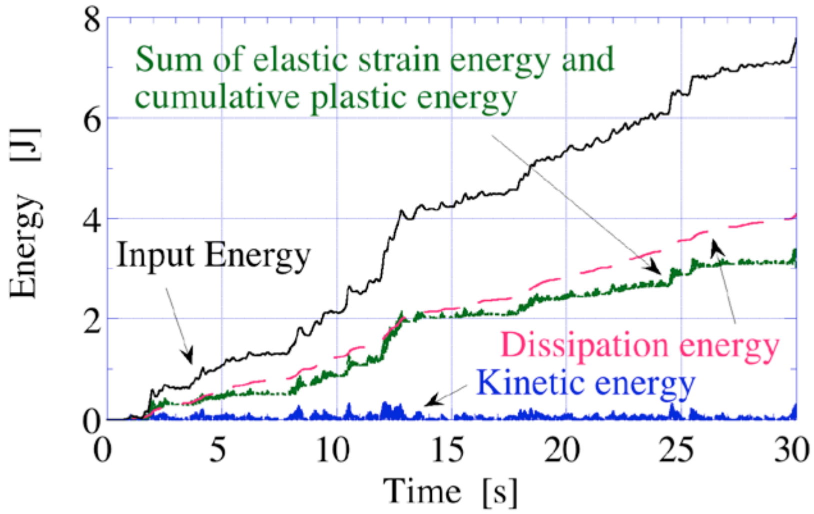

Figure 1. Behavior of energy

2.2. Energy Balance Equation for Multi-Degree-of-Freedom model

Some methods of energy balance equation for multi-degree-of-freedom model have been proposed. One is based on the equation of motion expressed in the matrix format [4]. However the method based on the matrix format cannot express energy of each mass. Thus authors have proposed another method that is focused on each mass point [5].

The energy balance equation focused on each mass point is obtained by same procedure as single-degree-of-freedom. In this method, relative relationship among mass points is focused on. The relative displacement of *i*th and *i*-1th mass point $x_i - x_{i-1}$ replaces with Y_i, the sum of displacement of *i*th mass point and ground $x_i + z_H$ replaces with Z_i. Equations (5)-(7) show the energy balance equation of *i*th mass point of *N* degree-of-freedom model.

$i = 1:$

$$\int_0^t m_1 \ddot{x}_1 \dot{x}_1 dt + \int_0^t c_1 \dot{x}_1^2\, dt + \int_0^t k_1 x_1 \dot{x}_1\, dt = -\int_0^t m_1 \ddot{z}_H \dot{x}_1\, dt - \int_0^t \sum_{k=2}^{N} \left(m_k \ddot{Z}_k \right) \dot{x}_1 dt \tag{5}$$

$i = 2, \cdots, N-1:$

$$\int_0^t m_i \ddot{Y}_i \dot{Y}_i\, dt + \int_0^t c_i \dot{Y}_i^2\, dt + \int_0^t k_i Y_i \dot{Y}_i\, dt = -\int_0^t m_i \ddot{Z}_{i-1} \dot{Y}_i\, dt - \int_0^t \sum_{k=i+1}^{N} \left(m_k \ddot{Z}_k \right) \dot{Y}_i\, dt \tag{6}$$

$i = N:$

$$\int_0^t m_N \ddot{Y}_N \dot{Y}_N dt + \int_0^t c_N \dot{Y}_N^2\, dt + \int_0^t k_N Y_N \dot{Y}_N\, dt = -\int_0^t m_N \ddot{Z}_{N-1} \dot{Y}_N\, dt \tag{7}$$

In the Eqns. (5)-(7), the 1st term of the left hand side is a kinetic energy, the 2nd term is a dissipation energy by viscous damping, the 3rd term is a sum of a elastic strain energy and the 1st term of the right hand side is an input energy. In the Eqns. (5) and (6), 2nd term of the right hand side can be construed as an transferred energy from upper to *i*th mass point. Energy of each individual mass point is clarified by using this method.

2.3. Hysteresis Energy

Hysteresis energy is known in the traditional fatigue failure field. The hysteresis energy is area of the hysteresis loop per cycle. In the elastic deformation, stress is directly proportional to strain, so that the hysteresis loop is not shaped. On the other hand, in the plastic deformation, the relationship between stress and strain is non-linear, so that the hysteresis loop is shaped. The area of the hysteresis loop is the total sum of the product of the stress and strain per cycle. Therefore hysteresis energy indicates quantity of energy absorption per cycle by plastic deformation.

3. RELATIONSHIP BETWEEN ENERGY AND FAILURE

3.1. Investigation by Vibration Experiment

In order to investigate relationships between energy and fatigue failure, a vibration experiment was carried out.

3.1.1. Experimental procedure
A simple single degree of freedom model shown in Fig. 2 was used as an experimental model. The experimental model consists of mass and a pole. Weight of the mass with an accelerometer was 0.204 kg. The length of the pole was 0.157 m, and cross-section of the pole was rectangle that has width of 0.012 m and breadth of 0.003 m. The experimental model was made of stainless steel Japanese Industrial Standard (JIS) SUS304. The nominal natural frequency and the damping ratio were 16.3 Hz and 0.73 %, respectively.

Random waves having predominant frequency similar to the natural frequency of the experimental model were input. The predominant frequency of the random wave was 13 to 16 Hz. The time length of the random wave was 30seconds, and the random wave was input repeatedly. So experimental models vibrate in resonance condition. An excitation was continued until fatigue failure of the experimental model. This excitation was repeated in various input amplitude, that is 18.8, 21.9, 25.5, 28.8, 31.1 m/s^2.

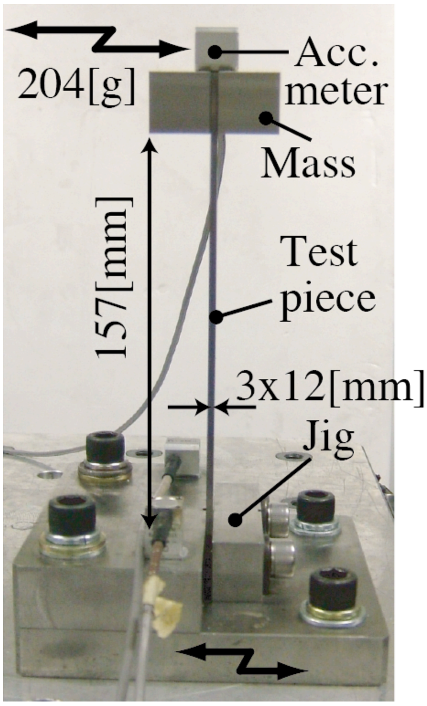

Figure 2. Experimental model

3.1.2. Experimental results
Experimental results were arranged from the viewpoint of energy and failure as shown in Fig. 3. The

relationships were put in order from the viewpoint of "time for failure and input energy for failure", and "increment of input energy and input energy for failure". From Fig. 3, it is confirmed that input energy for failure is proportional to time for failure, and is inversely proportional to increment of input energy. In other words, input energy for failure is inversely proportional to the maximum input acceleration. In addition, they are in good agreement with regression curves by power function. Therefore relationships between input energy and fatigue failure were confirmed.

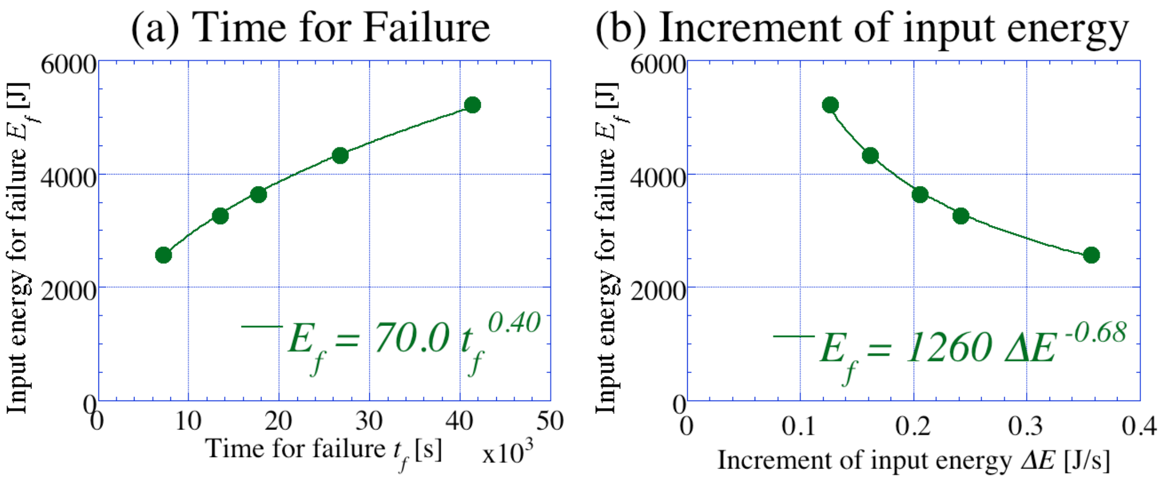

Figure 3. Relationships between energy and failure

3.2. Theoretical Investigation

Relationship between energy and fatigue failure is investigated from the hysteresis energy. Morrow [6] has reported a relationship between the hysteresis energy ΔW_h and a fatigue life N_f as shown in Eqn. (8).

$$\Delta W_h = aN_f^d \tag{8}$$

Where, a, d are constants and these have ranges of $a > 0$ and $-1 < d < 0$ [7] [8]. In addition, the total hysteresis energy to fatigue failure W_h is presented as Eqn. (9).

$$W_h = N_f \Delta W_h = aN_f^{d+1} \tag{9}$$

Eliminating fatigue life N_f from Eqns. (8) and (9) lead to the following equation.

$$W_h = \frac{\Delta W_h^{1+\frac{1}{d}}}{a^{\frac{1}{d}}} \tag{10}$$

In the above-mentioned experimental results, time for failure t_f is proportional to fatigue life N_f, and input energy for failure E_f is hysteresis energy to fatigue failure W_h. Therefore a relation between input energy for failure E_f and the time for failure t_f is estimated as following power function by referring to Eqn. (11).

$$E_f = pt_f^q \tag{11}$$

Where, p, q are constants that depend on material. It is supposed that a relation between an input energy E of energy balance equation and an increment of input energy ΔE is equivalent to the relation between the total hysteresis energy for fatigue failure W_h and the hysteresis energy ΔW_h shown in Eqn. (10). Thus the relation between the input energy E and the increment of input energy ΔE is assumed as following power function by referring to Eq. (10).

$$E_f = v\Delta E^w \tag{12}$$

Where, v is a positive proportional constant, w is a negative proportional constant. As shown in Fig. 3, relationships between energy and failure that were confirmed from the experiment were good agreement with regression curves calculated by Eqns. (11) and (12). Therefore relationships between energy and failure are expressed by power functions shown in Eqns. (11) and (12).

4. APPLICATION OF MINER'S LAW TO ENERGY

Relationships between energy and failure were confirmed by vibration experiment in section 3.1. However the input wave had the constant amplitude until failure, so that application method for various input level is required. Therefore, Miner's law is applied to energy balance equation in order to estimate the fatigue life for various input level. The Miner's law is known in the field of fatigue strength.

4.1. Application Method

In Miner's law, fatigue life is estimated by fatigue damage n_i / N_{fi}, and fatigue failure occurs when total sum of fatigue damage exceeds 1. Where, N_{fi} is a fatigue life when stress amplitude is σ_i, and n_i is the number of excitation when stress amplitude is σ_i. That is to say, the condition of the fatigue failure is the following;

$$\sum_i \frac{n_i}{N_{fi}} \geq 1 \tag{13}$$

For the energy balance equation, fatigue life against various input level is estimated by energy cumulative frequency $D_{Ei} = \Delta E_i\, t_i$ / E_{fi}, and fatigue failure occurs when energy cumulative damage D_{Ef} exceeds 1. Where E_{fi} is input energy for failure when increment of the input energy is ΔE_i, and t_i is time for failure when increment of the input energy is ΔE_i, and D_{Ef} is total sum of the energy cumulative frequency D_{Ei}. That is to say, the condition of the fatigue failure is the following;

$$D_{Ef} = \sum_i D_{Ei} = \frac{\Delta E_1 t_1}{E_{f1}} + \frac{\Delta E_2 t_2}{E_{f2}} + \cdots + \frac{\Delta E_i t_i}{E_{fi}} + \cdots \geq 1 \tag{14}$$

4.2. Vibration Test

In order to confirm proposed technique, a vibration experiment having various input level was carried out. In these experiments, input level was changed on every 3600 seconds. The experiment is conducted by two patterns of combinations of the input level. The maximum input acceleration of pattern 1 was changed as ascending order, 22.1 -> 25.4 -> 28.3 -> 31.7 m/s^2, and pattern 2 was changed as descending order, 26.4 ->23.6 -> 21.1 ->18.4 -> 26.6 -> 23.7 m/s^2. The same experimental condition as section 3.1 was applied to this experiment. Thus the input energy for failure E_{fi} when increment of the input energy ΔE_i is estimated by the power function in the Fig. 6 (b)

Figure 4 shows experimental results. The left vertical axis indicates the maximum input acceleration, and the right vertical axis indicates the energy cumulative damage. From Fig. 4, it is confirmed that the experimental model fractured when D_{Ef} = 1.16 in the pattern 1, and when D_{Ef} = 1.14 in the pattern

2. These results satisfy Eqn. (14). Therefore it was confirmed that application of Miner's law to energy balance equation is applicable and fatigue life estimation using energy balance equation are effective. In addition, this technique is valid for evaluation of soundness after an earthquake, because the energy cumulative damage indicates a margin to fatigue failure.

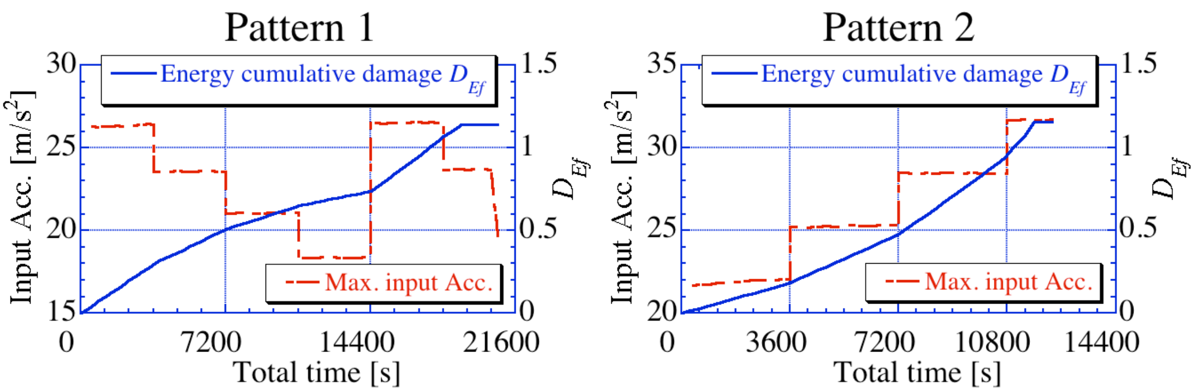

Figure 4. Confirmation of application of Miner's law

5. CONCLUSION

This paper describes cumulative damage evaluation based on the energy balance equation. The results are summarized as follows;

Energy balance equation for both single and multiple degree-of-freedom was introduced. The energy balance equation explains cumulative information of motion. Therefore it is suitable for evaluation of cumulative damage by earthquakes.

Input energy for failure is proportional to time for failure, and is inversely proportional to increment of input energy. These relationships are expressed by power functions.

Application of Miner's law to energy balance equation enables estimation of fatigue life for various input level. Moreover this technique enables evaluation of soundness after an earthquake, because the energy cumulative damage indicates a margin to fatigue failure.

In order to improve this method, quantitative estimation of energy for failure and failure point are required in the future.

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