Ground Motion of Arbitrary Inclusion and a Crack in Half Space under Incident SH-waves

Z.L. Yang, H.N. Xu & Y. Yang
College of Aerospace and Civil Engineering, Harbin Engineering University, Harbin,15001, China

SUMMARY:
The problem of SH-waves scattering caused by a subsurface arbitrary shape of inclusion and a nearby crack is studied in current paper based on the methods of conformal mapping, Green's function and multi-polar coordinates. Firstly, the wave field scattered by the inclusion under incident SH-waves is deduced according to zero-stress condition at horizontal interface. A suitable Green's function, the essential solution to the displacement field for the elastic space possessing an inclusion while bearing out-plane harmonic line loads at an arbitrary point, is then constructed to create a crack combined with the technique of "crack-division" method. Thus expressions of displacement and stress are established at the existence of the crack and the inclusion under incident SH-waves. Finally, some numerical examples are given to present variation of the ground motion of horizontal surface in the elastic half space containing arbitrary shape of inclusion and a crack with respect to different parameters.

Keywords: SH-waves, Inclusion, Crack, Green's function, Crack-division, Ground motion

1. INTRODUCTION

Elliptical inclusion commonly appears in matrix in earthquake engineering. Since nature is in eternal motion and change, the media, artificial materials or structures inevitably produce various defects and uneven distribution of the formation also give rise to large amounts of heterogeneity, such as small faults, joints, multiple lens, etc., which distribute arbitrarily. These structures with diverse distribution can be simplified to be cracks in engineering practice. Therefore, study on interaction of inclusion and cracks under SH wave is of great significance and can provide references for earthquake engineering, which has been reported by numerous specialists. In this paper, the earthquake resistance problem of SH-wave scattering caused by an elliptical inclusion and a crack in elastic half-space is investigated scrupulously.

2. STATEMENT OF THE PROBLEM

Figure 1 shows an elastic model with an elliptical inclusion and a crack under incident SH-wave with a incident angle α. Media Ⅰ and media Ⅱ have different material constants (ρ_1, μ_1; ρ_2, μ_2). Three coordinates are given in the figure and the relation of them is defined by

$$\begin{aligned} & x' = x, y' = y - h, x_1 = x\cos\beta + y\sin\beta, \\ & y_1 = y\cos\beta - x\sin\beta, h_{11} = \left(h_1 + b\sin\beta\right)/\cos\beta . \end{aligned} \tag{1}$$

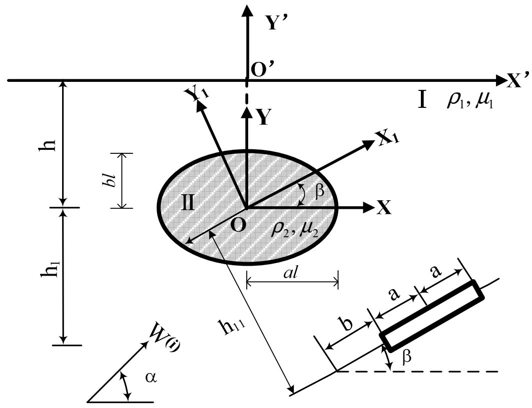


Figure 1. Half Space Model with an Elliptical Inclusion and a Crack under Incident SH-wave

3. THE SCATTERING WAVES AROUND THE ELLIPTICAL INCLUSION

3.1. Governing Equation

In an isotropic medium, to study the scattering of SH-wave is the easiest issue of elastic wave scattering. Inducing conformal mapping function $z = \omega(\eta) = R(\eta + m/\eta)$ ($\eta = Re^{i\theta}$, $R = (al + bl)/2$, $m = (al - bl)/(al + bl)$, al and bl represent the length of semi-major axis and semi-minor axis of the elliptical inclusion, respectively), the displacement W impacted by the incident wave should satisfy the governing equation

$$\frac{1}{\omega'(\eta)\overline{\omega'(\eta)}}\frac{\partial^2 W}{\partial\eta\partial\overline{\eta}} + \frac{k^2}{4}W = 0 \tag{2}$$

in which, $k=\omega/c_s$, ω is the circular frequency of the displacement function, $c_s = \sqrt{\mu/\rho}$ stands for the velocity of the shear wave. ρ and μ are the mass density and shear modulus of the medium respectively.

In polar coordinate, the corresponding stresses are given by

$$\tau_{rz} = \frac{\mu}{R|\omega'(\eta)|}(\eta\frac{\partial W}{\partial\eta} + \overline{\eta}\frac{\partial W}{\partial\overline{\eta}}), \quad \tau_{\theta z} = \frac{i\mu}{R|\omega'(\eta)|}(\eta\frac{\partial W}{\partial\eta} - \overline{\eta}\frac{\partial W}{\partial\overline{\eta}}) \tag{3}$$

3.2. The Scattering Wave in District Ⅰ

According to the symmetry of the scattering wave, multi-polar coordinates is applied to construct the scattering wave in the medium induced by the elliptical inclusion, which should satisfy the governing equation (2) and Sommerfeld radiation condition for infinite distance beside the zero-stress condition at the horizontal interface.

In η plane, $W_1^{(s)}$ can be expressed by

$$W_1^{(s)} = \sum_{n=-\infty}^{+\infty} A_n \left\{ H_n^{(1)}[k_1 |w(\eta)|] \left[\frac{w(\eta)}{|w(\eta)|} \right]^n + H_n^{(1)}[k_1 |w(\eta) - 2ih|] \left[\frac{w(\eta) - 2ih}{|w(\eta) - 2ih|} \right]^{-n} \right\} \tag{4}$$

where A_n are the unknown coefficients, determined by the boundary condition of the elliptical inclusion.

3.3. The Standing Wave in District II

In the mapping plane, the standing wave inside of elliptical inclusion is deduced as

$$W_{\mathrm{II}}^t = \sum_{n=-\infty}^{\infty} B_n J_n \left(k_2 |w(\eta)| \right) \left[\frac{w(\eta)}{|w(\eta)|} \right]^n \tag{5}$$

where B_n is the unknown coefficient determined by the boundary condition.

4. GREEN'S FUNCTION

The Green's function G_1 denotes an essential solution to the displacement field for an elastic half-space containing an elliptical inclusion while bearing out-plane harmonic linie loads at arbitrary point, which is expressed as $e^{-i\omega t}$ and satisfies the governing equation (2).

In a half elastic space, the incident wave $G^{(i)}$ excitated by the out-plane line load takes the form of

$$G_1^{(i)} = \frac{i}{4\mu_1} H_0^{(1)}(k_1 |w(\eta) - w(\eta_0)|) \tag{6}$$

The reflected wave field caused by horizontal interface can be indicated as $G^{(r)}$, then

$$G_1^{(r)} = \frac{i}{4\mu_1} H_0^{(1)}(k_1 \left| w(\eta) - \overline{w(\eta_0)} - 2ih \right|) \tag{7}$$

The scattering wave $G_{\mathrm{I}}^{(s)}$ excited by the inclusion and the standing wave G_{II}^t in the inclusion take the forms of Eq.(4) and Eq.(5), respectively.

In the complex plane$(z, \bar{z})$, the boundary condition can be expressed as

$$\begin{cases} G_1 = G_{\mathrm{II}}^t \\ \tau_{rz,1} = \tau_{rz,\mathrm{II}}^t \end{cases} \tag{8}$$

Substitution of Eqs.(6), (7) and the expressions of $G_{\mathrm{I}}^{(s)}$ and into Eq.(3), the corresponding stresses can be derived, then substituting these expressions into boundary conditions (8), the coefficients A_n and B_n will be solved.

The total wave field is

$$
\begin{aligned}
G_1 &= G^{(i)} + G^{(r)} + G_{\mathrm{I}}^{(s)} \\
&= \frac{i}{4\mu} H_0^{(1)}(k_1 \left| w(\eta) - w(\eta_0) \right|) + \frac{i}{4\mu} H_0^{(1)}(k_1 \left| w(\eta) - \overline{w(\eta_0)} - 2ih \right|) \\
&+ \sum_{n=-\infty}^{+\infty} A_n \left\{ H_n^{(1)}[k_1 \left| w(\eta) \right|] \left[\frac{w(\eta)}{\left| w(\eta) \right|} \right]^n + H_n^{(1)}[k_1 \left| w(\eta) - 2ih \right|] \left[\frac{w(\eta) - 2ih}{\left| w(\eta) - 2ih \right|} \right]^{-n} \right\}
\end{aligned} \tag{9}
$$

5. SCATTERING OF SH-WAVE BY ELLIPTICAL INCLUSION AND CRACK NEAR INTERFACE

This study can be treated as the problem of earthquake resistance. Showed as Figure 1, a steady SH-wave is incident from downside with an angle α in half-space, a reflected SH-wave $W^{(r)}$ occurs owing to the interface. Introducing a conformal mapping function $z = \omega(\eta)$, in mapping plane η, $W^{(i)}$ and $W^{(r)}$ take the forms of

$$
\begin{aligned}
W^{(i)} &= W_0 \exp\left\{ \frac{ik_1}{2} \left[\left(w(\eta) - ih \right) e^{-i\alpha} + \left(\overline{w(\eta)} + ih \right) e^{i\alpha} \right] \right\} \\
W^{(r)} &= W_0 \exp\left\{ \frac{ik_1}{2} \left[\left(w(\eta) - ih \right) e^{i\alpha} + \left(\overline{w(\eta)} + ih \right) e^{-i\alpha} \right] \right\}
\end{aligned} \tag{10}
$$

where W_0 is amplitude of incident wave, α represents the incident angle.

The scattering wave $W_{\mathrm{I}}^{(s)}$ and the standing wave W_{II}^{t} excited by the elliptical inclusion can be described as Eqs.(4) and (5), respectively. And the process of solving A_n and B_n is the same as that of the Green's function discussed in preceding paper

$$
\begin{cases}
W^{(i)} + W^{(r)} + W_{\mathrm{I}}^{(s)} = W_{\mathrm{II}} \\
\tau_{rz,\mathrm{I}} = \tau_{rz,\mathrm{II}}
\end{cases} \tag{11}
$$

The total wave field of domain I can be obtained

$$
W_{\mathrm{I}} = W^{(i)} + W^{(r)} + W_{\mathrm{I}}^{(s)} \tag{12}
$$

The total stresses can be also performed. If additional stresses which have same magnitude but opposite in direction are applied at the same point, the ultimate stresses will be zero. Therefore, when a pair of forces with the same magnitude but opposite in direction are loaded at the region where the crack will be created, the resultant forces will be zero, then a crack is created.

Then we can obtain the total wave field in domain I when a crack coexists with the inclusion

$$
W_{\mathrm{I}}^{(t)} = W^{(i)} + W^{(r)} + W_{\mathrm{I}}^{(s)} - \int_{(b,-h_{11})}^{(2a+b,-h_{11})} \tau_{\theta z,\mathrm{I}} G_1 dz_1 \tag{13}
$$

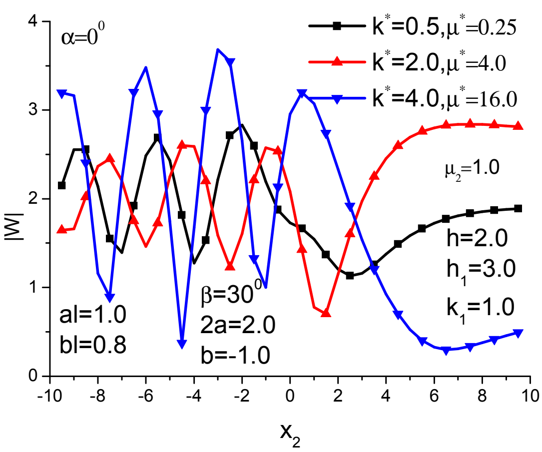
6. RESULTS AND DISCUSSION

Numerical examples are provided here to discuss influence rule of various parameters such as the wave number k_1, the incident angle α, the ratio of wave number $k^* = k_1/k_2$, the ratio of shear modulus $\mu^* = \mu_2/\mu_1$ *et al.* on gound motion of horozontal surface. The expression of surface displacement is difined as Eq. (13).

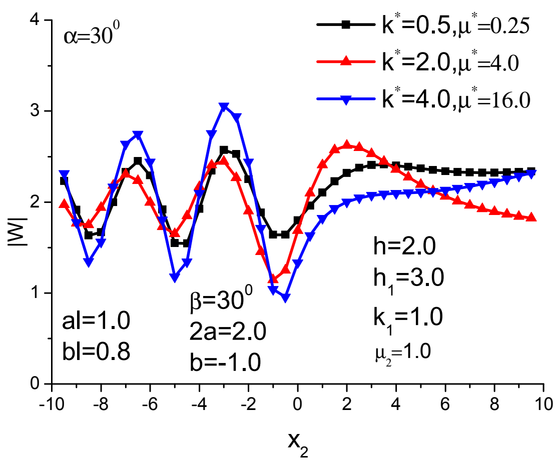
Figure 2 illustrates that the hader the inclusion is, the more strongly the surface displacement $|W|$ varies. Plotted by Figure 3, in the case of $k_1 = 0.1$, namely quasi-static, $|W|$ changes slightly and the magnitude keeps around 2.0, while k_1 increases, the variation $|W|$ shows more and more obvious oscillation characteristics. Under circumstances of $\alpha = 0^{\circ}$, 30° and 45°, with the increasing of k_1, $|W|$ of left-side varis more strongly, while SH wave is incident veritically, $|W|$ has symmetry change, as shown as Figure 3(b), (c) and (d).

From Figure 4(a) to Figure 4(e), it can be demonstrated that the crack angle β influences slightly on horizongtal surface displacement; in addition, as the burial depth h of the inclusion increases, the variation of $|W|$ tends to be stable. Seen from Figure 4(b), (d) and (f), as the crack angle increases, $|W|$ changes more and more distinctly and then tends to be stable with h_1. $|W|_{\min}$ appears at $x_2 = 0.0$ ($h_1 = 5.0$), about 1.45.

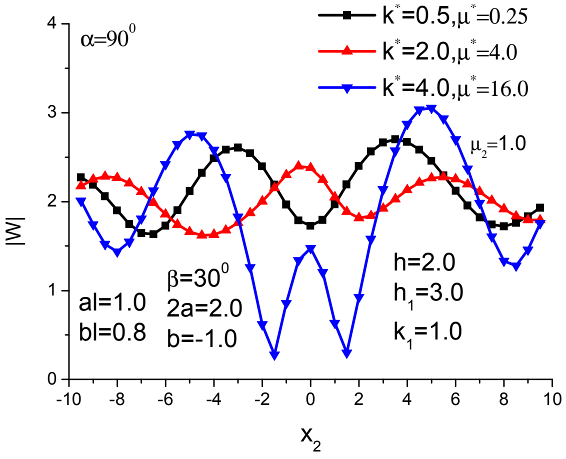
Shown as Figure 5, the increase of incident angle α, the variation curve of surface displacement $|W|$ prensents gradual amplification along x_2; besides, the increasing of the lengh of crack $2a$ gives rise to the augment of surface displacement $|W|$ of the side containing the crack, and $|W|_{\max}$ achieves 4.75 while SH wave is incident vertically. With bl/al increases, the amplitude of $|W|$ becomes great but not too much, which can be demonstrated in Figure 6.



(a)

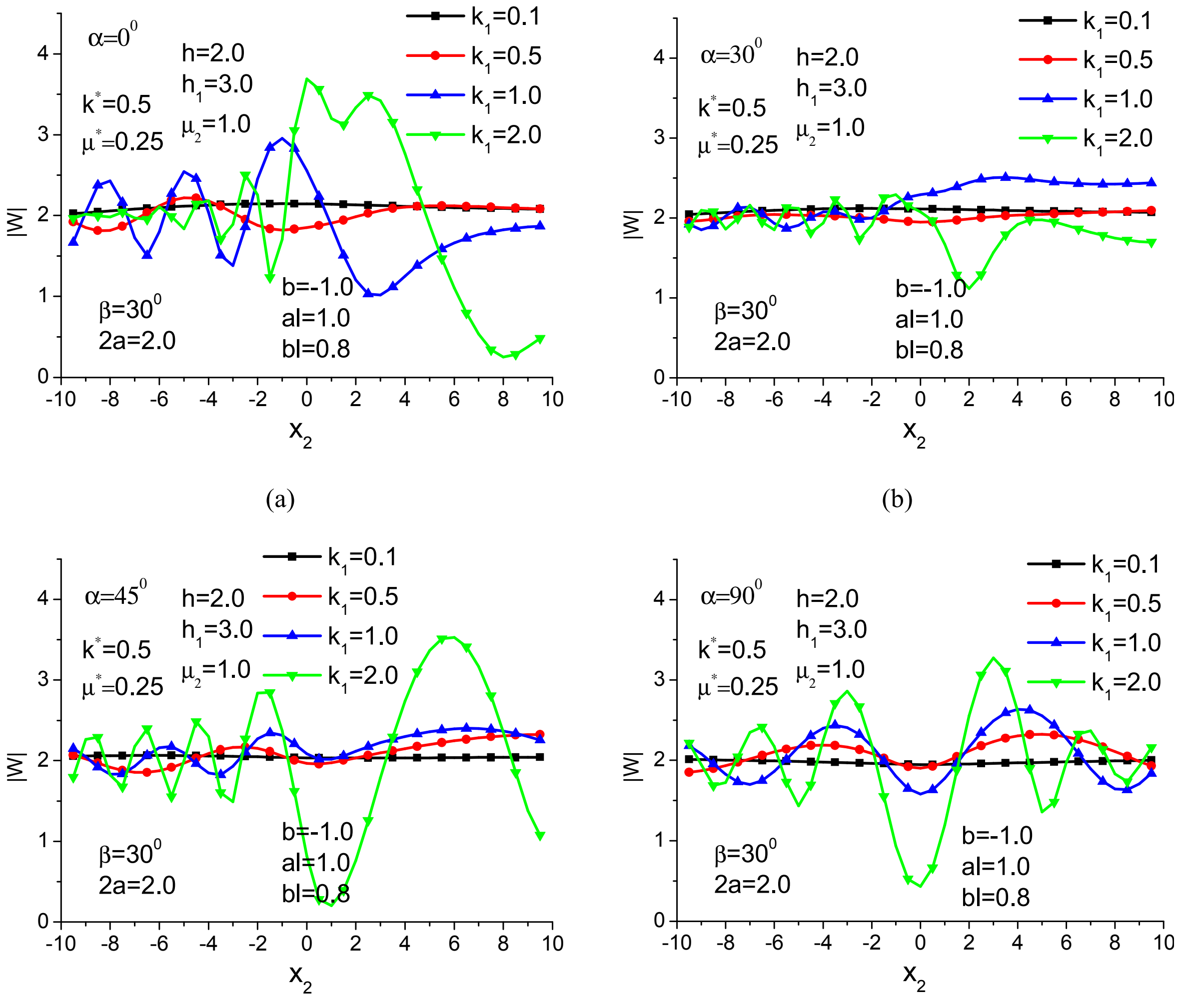


(b)



(c)

Figure 2. Variation of $|W|$ with the Ratio of Matrix to Inclusion



(a) (b)

(c) (d)

Figure 3. Variation of $|W|$ with k_1

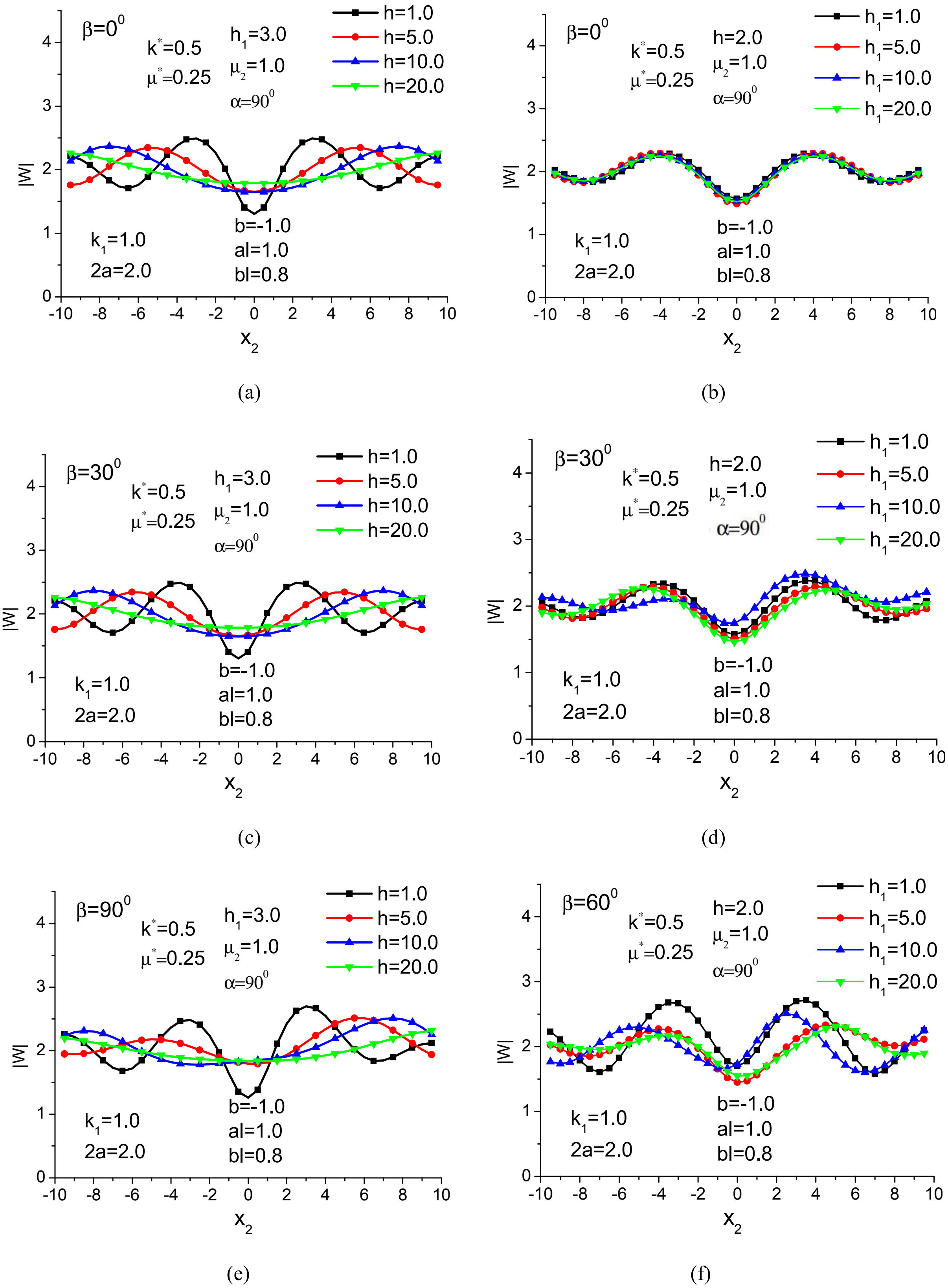
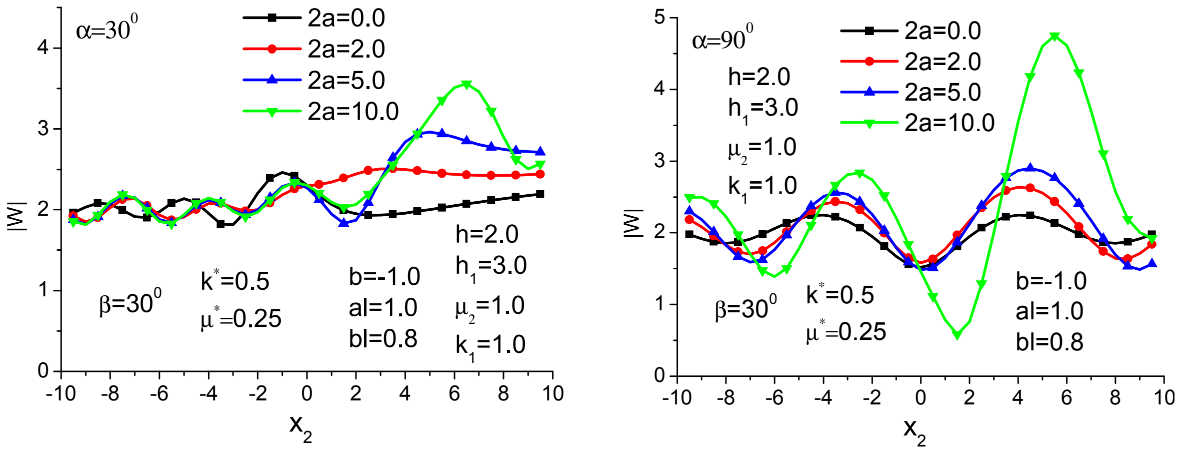


Figure 4. Variation of $|W|$ with h and h_1



(a) (b)

Figure 5. Variation of $|W|$ with $2a$

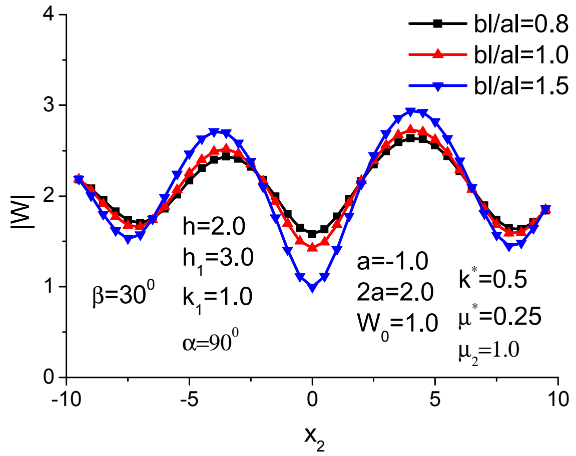


Figure 6. Variation of $|W|$ with bl/al

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