Effect of Diagonal Modes on Response Spectrum Analysis

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Summary:
Three dimensional structural models of many structures possessing symmetry in two horizontal directions, exhibit diagonal modes in the lateral directions. These diagonal modes occur in pair in horizontal directions, they are orthogonal to each other and the two diagonal modes have same natural period. Structures with diagonal modes, when analysed using response spectrum analysis (RSA), gives seismic response, which is inconsistent and it does not match with the seismic response obtained using time history analysis (THA). When a symmetric structure with diagonal modes is subjected to base excitation in X- horizontal direction, then in RSA, one gets response in Y-direction also. However, in THA one does not get response in Y-direction. It is shown that this inconsistency in response from RSA occurs due to the loss of sign of modal responses in the modal combination rules. On the other hand, in THA, the modal responses are added algebraically, hence, one does not get response in Y-direction, when base excitation is applied in X-direction. Effect of various parameters on diagonal modes is also studied. Simple, practical remedies to eliminate diagonal modes, without any loss of accuracy are suggested.

Keywords: Diagonal Modes, Base Reaction, Response Spectrum Analysis, Time History Analysis,

1. INTRODUCTION

Damages to structures during earthquakes continue to provoke more efforts from structural engineers towards better seismic performances. Research efforts are being made to understand earthquake loading properly and to make structural analysis more and more refined. With the availability of computing machines, analysis and design of structures is being carried out using computer software. In a typical software oriented structural analysis, structural model is prepared using appropriate elements. For a framed building, modeling comprises of beam and column and a rigid diaphragm to account for the in-plane rigidity of slab at each floor (Agarwal and Shrikhande 2006). Usually in computer oriented structural analysis, three-dimensional models of buildings are used. After achieving a reasonably good structural model, next stage is to use appropriate analysis method for seismic response. In India, IS 1893(Part 1): 2002, is the standard for calculating earthquake loads on the structures. In this Indian Standard, three methods of analysis are given.

In the first method, which is used for most of the buildings, static earthquake loads are obtained at each floor of building using empirical time period. This method, termed as equivalent static analysis (ESA), is very easy to use and is based on empirical time period and empirical distribution of earthquake loads on each floor along the height of the building. Next method given in IS 1893 is Response spectrum analysis (RSA), wherein, from the structural model of building, natural frequencies and natural modes are obtained. For this purpose, free vibration analysis is performed, wherein mass of structure are to be properly modeled. The mass of slab and mass corresponding to appropriate amount of imposed load are considered along with the mass of beam and column. Using natural frequencies and mode shapes, static earthquake loads and response in each mode are obtained. These modal responses are combined using any one of the combinational rules, i.e. Sum of square root of squares (SRSS), Combined quadratic combination (CQC) and Absolute sum. In the third method of analysis, i.e., time history analysis (THA), dynamic response is obtained by using either modal superposition method or numerical integration method. Here time history of ground acceleration is used and dynamic response in the form of time history of response is obtained. It is to be noted that if modal superposition method is used to obtain dynamic response, then modal responses are combined using algebraic sum.

The study presented in this paper pertains to RSA of three dimensional model of structures which possess diagonal modes. Certain structures with symmetry in two horizontal directions exhibit diagonal modes. These diagonal modes are characterized by the fact that they are diagonal in nature in plan, they occur in pair and two diagonal modes have same natural frequency. If such pair of diagonal modes is present, then in RSA, they lead to altogether different response of structure as compared to response obtained from THA (Nasurde, 2010). In the present study, a detailed discussion on when diagonal modes occur and by which factor they get influenced is presented. These diagonal modes could occur because of numerical and/or rounding off in numerical calculation of modal property by computer software. Possible remedies to overcome these diagonal modes are also discussed.

2. WHAT ARE DIAGONAL MODES?

When three-dimensional model of a structure is analyzed, one gets modes in all the three directions. Depending on geometry of structure, one gets modes which are explicitly in one of the three directions. In some cases, depending on torsional coupling, one gets modes which are combined in two lateral directions (Fig 2.1). In certain geometry, particularly these with square plan and square sizes of column have diagonal modes, i.e. modes has deformation in both the horizontal (or plan) direction as shown in Fig 2.2. These types of diagonal modes occur in pair, i.e. when using the two diagonal modes having same natural frequency and mass excited in those both the lateral directions.

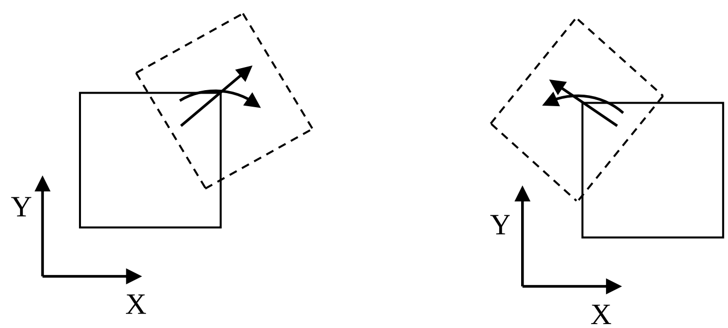


Figure 1. Coupled modes in plan

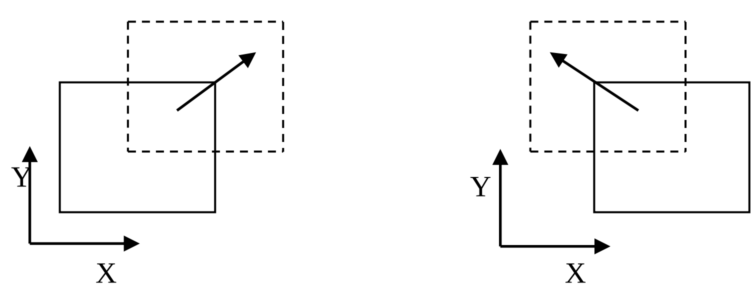


Figure 2. Diagonal modes in plan

The diagonal modes are described with the help of an example building. Consider a building plan with four columns and two floors (Fig 2.3). Column of size 300mm x 300mm and beam size of 230mm x 300mm and a load of 200kN is applied on all four junction points on both the floor levels. Grade of concrete is M25, poisons ratio is 0.17. Mass of only loads is considered and mass density of beam and column is taken as zero. Rigid diaphragm for in-plane rigidity is applied at floor levels. Results of free vibration analysis are shown in Table 2.1.

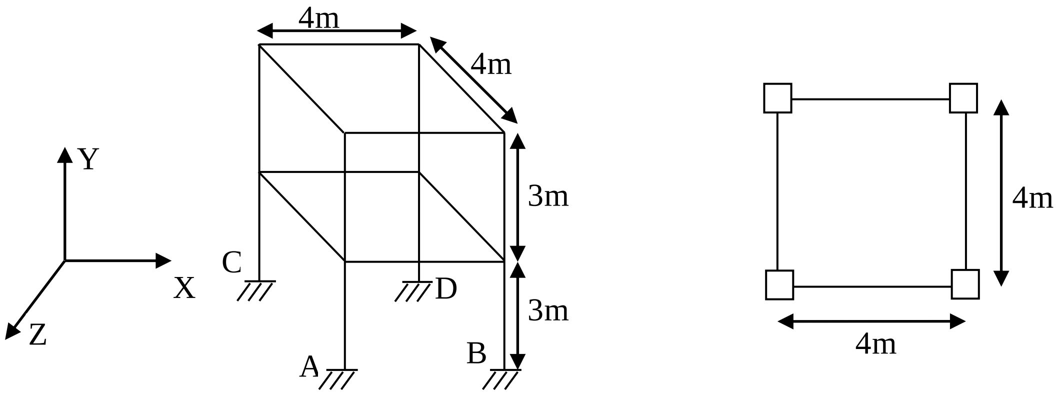


Figure 3. Geometric details of building with square columns

Table 2.1. Modal mass and time period of building with square columns

Mode No	Time Period (Sec)	Modal Mass (%)		
		M_X	M_Y	M_Z
1	0.85	69	18	0
2	0.85	18	69	0
3	0.75	0	0	0
4	0.24	12	0.2	0
5	024	0.2	12	0

From Table 2.1 it is seen that period of 1st and 2nd modes is same and modal mass in X- and Y-direction got interchanged in these two modes. These modes are the ones which are termed as diagonal modes. These diagonal modes are present in certain types of geometry only. For the building considered above, there is symmetry in both the horizontal directions and columns are square type. These diagonal modes will vanish if columns are of rectangular type. In the above building, if rectangular columns of 300 x 400 mm are used, then the free vibration results are as shown in Table 2.2. It is seen that in the 1st mode mass is excited only in X-direction and in the 2nd mode mass is excited in Y-direction only and diagonal modes are not present.

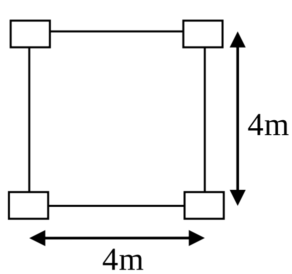


Figure 2.4. Plan of a building with rectangular columns

Table 2.2. Modal participating mass ratios and time period of a building with rectangular columns

Mode No	Time Period (Sec)	Modal Mass (%)		
		M_X	M_Y	M_Z
1	0.559	0	87	0
2	0.489	85	0	0
3	0.448	0	0	0
4	0.152	0	13	0
5	0.125	14	0	0

3. EFFECT OF DIAGONAL MODES ON RESPONSE SPECTRUM ANALYSIS

In the previous section it is seen that, diagonal modes are present for building with square plan and square columns, i.e., symmetry in both the horizontal directions exist. In these diagonal modes, which occur in pair, natural periods of both the modes are same and modal mass excited in two lateral directions gets interchanged. These are the typical characteristics of diagonal modes. Building shown in Fig. 3 is analyzed using RSA for seismic loading in X-direction. Seismic Zone IV of IS 1893 (Part 1):2002 with medium soil type is considered. The importance factor (I) is 1.0, the response reduction factor (R) is 5.0. Structure is analyzed using response spectrum analysis and first five modes are used in the analysis. Modal responses are combined using square root of sum of squares (SRSS) method. Results on base reaction at four supports (A, B, C and D in Fig. 2.3) are shown in Table 3.1.

Table 3.1. Base reactions from response spectrum analysis and time history analysis

Support	Base Reactions (kN)			
	Response Spectrum Analysis (RSA)		Time History Analysis (THA)	
	X	Y	X	Y
A	11.683	8.066	14.91	0
B	11.683	8.066	14.91	0
C	11.683	8.066	14.91	0
D	11.683	8.066	14.91	0

It is seen that when response spectrum is applied in X-direction, reaction is present in X-direction and as well as in Y-direction. This is because in modal combination, reactions are squared, summed and then added. So the negative sign gets cancelled and reaction is present in both lateral directions. It is to be noted that the building is symmetric and response spectra was applied in X-direction and yet in RSA reactions are present in X- as well as Y-direction. This issue of getting response in orthogonal direction, even though base excitation is applied in another direction will not occur in time history analysis (THA), because in THA modal responses are combined using algebraic summation. In order to demonstrate this, the above building is also analyzed using time history analysis. For this purpose, base acceleration time history of El-Centro EQ is chosen. This time history is scaled to get peak ground acceleration of 0.024g, which is same as that of RSA in zone IV i.e. (Z/2*I/R)g. Time history of base reaction at support A in X- and Y-direction is shown Fig 3.1. Maximum value of base reaction from THA is shown in Table 3.1. It is seen that in THA, there is no reaction in Y- direction.

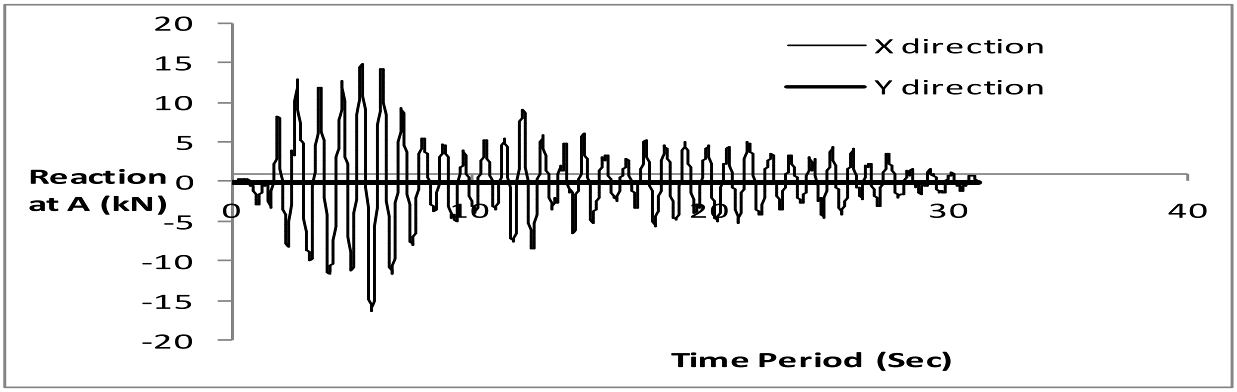


Figure 3.1. Time history for base reaction at joint A

4. PARAMETERS AFFECTING DIAGONAL MODES

Presence of diagonal modes is either due to structural characteristics or can be due to numerical errors/ rounding-off during the numerical calculations of modal properties. In order to ascertain the robustness of presence of diagonal modes, effect of various parameters on presence of diagonal modes is studied. Following parameters are varied.

1) Elastic Modulus E
2) Beam Sizes
3) Poisons Ratio µ
4) Loads on beams

4.1. Effect of Elastic Modulus

Same building is analysed for two different values of Young's modulus and results are shown in Table 4.1. It is seen that, when there is change in elastic modulus there is change in the percentage of modal mass excited, this makes the difference in base reaction. When the mass excited is nearly same in both the lateral directions, the base reaction is also approximately same in both directions. It is observed that the base reaction is also based on modal mass excited. This, with change in modulus, the diagonal modes do not vanish, but mass excited in the two lateral direction changes.

Table 4.1. Effect of elastic modulus on modal mass and base reaction

$Ex10^6$ kN/m^2	Mode No	Modal Mass (%)		Time Period (Sec)	Base Reaction (kN)	
		X	Y		X	Y
22.36	1	87	0.8	0.63	9.47	0.93
	2	0.8	87	0.63	9.47	1.31
	3	0	0	0.56	9.47	1.31
	4	5.7	6.4	0.18	9.47	1.51
	5	6.4	5.7	0.18	9.53	1.68
25	1	55	32	0.60	6.35	4.84
	2	32	55	0.60	7.34	6.84
	3	0	0	0.53	7.34	6.84
	4	13	0.8	0.17	7.48	6.85
	5	0.8	13	0.17	7.48	6.86

4.2. Effect of Beam Sizes

Same building is analysed for two different values of beam sizes and results are shown in Table 4.2. It is seen that, when there is change in beam size there is change in the percentage of modal mass excited, this makes the difference in base reaction. When the mass excited is nearly same in both the lateral directions, the base reaction is also approximately same in both directions. It is observed that the base reaction is also based on modal mass excited. This, with change in beam size, the diagonal modes does not vanish, but mass excited in the two lateral direction changes.

Table 4.2. Effect of beam sizes on modal mass and base reaction

Beam mm x mm	Mode No	Modal Mass (%)		Time Period (Sec)	Base Reaction (kN)	
		X	Y		X	Y
300 x 350	1	0.04	82	0.38	0.5	0.24
	2	82	0.04	0.38	10.6	0.35
	3	0	0	0.32	10.6	0.35
	4	4.9	8.7	0.10	10.6	0.35
	5	8.7	4.9	0.10	10.6	1.19
250 x 500	1	69	20	0.31	8.5	4.5
	2	20	69	0.31	8.8	6.4
	3	0	0	0.28	8.8	6.4
	4	4	6.7	0.09	8.8	6.5
	5	6.7	4	0.09	8.8	6.5

4.3. Effect of Poisons Ratio

Same building is analysed for two different values of poisons ratio and results are shown in Table 4.3. It is seen that, when there is change in poisons ratio there is change in the percentage of modal mass excited, this makes the difference in base reaction. When the mass excited is nearly same in both the lateral directions, the base reaction is also approximately same in both directions. It is observed that the base reaction is also based on modal mass excited. This, with change in poisons ratio, the diagonal modes do not vanish, but mass excited in the two lateral direction changes.

Table 4.3. Effect of poisons ratio on modal mass and base reaction

μ	Mode No	Modal Mass (%)		Time Period (Sec)	Base Reaction (kN)	
		X	Y		X	Y
0.15	1	0.3	87	0.60	0.03	0.59
	2	87	0.3	0.60	10.0	0.83
	3	0	0	0.53	10.0	0.83
	4	2.6	9.6	0.17	10.0	1.03
	5	9.6	2.6	0.17	10.1	1.20
0.25	1	87	0.5	0.60	10.0	0.79
	2	0.5	87	0.60	10.0	1.12
	3	0	0	0.53	10.0	1.12
	4	5.8	6.4	0.17	10.0	1.35
	5	6.4	5.8	0.17	10.1	1.54

4.4. Effect of Loads on the Structure

Building is analysed for two different values of loads are applied as four concentrated loads on both floors and results are shown in Table 4.4. It is seen that, when there is change in load there is change in the percentage of modal mass excited, this makes the difference in base reaction. When the mass excited is nearly same in both the lateral directions, the base reaction is also approximately same in both directions. It is observed that the base reaction is also based on modal mass excited. This, with change in load, the diagonal modes does not vanish, but mass excited in the two lateral direction changes.

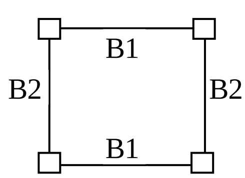
Table 4.4. Effect of loads on modal mass and base reaction

Load (kN)	Mode No	Modal Mass (%)		Time Period (Sec)	Base Reaction (kN)	
		X	Y		X	Y
100		55	32	0.60	6.3	4.8
	2	32	55	0.60	7.3	6.8
	3	0	0	0.53	7.3	6.8
	4	11	0.8	0.17	7.5	6.8
	5	0.8	11	0.17	7.5	6.9
200	1	69	19	0.85	10.9	5.7
	2	19	69	0.85	11.3	8.0
	3	0	0	0.75	11.3	8.0
	4	12	0.2	0.24	11.7	8.1
	5	0.2	12	0.24	11.7	8.1

5. REMEDIES FOR DIAGONAL MODES

From the above results, it is clear that if diagonal modes are depicted, then, in RSA, the results will be quite different than THA. In the presence of diagonal modes, the results from RSA are inconsistent. As mentioned earlier, the results are getting skewed due to modal combination rules, wherein, sign of modal responses is lost. If THA is used, then, even the building with diagonal modes does not show any skewed results. Thus, it will not be a bad idea, if diagonal modes can be somehow removed. In this context, one shall note that diagonal modes might be occurring due to numerical rounding-off error in the calculations of Eigen values and Eigen modes in the free vibration analysis. Here, simple remedy to eliminate diagonal modes is suggested. In this remedy, sizes of two opposite beams are changed slightly. Effects of this remedy on response are discussed next.

In the building described in section 2 (Fig. 2.3), sizes of two opposite beams are changed slightly as shown in Fig 5.1. All other dimensions and parameters are same. The depths of two beams are changed from 300 mm to 301 mm. The sizes of two opposite beams of all floor levels are changed. The modal characteristics of this model along with base reactions obtained from RSA are shown in Table 5.1. It is seen that the diagonal mode effect is completely removed and one gets modes predominantly in one direction only. In order to assess, what effect this change in beam size will cause to actual seismic response, THA of both the models, i.e., building with same beam sizes and building with different beam sizes is done and comparison of time history of base reaction is shown in Fig 5.2. It is seen that THA results of both the models are almost same. The maximum values of base reaction for two cases are shown in Table 5.2. As can be seen, results of both the modes are almost same. This shows that by slightly changing the sizes of two opposite beams, the diagonal modes are removed and there is no loss of accuracy in structural analysis results.



B1**:** 230 x 300 mm

B2**:** 230 x 301mm

Figure 5.1. Plan showing different beam sizes of a building

Table 5.1. Modal mass and time period results for structure with beams of different sizes

Mode No	Modal Mass (%)		Time Period (Sec)
	M_X	M_Y	
1	0	88	0.849
2	88	0	0.848
3	0	0	0.751
4	0	12	0.242
5	12	0	0.241

Table 5.2. Base reaction for RSA and THA for structure with beams of different sizes

Support	Base Reactions THA (kN)			
	Same Beams		Different Beams	
	X	Y	X	Y
A	14.91	0	14.91	0
B	14.91	0	14.91	0
C	14.91	0	14.91	0
D	14.91	0	14.91	0

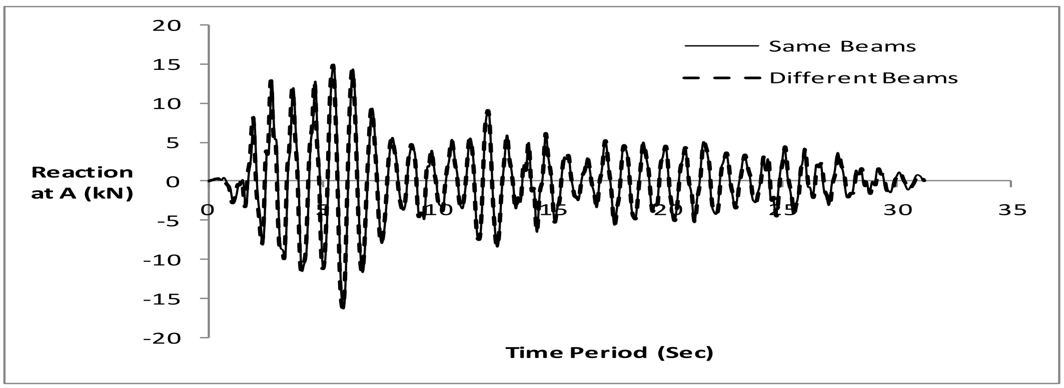


Figure 5.2. Comparison of time history at joint A for same size beams and different size beams.

6. DISCUSSION AND CONCLUSIONS

It is quite common to get diagonal modes in the free vibration analysis of three dimensional models of structures in the computer oriented structural analysis. These diagonal modes occur in structures which in plan possess symmetry in two horizontal directions. These diagonal modes occur in pair having same time period, and modal mass excited in x-direction of the first mode is same as the modal mass excited in y-direction of the second mode. When response spectrum analysis (RSA) of a structure with diagonal modes is performed with base excitation in X-direction, one gets response in X-direction and also in the perpendicular direction i.e. Y-direction. This is because in RSA, the modal responses lose their sign before they are added. In SRSS modal combination rule, modal responses are squared and then added, hence, they lose their sign Due to squaring the negative response of one diagonal mode becomes positive and gets added to positive response from other diagonal mode. If structure with diagonal modes is analysed using time history analysis, then one does not get response in perpendicular direction. This is so, because in THA, the modal responses are added algebraically. Thus, for structures with diagonal modes, the seismic response obtained using RSA is inconsistence and does not match with the results obtained from THA. In order to assess robustness of diagonal modes, a parametric study is performed. It is seen that if parameters like, Young's modulus, load on structure etc. are changed, then diagonal modes do not mass in two orthogonal directions may get changed but diagonal modes do not vanish. The diagonal modes could have occurred due to numerical rounding-off in the calculation of Eigen modes and Eigen values. It is desirable if these diagonal modes are eliminated. For this purpose, a very simple remedy in the form of changing slightly the depth of beams in any one direction is suggested. In the present example beam depth for beams parallel to Y-axis is changed from 300mm to 301 mm. With this small change, the diagonal modes vanish and there is almost no change in the response. This is ascertained by performing THA of model in which beam depth was changed slightly and results are compared with the model in which beam sizes were same.

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