Displacement-Based Design of Hysteretic Dissipative Braces for the Seismic Retrofitting of R.C. Framed Buildings

F. Mazza, M. Mazza & A. Vulcano
Dipartimento di Strutture, Università della Calabria, Rende (Cosenza), Italy

15 WCEE
LISBOA 2012

SUMMARY:
A design procedure is proposed for proportioning hysteretic dissipative braces in order to attain, for a specific level of seismic intensity, a designated performance level of a reinforced concrete (r.c.) framed building which has to be retrofitted. Exactly, a proportional stiffness criterion, which assumes the elastic lateral storey-stiffness due to the braces proportional to that of the unbraced frame, is combined with the Direct Displacement-Based Design, in which the design starts from a target displacement. Two different criteria are followed for distributing the stiffness and strength properties of dissipative braces, obtained on the whole at each storey, among the single braces. To check the effectiveness and reliability of the design procedure, a six-storey r.c. framed building is considered as a reference structure, which primarily designed in a medium-risk seismic region, has to be retrofitted as in a high-risk seismic region by insertion of hysteretic dissipative braces. Nonlinear dynamic analyses are carried out, under real and artificially generated ground motions, by a step-by-step procedure, assuming bilinear models to simulate the response of frame members and hysteretic dampers. The results show that the proposed design procedure is effective and reliable.

Keywords: R.c. framed buildings, Seismic retrofitting, Hysteretic dissipative braces, Displacement-based design, Nonlinear seismic analysis.

1. INTRODUCTION

The use of steel braces equipped with dissipative devices (e.g. hysteretic dampers, HYDs: as friction, FR, or metallic-yielding, YL, dampers) proved effective for seismic retrofitting of framed buildings (e.g., see Soong and Dargush 1997). New seismic codes (e.g., Italian code 2008, NTC08) allow for the use of dissipative braces for the seismic retrofitting of framed buildings, while only few codes provide simplified criteria for their design (e.g., FEMA 356, 2000). In a previous work (Mazza and Vulcano 2008a and 2008b) an effective procedure was proposed for design of hysteretic dissipative braces (HYDBs) following a Displacement-Based Design (DBD) approach (Bertero 2002, Priestley 1997), which combines pushover analysis of the actual structure with response spectrum analysis of an equivalent SDOF system. In particular, on the basis of a previous work (Vulcano 1994), the stiffness distribution of the HYDBs is selected assuming the same ratio between the lateral stiffness of the HYDBs and that of the unbraced frame at each storey (proportional stiffness criterion); moreover, the distribution law of the HYDB strength is assumed similar to that of the elastic force induced by the lateral seismic loads. However, further studies are needed to generalize and validate the design procedure.

An important point is selecting a suitable distribution of the stiffness and strength of the HYDBs among the dissipative braces actually provided at each storey. In the present work two different criteria are proposed: the first one aims to protect the weakest column(s) at each storey; the second one aims to protect also the nonstructural components, imposing the same drift ratio at each storey. To check effectiveness and reliability of the design procedure, a numerical investigation is carried out studying the nonlinear seismic response of a six-storey r.c. framed building, which, primarily designed according to a previous Italian seismic code (1996) for a medium-risk zone, has to be retrofitted by insertion of HYDBs for attaining performance levels imposed by NTC08 in a high-risk zone.

2. DISPLACEMENT-BASE DESIGN OF DISSIPATIVE BRACES

As mentioned above, the design of the dissipative braces is based on a proportional stiffness criterion, which assumes, at each storey, the same value of the stiffness ratio K^*_{DB}(=K_{DB}/K_F, K_{DB} being the lateral stiffness of the damped braces and K_F that of the unbraced frame). In the case of HYDs, K_{DB} can be expressed as for an in-series model depending on the brace stiffness, K_B, and the elastic stiffness of the damper, K_D:

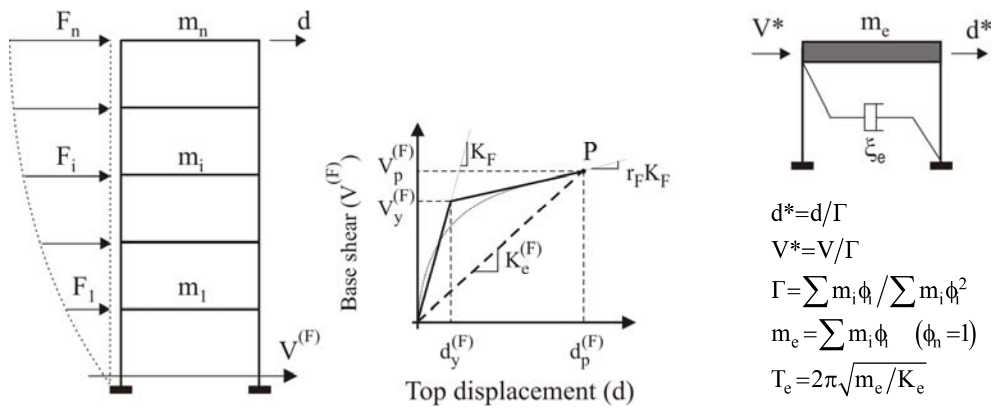
$$K_{DB} = \frac{1}{1/K_B + 1/K_D} \tag{2.1}$$

Note that Eqn. 2.1 specializes as K_{DB}=K_D, in the case of a FR damper ($K_D \rightarrow \infty$).

Moreover, the distribution law of the yield-load (N_y) is assumed similar to that of the elastic force induced in the braces by the lateral seismic loads (e.g., those corresponding to the first-mode shape). As proposed by Vulcano 1994, the selection of the N_y value at a generic storey can be restricted to the range ($0.5N_{max}$, N_{max}), where: the lower bound aims to avoid yielding of dampers under service gravity loads and moderate seismic (or wind) loads; the upper bound should avoid any yielding of frame members before yielding of dampers, as well as the occurrence of undesirable phenomena in the frame columns (e.g., buckling, brittle failure of r.c. columns, etc.). Due to the above assumptions, the yield-load is such to have at each storey the same value of the ratio N^*=N_y/N_{max}. The main steps of the proposed design procedure are summarized below with reference to diagonal braces with YL dampers.

Step 1: Pushover analysis of the unbraced frame and equivalent Single Degree of Freedom (ESDOF) system

Nonlinear static (pushover) analysis of the unbraced frame (which is supposed given), under constant gravity loads and monotonically increasing horizontal loads, is carried out to obtain the base shear-top displacement ($V^{(F)}$-d) curve (Fig. 2.1a). For this purpose, the smallest-capacity curve is selected between those corresponding to the lateral-load patterns considered along the height: e.g., a "uniform" pattern, proportional to the floor masses [m_1, m_2, ..., m_n]; a "modal" pattern, similar to the first mode shape $\boldsymbol{\phi}$=[ϕ_1, ϕ_2, ..., ϕ_n]^T multiplied by the floor masses.



(a) Framed structure and idealization of the capacity curve (b) ESDOF system

Figure 2.1. Pushover analysis

The unbraced frame can be represented by an ESDOF system (Fajfar 1999) characterized by a bilinear curve (V^*-d^*) derived from an analogous idealization of the $V^{(F)}$-d curve already obtained for the actual structure (Fig. 2.1b). Once the displacement corresponding to a selected performance level (d_p) is fixed, ductility $\mu_F(=d_p/d_y^{(F)}$; $d_y^{(F)}$=yield displacement), stiffness hardening ratio r_F and equivalent (secant) stiffness $K_e^{(F)}(=V_p^{(F)}/d_p$; $V_p^{(F)}$=base shear at the performance displacement) can be evaluated for the frame. Then, the equivalent viscous damping due to hysteresis $\xi_F^{(h)}$ can be calculated as

$$\xi_F^{(h)}(\%)=\frac{1}{4\pi}\frac{E_D}{E_S}=\kappa\left(63.7\frac{(\mu_F-1)(1-r_F)}{\mu_F\left[1+r_F(\mu_F-1)\right]}\right) \tag{2.2}$$

E_D being the hysteretic energy dissipated in a complete cycle of loading at maximum displacement (d_p) and E_S the elastic strain energy at the yielding point ($V_y^{(F)}$, $d_y^{(F)}$). The parameter κ, which accounts for the mechanical degradation, depends on the structural type (e.g. according to ATC40, 1996, κ can be assumed equal to 1/3 in case of poor structural behaviour).

Step 2: Equivalent viscous damping due to hysteresis of the damped braces (ξ_{DB})
Once the constitutive law of the equivalent damped brace is idealized as bilinear, the corresponding viscous damping, $\xi_{DB}=\xi_{DB}(\mu_{DB}, r_{DB})$, can be evaluated by an expression analogous to Eqn. 2.2 (but without κ). The ductility of the equivalent damping brace, μ_{DB}, can be assumed not less than μ_F (e.g., according to the design criteria specified above for selecting N_y, it may be assumed: $\mu_F \le \mu_{DB} \le 2\mu_F$), while the corresponding stiffness ratio, r_{DB}, can be expressed as

$$r_{DB}=\frac{1/K_B+1/K_D}{1/K_B+1/(r_D K_D)}=\frac{r_D\left(1+K_D^*\right)}{1+r_D K_D^*} \quad ; \quad K_D^*=\frac{K_D}{K_B} \tag{2.3a,b}$$

where r_D is the stiffness hardening ratio of the damper (e.g., specifically for a friction damper, $r_{DB}=r_D=0$) and the ratio $K^*_D(=K_D/K_B)$ can be reasonably assumed rather less than 1. Moreover, μ_{DB} can be expressed as (μ_D=damper ductility)

$$\mu_{DB}=\frac{1+(\mu_D-1)\left(1+r_D K_D^*\right)}{\left(1+K_D^*\right)} \tag{2.4}$$

Then, a K^*_D value should be selected such that the ductility demand to the damper, μ_D, be compatible with the deformation capacity of the damper itself.

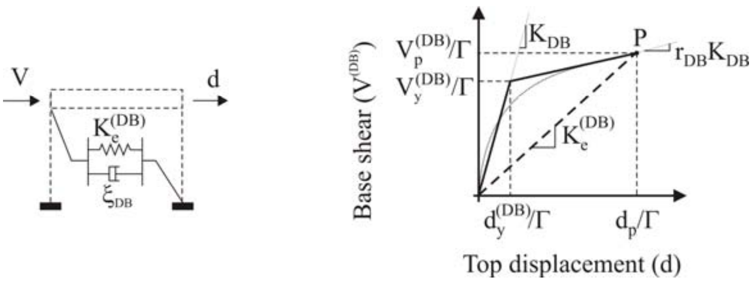


Figure 2.2. Equivalent system representing damped braces and its response idealized as bilinear

Step 3: Effective period of the frame with damped braces (DBF)
Assuming a suitable value of the elastic viscous damping for the framed structure (e.g. as commonly done, ξ_V=5%), the equivalent viscous damping of the damped braced frame (DBF) is:

$$\xi_e(\%)=\xi_V+\frac{\xi_{DB}V_p^{(DB)}+\xi_F^{(h)}V_p^{(F)}}{V_p^{(DB)}+V_p^{(F)}} \tag{2.5}$$

where $\xi_F^{(h)}$ and ξ_{DB} have been calculated in steps 1 and 2, respectively. Then, the effective period (T_e)

of DBF can be evaluated as the period corresponding to the performance displacement d_p, by means of the displacement response spectrum (S_D-T) for the viscous damping ξ_e. As noted in a previous work (Mazza and Vulcano 2010), the calibration of ξ_e is an open problem, because Jacobsen's equivalent damping approach (see Eqn. 2.2 for κ=1) can give an overestimation. For this purpose a procedure was proposed accounting for the hysteretic behaviour of an equivalent SDOF system representing the damped braced frame, but further studies are needed for an improvement in the calibration of ξ_e. Therefore, in the following discussion it will be assumed κ=0.33, even though the response of the r.c. members is simulated by a bilinear model (without degradation).

Step 4: Effective stiffness of the equivalent damped brace

Once the mass of ESDOF system, m_e, is calculated, the effective stiffness of DBF (K_e) and the effective stiffness required to the damped braces ($K_e^{(DB)}$) can be easily evaluated as

$$K_e = 4\pi^2 m_e / T_e^2 \quad ; \quad K_e^{(DB)} = K_e - K_e^{(F)} \tag{2.6a,b}$$

Step 5: Strength properties of the damped brace

The base shear-displacement curve representing the response of the damped braces of the actual structure ($V^{(DB)}$-d) is idealized as bilinear. Specifically, the base-shear contributions due to the damped braces of the actual structure at the performance and yielding points ($V_p^{(DB)}$ and $V_y^{(DB)}$, respectively) are:

$$V_p^{(DB)} = K_e^{(DB)} d_p \quad ; \quad V_y^{(DB)} = \frac{V_p^{(DB)}}{1 + r_{DB}\left(\mu_{DB} - 1\right)} \tag{2.7a,b}$$

It is worth mentioning that the equivalent viscous damping expressed by the Eqn. 2.5 is a function of the base-shear $V_p^{(DB)}$, which is initially unknown. As a consequence, an iterative procedure is needed to solve Eqns. 2.5-2.7.

Step 6: Design of the hysteretic damped braces of the actual structure

According to the proportional stiffness criterion, it can be reasonably assumed that a mode shape (e.g. the first-mode shape ($\{\phi_1, \ldots, \phi_n\}^T$) of the primary frame remains practically the same even after the insertion of damped braces. Then, the distribution of the lateral loads carried by the damped braces at the yielding point can be assumed as reported in Fig. 2.3, being m_i the mass at the generic storey i^{th}. This design criterion is preferable in the case of a retrofitting, because the stress distribution remains practically unchanged.

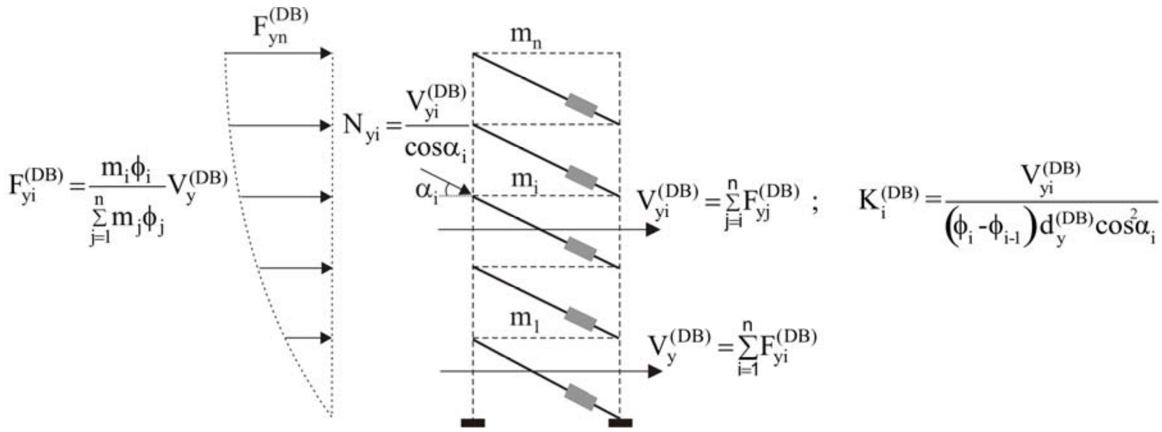


Figure 2.3. Quantities for design of diagonal braces with YL dampers

Finally, once the shear at a generic storey, $V_{yi}^{(DB)}$, is calculated, the quantities which are needed for designing the damped brace at that storey can be determined. Specifically, with reference to diagonal braces with YL dampers, the yield-load, N_{yi}, and the elastic stiffness of the damped brace (along the brace direction), $K_i^{(DB)}$, can be expressed as specified in Fig. 2.3, where $d_y^{(DB)}$ is the yielding displacement of the damped bracing system. Then, according to the value above assumed for K_D^* (step 2), the stiffnesses of brace (K_{Bi}) and damper (K_{Di}) at the i^{th} storey can be established according to the Eqn. 2.1.

The above procedure has been presented referring to a plane frame which represents a building in a given direction as a whole. Therefore, it is necessary to distribute the stiffness and strength properties of a damped brace so obtained among the braces actually provided at each storey. Two different approaches are presented in the next Section with reference to a six-storey r.c. framed building.

3. LAYOUT AND DESIGN OF THE UNBRACED AND DAMPED BRACED STRUCTURES

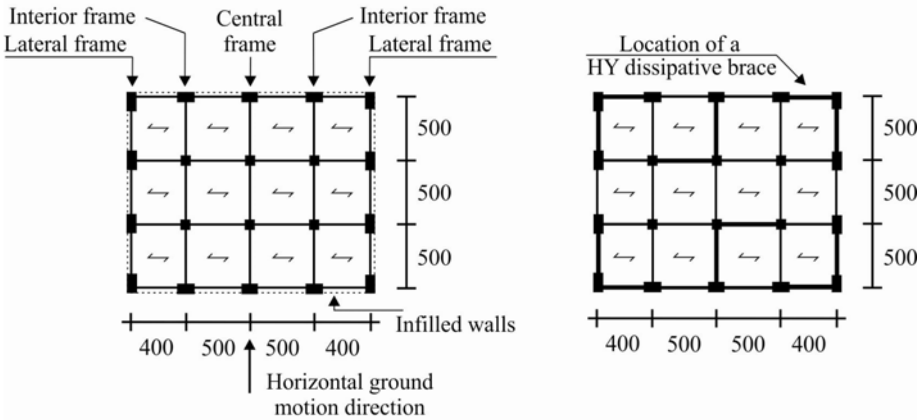
A typical six-storey residential building with a r.c. framed structure (Fig. 3.1a) is considered as primary structure. Because of the structural symmetry and assuming the floor slabs infinitely rigid in their own plane, the entire structure is idealized by an equivalent plane frame along the considered horizontal motion direction (see pseudo-spatial model in Fig. 3.2a), whose girders and columns have stiffness and strength properties so that the two lateral frames and the two interior frames could be represented as a whole. Length and cross-sections of the frame members are also shown in Fig. 3.2a, while floor masses and main dynamical properties are reported in Table 3.1.

The primary framed building is designed according to the Italian Seismic Code in force in 1996, for a medium-risk seismic region (seismic coefficient: C=0.07) and a typical subsoil class (main coefficients: R=ε=β=1). The gravity loads are represented by a dead load of 4.2 kN/m^2 at the top floor and 5.0 kN/m^2 at the other floors, and a live load of 2.0 kN/m^2 at all the floors. Masonry infill walls, regularly distributed in elevation along the perimeter (see Fig. 3.1a), are considered assuming an average weight of about 2.7 kN/m^2. A cylindrical compressive strength of 25 N/mm^2 for the concrete and a yield strength of 375 N/mm^2 for the steel are considered. The design is carried out to comply with the ultimate limit states. Detailing for local ductility is also imposed to satisfy minimum conditions for the longitudinal bars of the r.c. frame members: for the girders, a tension reinforcement ratio nowhere less than 0.37% is provided and a compression reinforcement not less than half of the tension reinforcement is placed at all sections; for a section of each column a minimum steel geometric ratio of 1% is assumed, supposing that the minimum reinforcement ratio corresponding to one side of the section be about 0.35%.

For the purpose of retrofitting the test structure from a medium-risk region up to a high-risk seismic region, diagonal steel braces equipped with YL dampers (HY dissipative braces) are inserted, at each storey, as indicated in Fig. 3.1b and Fig. 3.2b. The design of the dissipative braces is carried out according to the procedure described above, considering seismic loads provided by the recent NTC08 for a high-risk seismic region (return period: T_R=475 years; peak ground acceleration on rock: a_g=0.27g; maximum spectrum amplification coefficient: F_o=2.5) and subsoil class B on a level ground (site amplification factor: $S=S_SS_T$=1.13; PGA=1.13x0.27g=0.31g). The following values are assumed for design: μ_F=1.5, μ_D=10, κ=0.33. Properties of the dissipative braces are evaluated supposing a brace rigid enough that its deformability could be neglected (then, according to Eqn. 2.1, it can be assumed $K_{DB}\cong K_D$). Two different criteria are used to distribute the stiffness and strength properties of the damped braces (obtained as a whole) among the actual braces provided into the lateral and central frames along the assumed direction of the ground motion (see Fig. 3.1b and Fig. 3.2b).

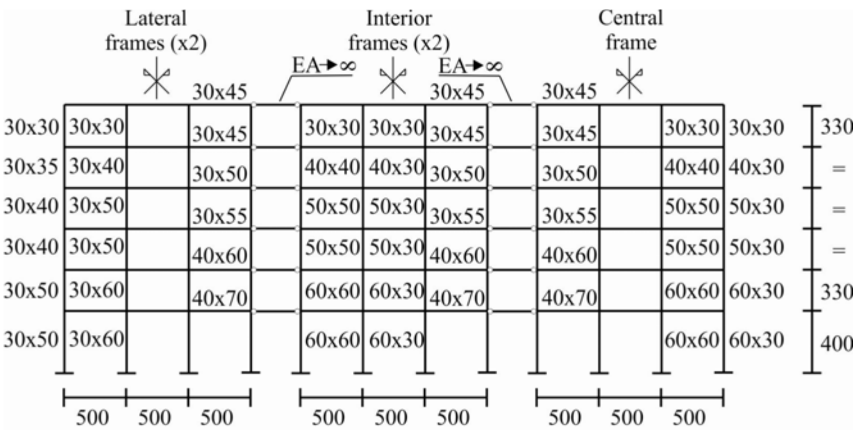
According to the first criterion, which is consistent with the design procedure described in Section 2, the distribution of the global stiffness between the braces of a storey is obtained assuming the contribution of the dissipative braces as proportional to the ratio between ultimate shear and design shear (V_u/V_d), calculated for the weakest column of the considered plane frame at that storey. Then, the strength distribution is assumed proportional to the stiffness distribution.

Alternatively, the second criterion, which is a variant of the above design procedure combined with the first criterion, aims to get on the whole a regular damped braced structure in terms of stiffness and strength. For this purpose, an inverted triangular mode shape, consistent with a drift ratio constant at each storey, is considered. To make comparable the two criteria, both the sum of all the storey stiffnesses $K_i^{(DB)}$ and the yield shear $V_{y1}^{(DB)}$ at the first storey were assumed the same for both the criteria. The distribution of stiffness properties among the damped braces of a storey is made as for the first criterion; then strength properties are distributed at each storey assuming an elastic behaviour under the lateral forces according to the mode shape assumed above. The properties of the HYDBs obtained according to the proposed two criteria are reported in Table 3.2. The marks DBF(1) and DBF(2) identify the structures designed according to the two criteria.

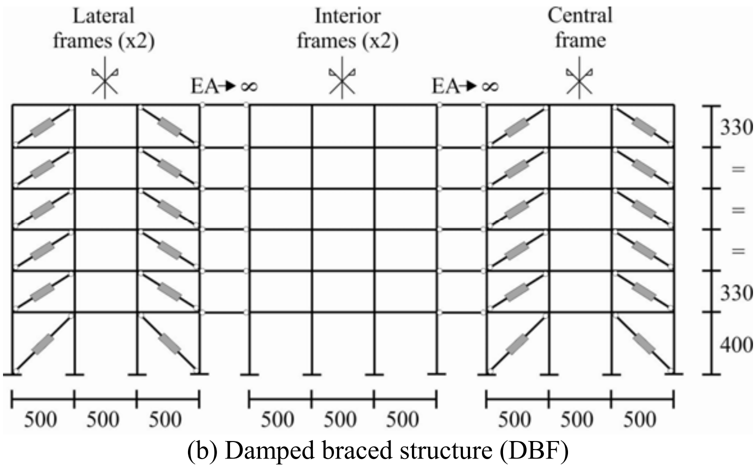


(a) Unbraced framed structure (UF) (b) Damped braced structure (DBF)

Figure 3.1. Plan of the reference structures (dimensions in cm)



(a) Unbraced framed structure (UF)



(b) Damped braced structure (DBF)

Figure 3.2. Pseudo-spatial models of the reference structures in Fig. 3.1 (dimensions in cm)

Table 3.1. Masses and dynamical properties of the primary framed structure

Storey	Floor masses ($kN\ s^2 / m$)	First-mode shape
6	171	1.00
5	245	0.83
4	257	0.63
3	264	0.44
2	285	0.26
1	301	0.13

Mode	Period (s)	Effective masses (% total mass)
1	0.762	74.1
2	0.311	16.2
3	0.193	5.6

Table 3.2. Properties of the hysteretic dissipative braces according to the proposed two criteria

(a) First criterion (structure DBF(1))

	$V_{yi}^{(DB)}$ (kN)		$K_i^{(DB)}$ (kN/m)	
Storey	Central frame	Exterior frames (x2)	Central frame	Exterior frames (x2)
6	56.33	50.13	74634	66411
5	131.16	105.94	144710	116891
4	193.43	148.73	231659	178125
3	233.94	181.83	285272	221727
2	292.30	186.29	514332	327799
1	323.14	188.88	553559	323556

(b) Second criterion (structure DBF(2))

	$V_{yi}^{(DB)}$ (kN)		$K_i^{(DB)}$ (kN/m)	
Storey	Central frame	Exterior frames (x2)	Central frame	Exterior frames (x2)
6	28.39	25.26	11707	104116
5	153.53	124.01	227776	183988
4	167.20	128.56	302062	232258
3	274.48	213.34	361247	280778
2	127.05	80.97	395406	252004
1	323.14	188.88	351956	205719

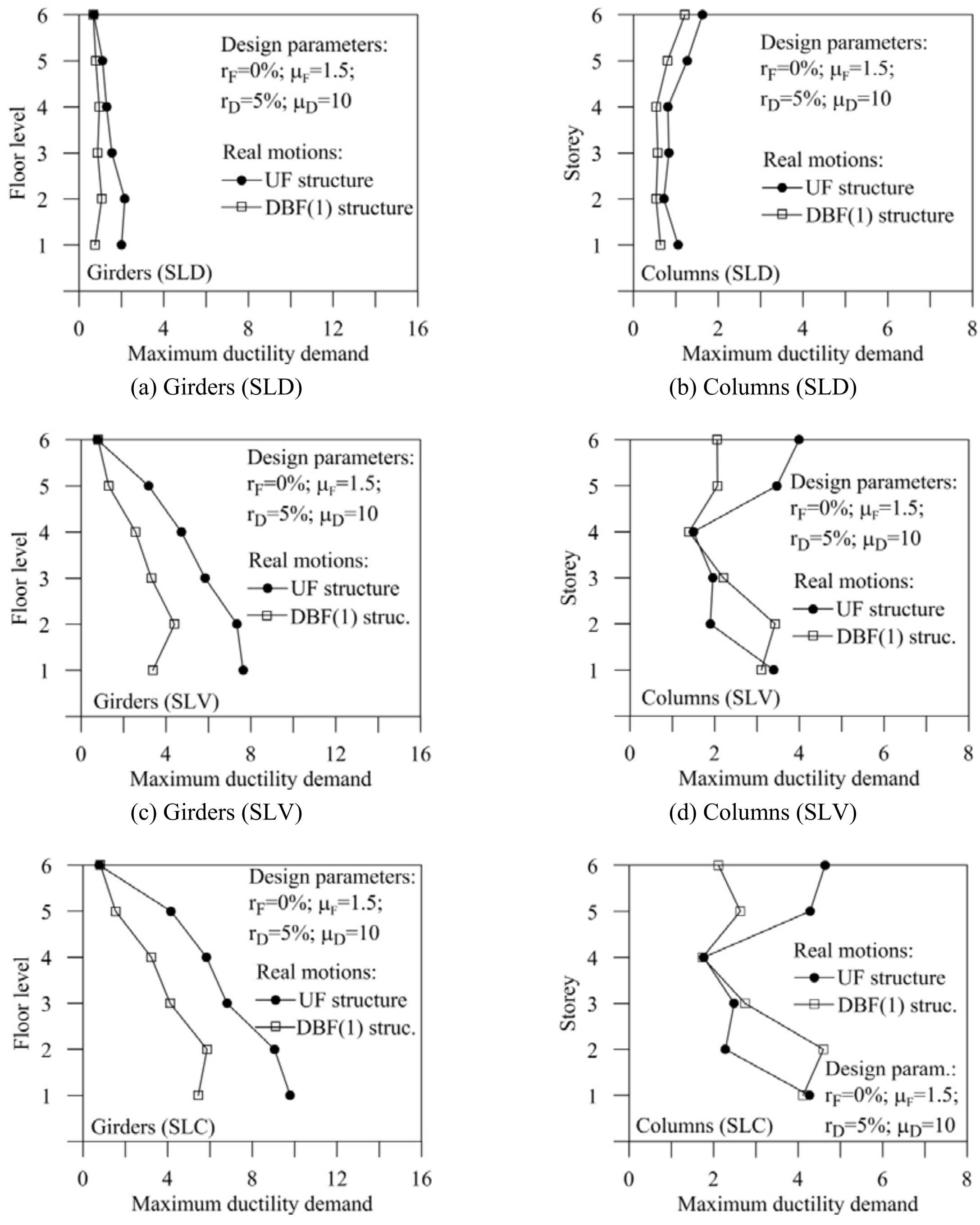
4. NUMERICAL RESULTS

To check the effectiveness and reliability of the design procedure and criteria illustrated above, a numerical investigation is carried out evaluating the nonlinear dynamic response of the primary and damped braced structures considered in Section 3 when subjected to sets of real and artificial ground motions. More precisely, sets of seven real motions selected according to the procedure proposed by Iervolino et al. 2008, and sets of three artificial motions generated as proposed by Gasparini and Vanmarcke 1976, are considered in order to match (on the average) the design spectra assumed by NTC08 for different limit states (damage, SLD; life safety, SLV; collapse, SLC). The nonlinear dynamic analyses are carried out by a step-by-step procedure, assuming elastic-perfectly laws to simulate the response of the r.c. frame members and hysteretic dissipative braces; in particular for the columns, the effect of the axial load on the ultimate moment is taken into account. Details on this procedure can be found in previous works (Vulcano 1981, Mazza and Vulcano 2008a). All the following results are obtained as an average of those separately obtained for the sets of real or artificial motions corresponding to a limit state.

The maximum ductility demand to girders and columns of structure DBF(1) under the set of real motions for different limit states is shown in Fig. 4.1. As can be observed, the insertion of damped braces is effective in reducing the ductility demand, even though for SLV and SLC (see Fig. 4.1d and Fig. 4.1f) columns of the second and third storeys exhibit a ductility demand greater than that obtained in the unbraced frame.

A comparison of curves obtained for real and artificial motions (SLV) is reported in Fig. 4.2. In particular, the curves in Figs. 4.2a and 4.2b have been obtained for DBF(1) as an average of the maximum values for all the critical (end) sections of the frame members considering both the loading directions (i.e., four values are considered for each member). It is interesting to note that, although the distribution of the ductility demand to the UF members is different for the two kinds of motions due to the different frequency content, the target values assumed for designing the damped braces (i.e., μ_F=1.5, μ_D=10) are close enough to the obtained values, with results lightly more conservative for dampers (Fig. 4.2d). However, this depends also on the value selected for the equivalent damping ratio (ξ_e), because it has been assumed κ=0.33 to take into account the degradation of r.c. members, but a bilinear model was used for the analyses. On the other hand, this can account for the overestimation of

Jacobsen's method and the scattering of spectral values for real motions, which can be higher than the design spectral values.



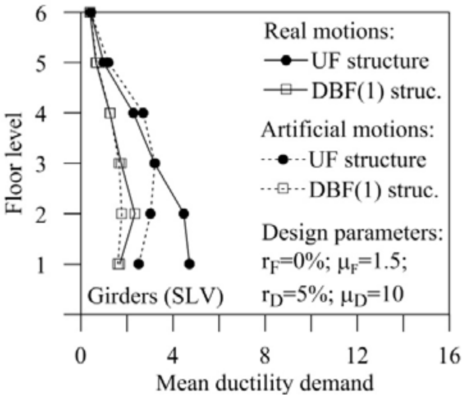
(a) Girders (SLD) (b) Columns (SLD)

(c) Girders (SLV) (d) Columns (SLV)

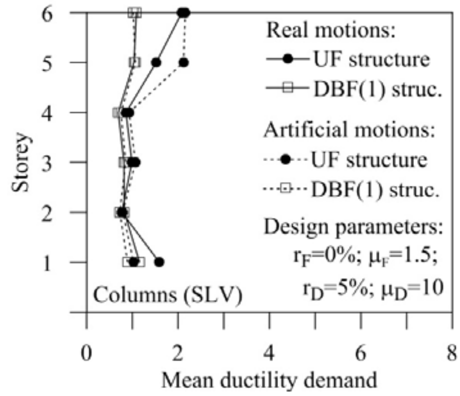
(e) Girders SLC (f) Columns (SLC)

Figure 4.1. Maximum ductility demand to frame members of UF and DBF(1) under real motions

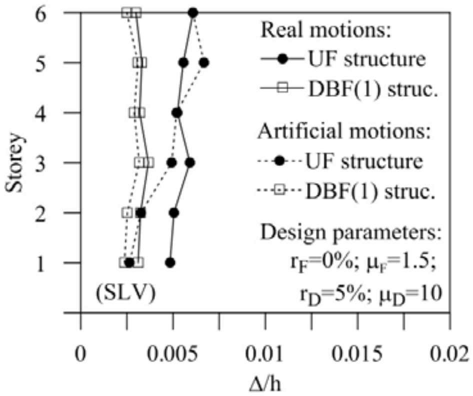
A comparison between curves obtained for structures DBF(1) and DBF(2) is reported in Fig. 4.3. As shown, the results are comparable with regard to ductility demand to frame members and drift ratio (Figs. 4.3a, b, c), while the ductility demand to dampers (Fig. 4.3d) is greater at the second and top storeys of structure DBF(2), where the strength of dampers is rather low in comparison with that at other storeys (see Table 3.2b). However, it should be noted that the primary structure is rather regular and some benefit (i.e., uniform drift ratio and consequent control of damage in nonstructural components as infill panels, plants, etc.) in using the second criterion does not appear evident.



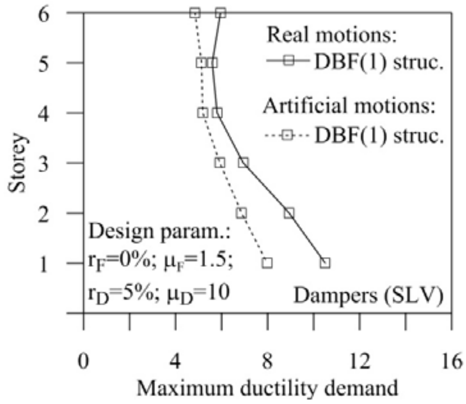
(a) Average ductility demand to girders



(b) Average ductility demand to columns

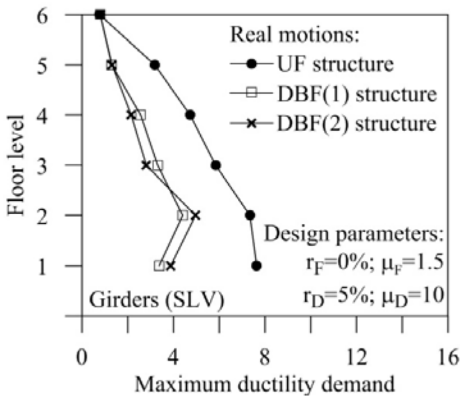


(c) Maximum drift ratio

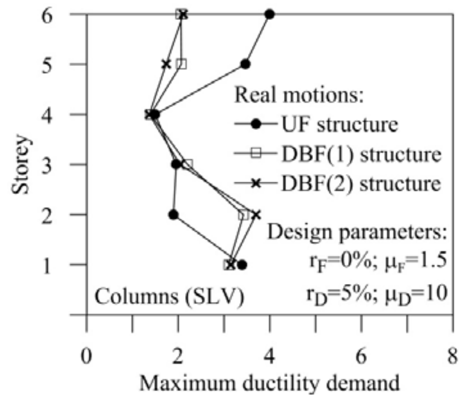


(d) Maximum ductility demand to hysteretic dampers

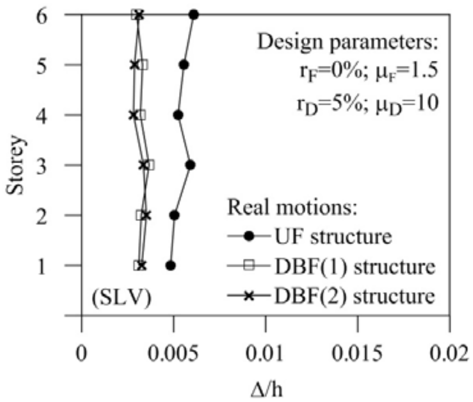
Figure 4.2. Comparison of results for real and artificial motions (SLV)



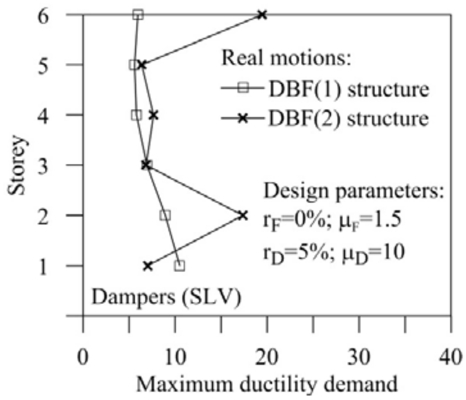
(a) Maximum ductility demand to girders



(b) Maximum ductility demand to columns



(c) Maximum drift ratio



(d) Maximum ductility demand to hysteretic dampers

Figure 4.3. Comparison of results obtained for DBF(1) and DBF(2) subjected to real motions (SLV)

5. CONCLUSIONS

The results discussed above showed the effectiveness and reliability of the proposed design procedure for proportioning the dissipative braces inserted for retrofitting a framed building. However, under strong ground motions (as for SLV and SLC) some columns can undergo ductility demand greater than that in the unbraced structure, due to high variation of the axial force inducing a reduction of the flexural capacity (e.g., for corner columns, subjected to rather low gravity loads). Both the criteria proposed for distributing the stiffness and strength properties of the dissipative braces among the actual braces of a storey led to comparable ductility demand to frame members and drift ratio. But the second criterion led to some peak values of the ductility demand to dampers; on the other hand, some benefit coming from this criterion (e.g., control of damage in nonstructural components) did not appear evident because the considered primary structure was rather regular. Further studies are needed before using the proposed design procedure in case of irregular primary buildings. Moreover, the reliability of the procedure can be improved by a better calibration of the equivalent viscous damping, considering also degrading models for simulating the hysteretic response of r.c. members.

AKCNOWLEDGEMENT
The present work was financed by RELUIS (Italian network of university laboratories of earthquake engineering), according to “convenzione D.P.C. – RELUIS 2010-2013, task 2.3.2”.

REFERENCES

Applied Technology Council (1996). Seismic Evaluation and Retrofit of Concrete Buildings, *Report no. ATC 40,* **Vol. 1**, Redwood City, CA.

Bertero, V.V. (2002). Innovative approaches to earthquake engineering. Edited by G. Oliveto, University of Catania (Italy), WIT Press, 1-84.

Fajfar, P. (1999). Capacity spectrum method based on inelastic spectra. *Earthquake Engineering and Structural Dynamics* **28:** 979-993.

Federal Emergency Management Agency (2000). FEMA 356. Prestandard and commentary for the seismic rehabilitation of buildings. Prepared by American Society of Civil Engineers, Reston, Virginia.

Gasparini, D. and Vanmarcke E. (1976). Simulated earthquake motions compatible with prescribed response spectra. *Massachusetts Institute of Technology, Department of Civil Engineering*, U.S.A..

Iervolino, I., Maddaloni, G. and Cosenza E. (2008), Eurocode 8 compliant record sets for seismic analysis of structures. *Journal of Earthquake Engineering* **12**(1): 54-90.

Italian Ministry of Public Works (1996). Norme tecniche per le costruzioni in zone sismiche e relative istruzioni. D.M. 16-01-1996 e Circolare M.ro LL.PP. 10-04-1997, n. 65/AA.GG..

Italian Ministry of Infrastructures (2008). Nuove norme tecniche per le costruzioni e relative istruzioni. D.M. 14-01-2008 e Circolare 02-02-2009, n. 617/C.S.LL.PP..

Mazza, F. and Vulcano, A. (2008a). Displacement-based seismic design procedure for framed buildings with dissipative braces. (a) Part I: Theoretical formulation; (b) Part II: Numerical results. *Seismic Engineering International Conference commemorating the 1908 Messina and Reggio Calabria Earthquake (MERCEA08),* Reggio Calabria, Italy. American Institute of Physics Conference Proceedings, U.S.A, **Vol. 1020** (part two).

Mazza, F. and Vulcano, A. (2008b). Displacement-based design of dissipative braces at a given performance level of a framed building. *Procs. of the 14th World Conference on Earthquake Engineering*, Beijing, China, paper No. 14-53.

Mazza, F. and Vulcano, A. (2010). Equivalent viscous damping for displacement-based aseismic design of damped braced frames. *Procs. of the 14th European Conference on Earthquake Engineering*, Skopije, Republic of Macedonia, paper No. 336.

Priestley, M.J.N. (1997). Displacement-based seismic assessment of reinforced concrete buildings. *Journal of Earthquake Engineering* **1(1):** 157-192.

Soong, T.T. and Dargush, G.F. (1997). *Passive energy dissipation systems in structural engineering.* John Wiley & Sons Ltd, Chichester, England.

Vulcano, A. (1981). Analisi sismica di strutture intelaiate piane in campo elasto-plastico. *Procs. 1° Convegno di Ingegneria Sismica in Italia,* Udine, 1980; CISM (International Centre for Mechanical Sciences) Courses and Lectures, n. 271, Springer Verlag, Wien-New York.

Vulcano, A. (1994). Design of damped steel bracing systems for seismic control of framed structures. *Procs. of the 10th European Conference on Earthquake Engineering,* Vienna, Austria, **Vol. 4:** 1567-1572.