Accurate application and higher-order solutions of the SAC/FEMA probabilistic format for performance assessment

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SUMMARY:
The SAC/FEMA probabilistic framework is based on a pioneering closed-form expression to analytically estimate the value of the risk integral convolving seismic hazard and structural response. Despite its immense practicality, implementation has been hindered by reduced accuracy due to a number of approximations needed to achieve a simple form, the most significant being the power-law approximation of the seismic hazard curve. To mitigate this problem, two approaches are hereby offered, namely (a) selecting an appropriately-biased power-law fit and (b) offering a novel closed-form expression involving a higher order approximation. Where blind application of the original format could involve error in excess of 100% for the predicted mean annual frequency of limit-state exceedance, biased fitting reduces it to less than 25% while for the new closed-form it remains consistently below 10%. Although other sources of error still remain, the robustness achieved opens new avenues of application for this popular format.

Keywords: seismic performance evaluation, probabilistic methods, SAC/FEMA, closed-form solution, hazard

1. INTRODUCTION

The SAC/FEMA project grew out of the destruction wrought by the Northridge 1994 earthquake to improve the performance of steel moment-resisting frame buildings. Its results have been summarized in a series of documents and guidelines, the most well-known being FEMA-350/351 (SAC/FEMA 2000a,b). One of the enduring legacies of the work generated by this project is the popularization of the concept of evaluating the seismic performance of a structure in terms of the mean annual frequency (MAF) of limit-state exceedance by Cornell et al. (2002). Equally important is the introduction by the same authors of the only closed-form solution for estimating the probabilistic integral and a simple checking format similar to the familiar LRFD.

The SAC/FEMA MAF format offers a simple expression to convolve the seismic hazard with the structural response and derive simple estimates of the mean annual rate (or the mean return period) of exceeding any limit-state that can be defined on the structural response, including both the effects of epistemic uncertainty and aleatory variability with a user-selected level of confidence. Thanks to the simplicity of this formulation, it has found widespread recognition and it has been used by prominent researchers as a basis for performance assessment of structures, e.g., Lupoi et al. (2002) and Fajfar & Dolsek (2012). At its basis, it can also be thought to have become the core of the highly influential Pacific Earthquake Engineering Research (PEER) Center probabilistic framework.

Nevertheless, it has also been criticized for its lack of accuracy (e.g., Aslani & Miranda 2005, Bradley & Dhakal 2008). The main issue is the adequacy of the approximations used in deriving the closed-form expression, thus limiting its predictive ability. The most important of them is the power law fit of the seismic hazard curve that is only locally accurate and, when not properly fitted, can introduce massive errors. Following in the steps of the original derivation, we aim to rectify this by offering two improvements, namely using the original expression with a biased hazard fit and employing a second-order hazard fit in the log-log domain to derive a novel closed-form solution.

2. THE SAC/FEMA FORMAT

Estimating the probability of violating a certain performance level or limit-state starts with the estimation of a site's seismic hazard. By adopting a Poisson model for earthquake occurrence, probabilistic seismic hazard analysis (Cornell 1968) offers a representation of site hazard by the hazard curve function $H(s)$ in terms of s, the adopted seismic intensity measure (IM), versus its mean annual frequency (MAF) of exceedance. Let then D, C be scalar demand and capacity characteristics of the structure, respectively. They can be expressed either in IM or engineering demand parameter (EDP) terms to be used to check for violating the limit-state LS. Thus, in the absence of uncertainty, failure is simply checked as $C < D$, or the capacity being less than the demand. For example this could be cast as the maximum interstory drift demand of the structure being more than a limiting value of, say, 1% (EDP basis), or the first-mode spectral acceleration of the ground motion excitation being higher than 0.4g (IM basis). In the presence of uncertainty, the conditional failure probability, also known as the fragility, $P(C < D|s)$ is used instead. By convolving with the seismic hazard, the mean annual frequency (MAF) of limit-state exceedance λ_{LS} can be estimated via any of the following three integrals (Jalayer 2003):

$$\lambda_{\mathrm{LS}} = \int_0^{+\infty} P(C < D \mid s)\,\left|\mathrm{d}H(s)\right| = \int_0^{+\infty} P(C < D \mid s)\left|\frac{\mathrm{d}H(s)}{\mathrm{d}s}\right|\mathrm{d}s = \int_0^{+\infty} \frac{\mathrm{d}P(C < D \mid s)}{\mathrm{d}s}\,H(s)\,\mathrm{d}s\,. \tag{2.1}$$

To avoid the tedious numerical integration, Cornell et al. (2002) have shown that a closed-form solution may be derived by making a series of rational assumptions or approximations. First, a local power law fit for the (mean) seismic hazard curve is adopted (see also Kennedy & Short 1994):

$$H(s) \approx k_0\, s^{-k_1} = k_0 \exp(-k_1 \ln s)\,, \tag{2.2}$$

where k_0 and k_1 are positive real numbers. If the capacity C and demand D of the structure in Eqn. 2.1 are expressed in terms of the IM, then we have the IM-based format. Assuming that the IM-capacity is lognormally distributed with median $\hat{s}_c$ and dispersion (standard deviation of the log of the data) β_{Sc}, then the MAF of the limit-state can be approximated as:

$$\lambda_{\mathrm{LS}} = H(\hat{s}_c)\exp\left(\frac{1}{2}k_1^2\beta_{Sc}^2\right). \tag{2.3}$$

If, instead, the capacity C in Eqn. 2.1 is represented as an EDP value that when exceeded by the seismic EDP demand D signals violation, further approximations are needed. Accordingly, it is assumes that the EDP capacity follows a lognormal distribution with median $\hat{\theta}_c$ and dispersion $\beta_{\theta c}$. If, additionally, the EDP demand given the IM is also lognormal with a constant dispersion of $\beta_{\theta d}$ and a conditional median demand provided by a power law:

$$\hat{\theta}(s) \approx a\, s^b\,, \tag{2.4}$$

where a, b are positive real numbers, then the closed form approximation of SAC/FEMA becomes

$$\lambda_{\mathrm{LS}} = H\left[\left(\frac{\hat{\theta}_c}{a}\right)^{\frac{1}{b}}\right]\exp\left[\frac{k_1^2}{2b^2}\left(\beta_{\theta c}^2 + \beta_{\theta d}^2\right)\right]. \tag{2.5}$$

In cases where instead of estimating the MAF one is interested in simply checking whether the structure violates a certain limit-state, a convenient method is using the Demand-Capacity Factor Design Format (DCFD). This was introduced by Cornell et al. (2002) for safety checking in a manner resembling the popular Load and Resistance Factor Design (LRFD) format. Thus, if we wish to verify whether the structure violates a limit-state consistent with a performance objective P_o (e.g., the typical 10% in 50yrs for Life Safety corresponds to a P_o = ln(1 - 0.10)/50 = 0.00211) then we need to estimate the median demand $\hat{\theta}_{Po}$ and its dispersion $\beta_{\theta d}$ that correspond to P_o. This entails performing a set of nonlinear dynamic analyses with several ground motion records at intensity level $s_{Po} = H^{-1}(P_o)$, consistent with the performance objective. Then, given our earlier assumptions about the lognormality of EDP capacity and of the conditional EDP demand, safety can be verified as

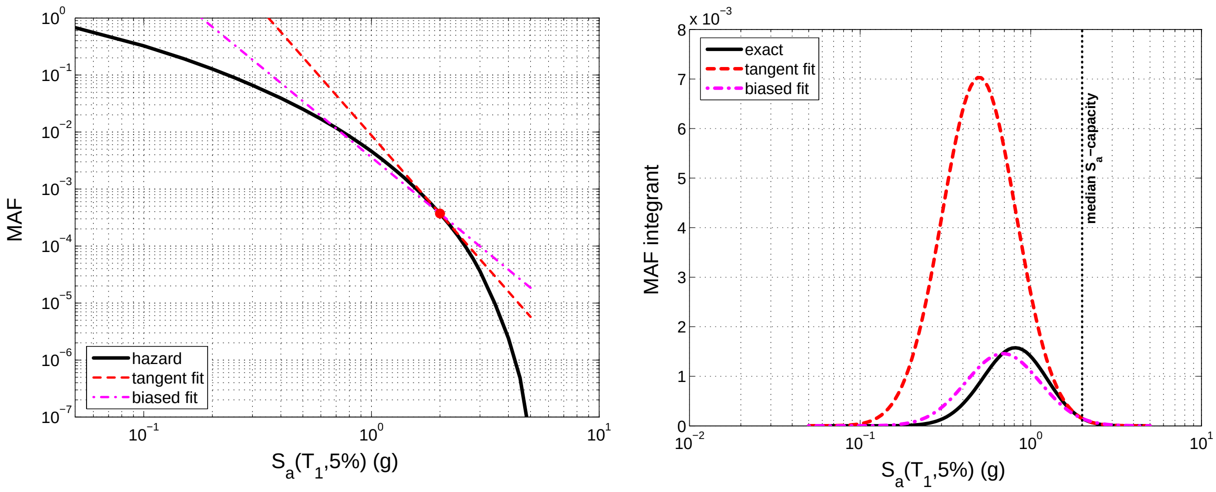
$$\hat{\theta}_c \exp\left(-\frac{k_1}{2b}\beta_{\theta c}^2\right) \geq \hat{\theta}_{Po} \exp\left(\frac{k_1}{2b}\beta_{\theta d}^2\right). \tag{2.6}$$

An additional exponential factor can be added to the right side of Eqn. 2.6 to offer a choice of treating the effect of epistemic uncertainties at the desired confidence level.

Summing up, it is obvious that the IM-based format needs only one approximation while the EDP-based formats need an additional two. In both cases it is the common local hazard fit that creates most problems, due to the rapid monotonically decreasing nature of the hazard function $H(s)$. We intend to offer two complementary ways of handling it that will mitigate any accuracy problems.

3. BIASED HAZARD FITTING

Approximating the curved seismic hazard function by a straight line in log-log space (Fig. 3.1a) can be a tricky endeavour. Jalayer (2003) proposed locally fitting Eqn. 2.2 as a tangent at the median IM-capacity. By construction this will always assure a conservative fit due to the concave shape of the hazard curve (Fig 3.1a). Unfortunately, as the hazard curvature and the capacity dispersion increase, this approach results in a massive overestimation of the integrand (Fig. 3.1b). If we adopt an IM-basis and assume the third form of Eqn. 2.1, the MAF integrand is simply the PDF of the IM capacity (essentially symmetric around the median in log-log) and multiplied by the geometrically decreasing hazard value. Thus, most of the contribution to the MAF integral (Eqn. 2.1) comes from the higher frequency earthquakes to the left of the median capacity (Bradley & Dhakal 2008, Eads et al. 2012) as shown in the high-curvature high-dispersion example of Fig. 3.1b.



(a) hazard curve fits (b) MAF integrands

Figure 3.1. The tangent and the biased power law fits and the corresponding MAF integrands for a median S_a capacity of 2g with 0.5 dispersion at the Van Nuys site (T_1 = 0.7s)

Thus, it makes sense to introduce a biased fit of Eqn. 2.2, a fact originally recognized by Dolsek and Fajfar (2008). They suggested performing a linear regression in the region of $[0.25\hat{s}_c, 1.25\hat{s}_c]$, consistently reducing the error of the hazard fit. Since this requires a regression and does not take into account the dispersion of capacity, we instead propose an even simpler solution that we will term the left-weighted, right-biased fit (pun intended). The seismic hazard is simply approximated by a secant line that passes through the median S_a-capacity but adopts a k_1-slope determined at points 0.5 and 1.5 standard deviations away. Thus, for the IM-based format of Eqn. 2.3 we have:

$$k_1 = -\frac{\ln H(s_2) - \ln H(s_1)}{\ln s_2 - \ln s_1}, \tag{3.1}$$

$$k_0 = H(\hat{s}_c) \cdot \hat{s}_c^{\,k_1}, \tag{3.2}$$

$$s_i = \hat{s}_c \exp(c_i \beta_{Sc}), \tag{3.3}$$

where, $c_{1,2} = -0.5, -1.5$. An example of its efficiency appears in Fig. 3.1a where it accurately captures the MAF integrand in a high-curvature high-dispersion situation. In this case, the exact MAF is 0.0015, with the tangent fit producing an exaggerated estimate of 0.0050 and the biased fit a very close approximation of 0.0014.

For application to the EDP-based format of Eqn. 2.5, the same idea is used, only now the dispersion employed is the total dispersion of EDP demand and capacity, divided by the slope (in log-log) of the IM-EDP relationship:

$$s_i = \left(\frac{\hat{\theta}_c}{a}\right)^{\frac{1}{b}} \exp\left(c_i \frac{\sqrt{\beta_{\theta c}^2 + \beta_{\theta d}^2}}{b}\right). \tag{3.4}$$

When employing the DCFD format of Eqn. 2.5, another slight modification is needed. Now, hazard fitting has to be performed at s_{po}, as this is the intensity where the analyses are performed. Ideally, we would want such the hazard fit to remain close to the IM value corresponding to the capacity, as in Eqn. 3.4 above. When the difference in these intensities is substantially in favour of the unfactored capacity (the opposite guarantees failure), the point is mute anyway as any reasonable fit will correctly suggest a safe result. When they are close though, then the accuracy of Eqn. 2.5 becomes critical and it helps to shift the c_i values closer to zero (i.e. shift the fit closer to the median IM capacity), say by 0.5 standard deviations. Thus, $c_{1,2} = 0.0, -1.0$ will be used together with the expression

$$s_i = s_{Po} \exp\left(c_i \frac{\sqrt{\beta_{\theta c}^2 + \beta_{\theta d}^2}}{b}\right). \tag{3.5}$$

4. HIGHER ORDER CLOSED FORM SOLUTIONS

While the biased fit is a definite improvement over the tangent fit of the power law approximation, it still does not take into account the curvature of the seismic hazard function. It is therefore not difficult to realize that its performance will degrade when higher curvatures are present. Employing a second-order polynomial fit in log-space can partially resolve this problem. Thus, letting

$$H(s) \approx k_0 \exp(-k_2 \ln^2 s - k_1 \ln s), \tag{4.1}$$

we shall proceed to derive new closed-form solutions by closely following the path laid out by Cornell et al. (2002).

4.1. MAF format on IM-basis

Let us first assume that demand and capacity are expressed in an IM-basis. Then, the third form of Eqn. 2.1 can be integrated analytically to become

$$\lambda_{\mathrm{LS}} = \sqrt{p}\; k_0^{1-p}\left[H(\hat{s}_c)\right]^p \exp\left(\frac{1}{2} p k_1^2 \beta_{Sc}^2\right) = \sqrt{p}\; k_0^{1-p}\left[H(\hat{s}_c)\right]^p \exp\left(\frac{k_1^2}{4k_2}(1-p)\right), \tag{4.2}$$

where

$$p = \frac{1}{1+2k_2\beta_{Sc}^2} \tag{4.3}$$

is a positive real number that varies within (0,1],

The first form of the solution (Eqn. 4.2) shows that for zero curvature ($k_2 = 0$), i.e., for the classic power law fit of Eqn. 2.2, $p = 0$ and the approximation reverts back to the SAC/FEMA original of Eqn. 2.3. The second form is valid only for $k_2 \neq 0$, which, by the way, is always the case anywhere on a realistic hazard curve.

Introducing the effect of epistemic uncertainty is equally simple. Uncertainty in the hazard curve is approximately included by using the mean hazard function $\overline{H}(s)$ and uncertainty in capacity is taken into account by having the dispersion incorporate both epistemic and aleatory contributions. Hence, employing the square root sum of squares rule as in SAC/FEMA to combine dispersions we only need to replace p with its respective counterpart p':

$$p' = \frac{1}{1+2k_2(\beta_{Sc}^2+\beta_{USc}^2)}, \tag{4.4}$$

with β_{USc} being the dispersion due to uncertainty in the S_a capacity. This would in turn produce the mean (regarding epistemic uncertainty) estimate of the MAF. If instead a certain percentile value reflecting, e.g., the 90% confidence level in the MAF is required, we need to define the associated dispersion in the MAF due to epistemic uncertainty as:

$$\beta_{TUSc} = k_1 \beta_{USc}\sqrt{p'p}\,. \tag{4.5}$$

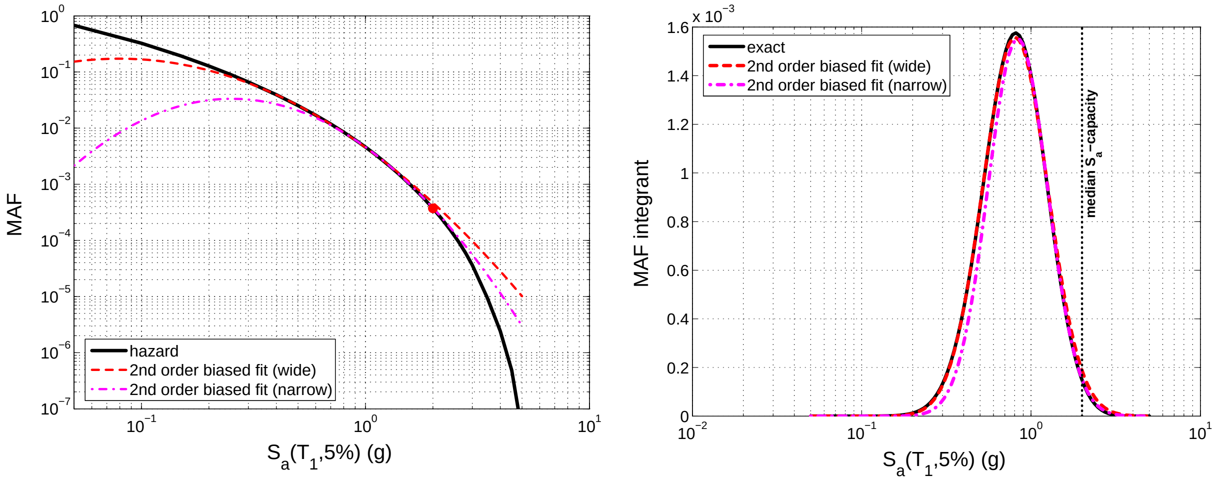
Then, let K_x be the standard normal variate corresponding to the desired confidence level. Formally, let $K_x = \Phi^{-1}(x)$, where Φ^{-1} is the inverse CDF of a standard normal variable (Benjamin & Cornell 1970), readily available in any probability textbook or spreadsheet program. For example, $K_x = 1.28$ for a 90% confidence level estimate. It follows that

$$\lambda_{\mathrm{LS}}^x = \sqrt{p}\; k_0^{1-p}\left[\overline{H}(\hat{s}_c)\right]^p \exp\left(\frac{1}{2} p k_1^2 \beta_{Sc}^2 + K_x \beta_{TUSc}\right). \tag{4.6}$$

For $k_2 = 0$, Eqn. 4.6 reverts again back to the SAC/FEMA original derivation.

When coupled with a biased fit concept, the new format promises excellent results. Using Eqn. 3.3 with $c_{1,2,3} = -0.5, -1.5, -3.0$ offers a useful set of interpolation points for Eqn. 4.1. Still, the selection can be quite more flexible than before and still produces a very good result. An example appears in Fig. 4.1a where the second-order power law has been bias-fitted over a narrow or a much wider interval than the one suggested above. While the fits themselves seem to differ markedly, they both perform equally well in the region that matters, as seen in Fig. 4.1b. Therein, the MAF integrands

(Eqn. 2.1) are nearly identical, easily producing MAF estimates that match the exact value of 0.0015 within 1%.



(a) hazard curve fits (b) MAF integrands

Figure 4.1. Two biased 2nd order power law fits, on a narrow or a wide interval, and the corresponding MAF integrands for a median S_a-capacity of 2g with 0.5 dispersion at the Van Nuys site ($T_1 = 0.7$s)

4.1. MAF format on EDP-basis

To utilize the second-order fit in an EDP-based format, we let

$$q = \frac{1}{1 + 2k_2\beta_{\theta d}^2 / b^2}, \quad \phi = \frac{1}{1 + 2k_2(\beta_{\theta d}^2 + \beta_{\theta c}^2)/b^2} \tag{4.7}$$

and define the IM-level corresponding to the median EDP-capacity according to the approximation of Eqn. 2.4 as

$$S_{\hat{\theta}c} = \left(\frac{\hat{\theta}_c}{a}\right)^{\frac{1}{b}}. \tag{4.8}$$

Then Eqn. 2.1 becomes:

$$\lambda_{\mathrm{LS}} = \sqrt{\phi}\; k_0^{1-\phi}\left[H(S_{\hat{\theta}c})\right]^{\phi} \exp\left(\frac{1}{2b^2} q k_1^2 (\beta_{\theta d}^2 + \phi\beta_{\theta c}^2)\right) = \sqrt{\phi}\; k_0^{1-\phi}\left[H(S_{\hat{\theta}c})\right]^{\phi} \exp\left(\frac{k_1^2}{4k_2}(1-\phi)\right). \tag{4.9}$$

Again the first form of the above format turns into the SAC/FEMA original when $k_2 = 0$, since $q = p$ =1. Introducing epistemic uncertainty again involves using the mean hazard curve $\overline{H}(s)$ and the updated value of φ' for estimating the overall mean value of the MAF:

$$\phi' = \frac{1}{1 + 2k_2(\beta_{\theta d}^2 + \beta_{\theta c}^2 + \beta_{U\theta d}^2 + \beta_{U\theta c}^2)/b^2}, \tag{4.10}$$

where $\beta_{U\theta d}$ and $\beta_{U\theta c}$ are the demand and capacity dispersions, respectively, due to the epistemic uncertainty.

If instead we are interested in a specific x-fractile value or x-confidence level for the MAF, then we need to use the standard normal variate $K_x = \Phi^{-1}(x)$. Hence, the corresponding MAF becomes:

$$\lambda_{\mathrm{LS}}^{x} = \sqrt{\phi}\, k_0^{1-\phi}\left[\bar{H}(S_{\hat{\theta}c})\right]^{\phi} \exp\left(\frac{k_1^2}{4k_2}(1-\phi) + K_x \beta_{TU\theta}\right). \tag{4.11}$$

The total uncertainty in the MAF is:

$$\beta_{TU\theta} = \frac{k_1}{b}\beta_{U\theta}\sqrt{\phi'\phi}, \quad \text{where } \beta_{U\theta} = \sqrt{\beta_{U\theta d}^2 + \beta_{U\theta c}^2}\ . \tag{4.12}$$

In all cases, biased fitting is suggested with interpolation points from Eqn. 3.4 for $c_{1,2,3}$ = –0.5, –1.5, –3.0.

4.3. DCFD format

If we are interested in simply checking whether the structure violates a certain limit-state, the Demand-Capacity Factor Design (DCFD) format becomes convenient. To derive the improved format for the seismic hazard fit of Eqn. 4.1, we set λ_{LS} less than or equal to the performance objective P_o. A second order expression is formed whose solution can be conservatively approximated as:

$$\frac{\hat{\theta}_c}{a}\exp\left(\frac{bk_1}{2k_2}\right) \geq \left[\frac{\hat{\theta}_{po}}{a}\exp\left(\frac{bk_1}{2k_2}\right)\right]^{1/\sqrt{\phi}}. \tag{4.13}$$

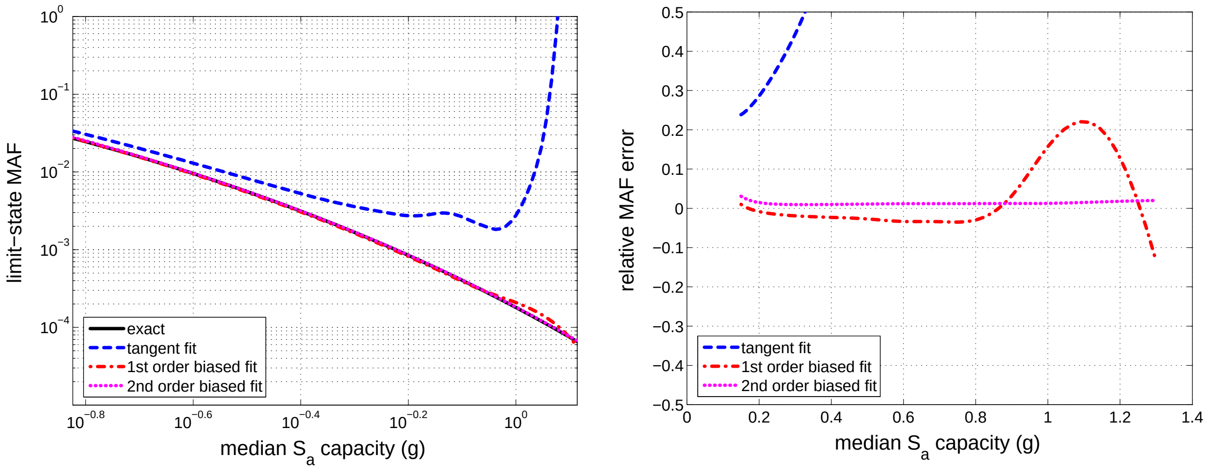
If checking at a certain confidence level x% is desired, by including the effect of epistemic uncertainty we come up with the following expression:

$$\frac{\hat{\theta}_c}{a}\exp\left(\frac{bk_1}{2k_2}\right) \geq \left[\frac{\hat{\theta}_{po}}{a}\exp\left(\frac{bk_1}{2k_2} + \sqrt{\frac{K_x\beta_{TU\theta}}{k_2}}\right)\right]^{1/\sqrt{\phi}}. \tag{4.14}$$

While certainly this is not as simple a format as the one used in FEMA 350/351, it nevertheless has the potential to deliver superior accuracy. This is because checking via DCFD formats is performed at the seismic intensity level consistent with P_o and not at the median IM-capacity. The local nature of the original power law fit may introduce severe accuracy problems compared to the broad-range applicability of the second-order fit used in Eqns. 4.13 and 4.14. Thus, these useful expressions can deliver much for practical applications. For improved accuracy, again, shifted bias fitting is suggested by using the interpolation points of Eqn. 3.5 for $c_{1,2,3}$ = 0.0, –1.0, –2.5.

5. ILLUSTRATIVE APPLICATION

To showcase the improvements brought by the biased fitting and the second-order approximation, we need a level playing field. To avoid any situation that would favour one over the other, we will adopt full compliance with all but one of the assumptions required by the SAC/FEMA approach. Thus, the median EDP demand is defined as a power law function of $S_a(T_1)$ and it has constant dispersion due to epistemic and aleatory sources, regardless of the intensity. The marked exception from adherence to theory will be the use of the highly-curved seismic hazard of the Van Nuys site in Los Angeles CA that is expected to severely test the proposed approximations. Obviously, the accuracy achieved in all subsequent analyses may further degrade in real-structure situations, wherever the EDP versus IM relationship does not adhere well to the above two assumptions.



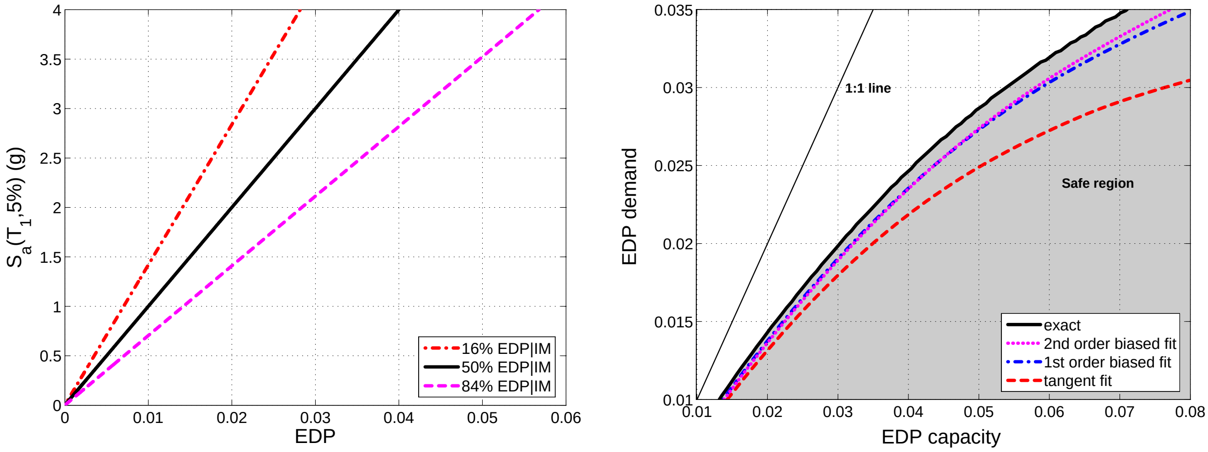
(a) estimated MAFs (b) MAF relative error

Figure 5.1. Estimates of limit-state MAFs and corresponding relative errors for varying median S_a-capacities and a dispersion of 0.5 at the Van Nuys site ($T_1 = 2.4$s)

For the MAF format, the above setup makes the IM-basis and the EDP-basis exactly equivalent, therefore results for the simpler IM-format will be shown. A typical example of the estimates obtained by numerical integration and the closed form solutions for the 1st order and 2nd order hazard fits appears in Fig. 5.1. For this case, the IM-dispersion is a constant 0.5 while a continuum of limit-states is used, having median S_a-capacities ranging from 0.15g up to 1.3g. Due to the nature of the seismic hazard (see for example Fig. 3.1), as the median capacity increases, so does the local curvature of the hazard curve. Thus, it is not surprising that the MAF estimates of the 1st order tangent fit widely miss the exact value (sometimes by several orders of magnitude) for practically any capacity beyond 0.2g for this severe case. Even for low capacities, the error observed is more than 20% of the numerical integration result (Fig. 5.1b). On the other hand, the 1st order biased fit achieves errors less than 10%, except for some areas beyond 1g. Predictably, its accuracy degrades as curvature increases. This is in contrast to the 2nd order fit results that manage an excellent prediction with less than 2% error practically regardless of curvature. Further results (not shown herein) provide ample evidence that the improved format rarely displays error in excess of 5% unless dispersions higher than 0.8 are used.

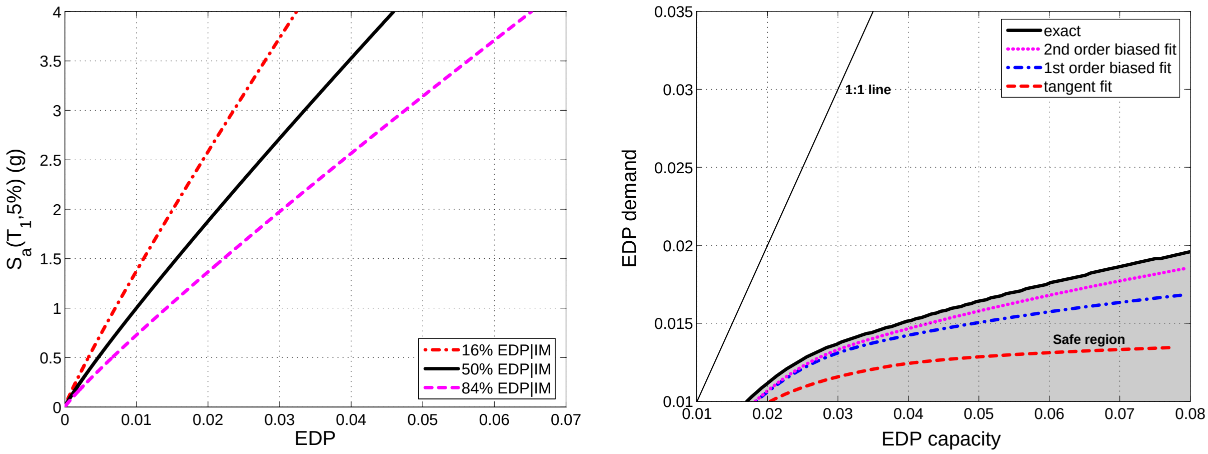
For testing the DCFD format we chose two fictional structures. Building A has a first-mode period of T_1 = 0.7s and its median structural EDP response follows $\hat{\theta} = 0.01 S_a^{1.0}$ (Fig. 5.2a). Building B has T_1=2.4s and is characterized by $\hat{\theta} = 0.01 S_a^{1.1}$ (Fig 5.3a). The longer period of the second structure engages a seismic hazard curve with higher local curvatures, resulting in a more severe test. In both cases the constant demand and capacity dispersions are $\beta_{\theta d} = \beta_{\theta c} = 0.35$. These values could be attributed either to aleatory randomness only or both aleatory and epistemic uncertainty sources, a choice that has no impact on the (mean MAF based) results. To properly visualize the discriminatory ability of the DCFD format to tell apart safe from unsafe situations, we will display the edge of the safe region in terms of the median demand $\hat{\theta}_{po}$ versus the median capacity $\hat{\theta}_c$, i.e., as determined by assuming equality in Eqns. 3.6 and 4.13. To serve as the basis of comparison, we will also plot the edge corresponding to the exact result of P_o being equal to the numerically estimated MAF. In all cases, safety checking will be performed at the level of the mean estimates consistent with Eqns. 3.6 and 4.13, corresponding to a confidence level somewhere above 50%.

Fig. 5.2b shows the boundary of the safety region between demand and capacity for building A. Clearly, the tangent fit is off the mark with its error predictably increasing for higher capacities (i.e., higher hazard curvature). On the other hand, the biased 1st and 2nd order fits offer a relatively good, slightly conservative approximation. For example, a median capacity of 0.050 should be able to resist an EDP demand of exactly 0.028, but this is estimated as 0.027 by the two improved methods. The original tangent fit instead would only accept a median demand lower than 0.025, obviously restricting the estimated structure's ability to absorb damage. The differences in Fig. 5.3b tell a worse story. At first glance, they may not look particularly excessive but the slopes can be deceiving. If a median demand of 0.015 is recorded, then the structure should in reality have an EDP capacity of 0.038 to resist it successfully. If this is approximated by a tangent fit DCFD format, then the needed capacity is off the charts, practically infinite. A biased 1st order fit manages a better approximation at 0.050, while the 2nd order fit achieves a much improved estimate of 0.42. In general it seems that as the safety boundary veers further away from the 1:1 demand/capacity ratio, fitting accuracy matters more.



(a) fractile IDA curves (b) allowable EDP demand versus capacity

Figure 5.2. The DCFD comparison results for Building A, a low-curvature, 'good case' (T_1 = 0.7s, EDP = $0.01S_a^{1.0}$, dispersions of 0.35 for demand and capacity)



(a) fractile IDA curves (b) allowable EDP demand versus capacity

Figure 5.3. The DCFD comparison results for Building B, a high-curvature, 'bad case' (T_1 = 2.4s, EDP = $0.01S_a^{1.1}$ dispersions of 0.35 for demand and capacity)

6. CONCLUSIONS

A concise investigation of possible improvements for the SAC/FEMA closed form solution has been presented. Two hazard fitting approaches have been proposed, offering increased accuracy at a negligible cost. The first is a left-weighted, right-biased fitting of the 1st order power-law function. The second is a biased fitting of a 2nd order power-law together with a novel closed form expression. The latter is shown to be remarkably efficient, nearly zeroing-out the excessive errors due to the curvature of the seismic hazard. The improvement is so overwhelming that we can emphatically say that tangentially fitting a power-law function in log-space should be avoided at all costs.

AKCNOWLEDGEMENT

Financial support for the research was provided by the European Research Executive Agency via Marie Curie grant PCIG09-GA-2011-293855. I also wish to gratefully acknowledge the guidance and friendship of the late Dr. H. Krawinkler who was a source of inspiration throughout my academic career.

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