A Performance Modeling Strategy based on Multifiber Beams to Estimate Crack Openings in Concrete Structures

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SUMMARY
Crack opening computation is one of the most challenges for concrete structures applications. This work deals with the development of a simplified modeling strategy using fiber beams 3D in order to estimate crack openings in concrete structures. In contrast to a detailed macroscopic finite element modeling, multifiber beams theory offers a flexibility of calculation thanks to the small number of freedom degrees. Constitutive laws based on damage and plasticity are introduced in order to take into account of non linearity. For concrete, the damage model developed in (Matallah *et al*, 2009) is implemented on the finite Element Code Cast3M into fiber element. This model is able to take into account the unilateral effect. A macroscopic post-processing method developed previously by the authors (Matallah et al 2010) is also adapted to multifiber modeling in order to estimate crack openings. Beams under cyclic loadings are considered for validation. The numerical results are compared to those given by the experimentation.

KEYWORDS: *Crack opening, damage, plasticity. multifiber beams.*

1. INTRODUCTION

Reinforced concrete structures are usually designed to allow cracking under service loadings. Numerical modeling of crack initiation and propagation is therefore important regarding structural safety and durability. Several damage and plasticity/damage models were developed in order to model the complex behavior of concrete (Matallah *et al* 2009, Raguneau *et al* 2008). Based on continuous approach, these models are not able to describe crack openings. A classical continuous finite element computation gives a distribution of internal variables (damage, plasticity...) over a finite element. Crack opening values are not directly available. A practical method was developed by the authors in order to extract a crack opening from a continuous finite element computation (Matallah *et al*, 2010).

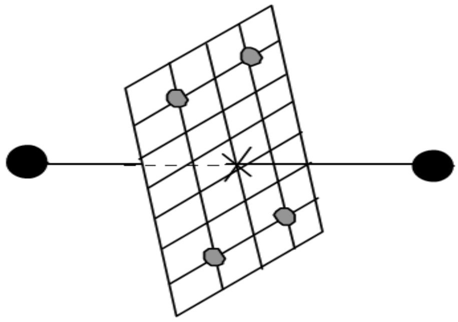
In order to design the non linear behavior of reinforced concrete structures under seismic loading, many approaches could be adopted. The main refined efficient approach would be to perform a non linear time history analysis using sophisticated non linear model and a 2D or 3D spatial discretization. However, this approach is not advantageous regarding the excessive computational cost. Multifiber beam theory, in contrast to a detailed finite element modeling approach, offers a flexibility of calculation thanks to the small number of freedom degrees. Constitutive laws based on damage and plasticity are introduced in order to take into account of non linearity.

The main purpose of this study is to extend the applicability of the practical method proposed in (Matallah et al 2010) to evaluate crack openings in concrete structures using a multifiber beam theory. The non linear damage model OUF (Matallah *et al*, 2009) is implemented in cast3M (Verpaux *et al*, 1998) into fiber element. This model is able to take into account of the unilateral effect. Firstly, we recall the main fundamental basis of the multifiber beam theory. Secondly, the OUF damage model formulation and the practical method (the “OUVFISS” procedure) basis are recalled. The OUF Model is implemented into fiber beam. Structural applications are considered for validation. Beam under cyclic loadings are simulated and the crack opening values are compared to those given by experimental data.

2. MULTI-FIBER MODELING

In order to perform a 3D non linear analysis of concrete structures behavior, the multi-fiber approach is adopted (Guedes *et al*, 1994). Even if the spatial representation is simplified, the inelastic behavior of concrete is still well represented. The multifiber beam approach allows taking into account of non linear constitutive behavior.

In the present study, the Timoshenko beam element is used. In contrast with Euler-Bernoulli hypothesis, it takes into account of the shear effects. The cross section remains plan but not necessary perpendicular to the neutral axis. The element cross section is described using 2 dimensional elements (3 nods or 4 nods). Each fiber is associated with a non linear uniaxial law which represents the non linear concrete and/or steel behavior.



Beam Element : $(u,\theta) \Rightarrow (\varepsilon_0,\phi,\gamma)$ $\qquad (M,N,T)$

$\Downarrow \qquad\qquad \Uparrow$

At the fiber level : $(\varepsilon,\gamma) \quad \Rightarrow \quad (\sigma_{xx},\tau_{xy},\tau_{xz})$

Figure 2.1 Description of a multi-fiber beam model

The hypothesis of plane sections (Timoshenko theory) allows expressing the displacements $u(x, y, z)$, $v(x, y, z)$, $w(x, y, z)$ of any node of the beam as a function of the "sections variables" :$u_s(x), u_y(y), u_z(y), \theta_{sx}, \theta_{sy}, \theta_{sz}$. The deformation field takes the following form

$$\epsilon_{xx} = u'_s(x) - y\theta'_{sz}(x) + z\theta'_{sy}(x) \tag{2.1}$$

$$2\varepsilon_{xy} = v'_s(x) - \theta_{sz}(x) - z\theta'_{sx}(x) \tag{2.2}$$

$$2\varepsilon_{xz} = w'_s(x) + \theta_{sy}(x) + y\theta'_{sx}(x) \tag{2.3}$$

The element has higher order interpolation functions to avoid shear locking phenomena. More details about the multifiber Timoshenko element are given in (Guedes *et al*, 1994)(Mazars *et al,* 2006).

3. NUMERICAL IMPLEMENTATION OF AN INELASTIC-DAMAGE MODEL USING MULTIFIBER BEAMS

The inelasticity damage base model developed by (Matallah *et al,* 2009) has been adopted. The model is able to describe the unilateral effect. Inelasticity is described by a tensor variable ε^{uco} which describes the cracking effect. Damage is described by an isotropic variable D. The model is based on a new combined formulation where damage is considered as a multiplier of the Unitary Crack Opening variable

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - D\varepsilon^{uco}_{kl}) \tag{3.1 }$$

To take into account the crack closure effect, the Unitary Crack Opening variable is related to the inelastic strain tensor using a scalar variable called the Crack Opening Indicator "S".

$$\varepsilon^{uco}_{ij} = S\varepsilon^{in}_{ij} \tag{3.2}$$

When cracks are closed, the function S should take the value 0. The material recovers its stiffness completely, but damage and inelastic strains are retained

$$\forall D, \forall \varepsilon_{ij}^{uco} S = 0 \Rightarrow \sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{3.3}$$

In order to implement the model into multifiber in Cast3M, only the uniaxial form is needed. The Uniaxial stress-strain form of the damage model is given by :

$$\forall\, \varepsilon^{in}, \forall\, D, \sigma = E\left(\varepsilon - DS\varepsilon^{in}\right) \tag{3.4}$$

S : The Crack opening Indicator

If S=1 Crack completely open : $\varepsilon^{uco} = S\varepsilon^{in} = \varepsilon^{in}$ (3.5)

If S=0 Crack completely closed : $\varepsilon^{uco} = S\varepsilon^{in} = 0$ (3.6)

If : S=1, the effective stress takes the following form:

$$\tilde{\sigma} = E\left(\varepsilon - \varepsilon^{in}\right) = E(\varepsilon - \varepsilon^{uco}) \tag{3.7}$$

For numerical consideration, the inelastic behavior and the damage effect are decoupled. Firstly, the inelastic behavior is considered where the inelastic strain tensor is calculated using the standard plasticity theory (normality rule). The inelastic strain tensor is given by

$$\dot{\varepsilon}^{in} = \dot{\lambda}\frac{\partial F}{\partial \tilde{\sigma}} \tag{3.8}$$

The loading function F is composed from two Drucker–Prager yield contours. One to limit the tensile stress and the other to model the compression and(compression–compression) regime in bi-axial stress. A non associated rule is adopted (Matallah *et al*, 2009). The damage is function of the inelastic strain

$$\dot{D} = \dot{\lambda}\xi\exp(-\xi p) \tag{3.9}$$

$\dot{\lambda}$ Is the inelastic multiplier, ξ is a constant parameter and p is the accumulated inelastic strain.

The inelastic multiplier is given by

$$\dot{p} = -\dot{\lambda}\frac{\partial F}{\partial R} = \dot{\lambda} \tag{3.10}$$

The model is implemented in Cast3M using the Generalized Cutting Plan Algorithm (Jirasek , 2001).

4. EVALUATION OF CRACK OPENINGS

A continuous modeling approach is used for the assessment of the crack openings. This procedure developed by (Matallah *et al*, 2010) is implemented in Cast3M (OUVFISS procedure). This post-processing method is based on an energetic regularization. I e, the crack is assumed to be localized in a band with a certain width over which we consider that micro-cracks are uniformly distributed.

The band width “h” is introduced to avoid spurious mesh sensitivity.

The energy dissipated per unit width (or unit length) is a constant and given by

$$G_f = \int_0^\infty \sigma\, d\delta \tag{4.1}$$

Where $d\delta$ is the crack opening displacement. The material behavior in the fracture process zone is characterized in a smeared manner through a strain softening constitutive relation. The crack opening displacement tensor is taken as the fracture strain accumulated over the width h of the finite element.

$$\delta_{ij} = h * \varepsilon_{ij}^{uco} \tag{4.2}$$

Where we consider ε_{ij}^{uco} the fracture strain tensor (the Unitary Crack Opening strain tensor Matallah *et al.* 2009). In the fracture zone, the total strain consists of an elastic portion and a fracture portion. The fracture energy represents the area under the stress-strain curve.

$$G_f = h\int_0^{\infty} \sigma \, d\varepsilon \tag{4.3}$$

We consider that each element is crossed by one crack. The total strain in the fracture element is written as:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{uco} \tag{4.4}$$

This gives after multiplication by the elastic stiffness tensor:

$$\tilde{\sigma}_{ij} = C_{ijkl}\varepsilon_{kl} = C_{ijkl}\varepsilon_{kl}^{e} + C_{ijkl}\varepsilon_{ij}^{uco} = \sigma_{ij} + \sigma_{ij}^{in} \tag{4.5}$$

From a finite element computation using a non linear model, we get a nominal (total) stress σ. The effective stress is computed using the elastic total strains. The inelastic stress is given by:

$$\sigma_{ij}^{in} = \tilde{\sigma}_{ij} - \sigma_{ij} \tag{4.6}$$

The fracture strain tensor ε_{ij}^{uco} is related to the inelastic stress tensor

$$\varepsilon_{ij}^{uco} = C_{ijkl}^{-1}\, \sigma_{ij}^{in} \tag{4.7}$$

The crack opening displacement normal to the crack is given by :

$$\delta_n = n_i\, \delta_{ij}\, n_j \tag{4.8}$$

Where δ_{ij} is the crack opening displacement tensor. Crack openings are computed considering that each cracked element is crossed by one crack. An average value is therefore given by :

$$\delta_n = \int_{element} \varepsilon_n^{uco}\, dn = \int_{element} n_i\, \varepsilon_{ij}^{uco}\, n_j\, dn \tag{4.9}$$

n : is the direction normal to the crack.

This method could be applied to all non linear damage or damage-plastic model. The inelastic damage based model exposed above is used. Nevertheless, the approach can only be applied if the model allows fracture energy regularization. Thus, for this purpose, for the model proposed, a linear hardening law is supposed for tension loading. So, the relation between ε^{uco} and the strain tensor is linear and it is given by

$$\varepsilon^{uco} = \varepsilon^{fr}\, \frac{\varepsilon - \varepsilon^{d0}}{\varepsilon^{fr} - \varepsilon^{d0}} \tag{4.10}$$

Where ε^{fr} is the fracture strain variable and ε^{d0} is the strain threshold. The fracture energy is given by :

$$G_f = \int_0^{\infty} \sigma\, d\delta = h\int_0^{\infty} E(\varepsilon - D\varepsilon^{uco})\, d\varepsilon^{uco} \tag{4.11}$$

With the linear hardening law, this equation gives

$$\frac{G_f}{h} = \frac{1}{2}\varepsilon^{fr} f_t + \frac{E}{\xi^2} \tag{4.12}$$

f_t is the tensile strength.

5. APPLICATION FOR CONCRETE STRUCTURES

5.1. Reinforced Concrete Beams under Static and Cyclic Loadings

The simulation is carried out on the LMT beam which has been proposed in the French national Project MEFISTO. The beam geometry is (15*20*170) cm^3. For reinforcement, four bars of 12mm diameters are symmetrically located in the reinforced concrete beam (Fig. 5.1). The beam is subjected to several displacements loading cycles. For each load level, three sets of cycles are imposed making it possible the stabilization of the crack processing (Fig. 5.2).

The modeling is performed using the Multifiber Timoshenko beam elements. The concrete constitutive law is based on the principles of damage mechanics (The damage model exposed above). The behavior of steel is modeled by the Pinto-Menegotto laws (Menegotto *et al*, 1973). The concrete model parameters are given in Table 5.1. For steel, the Young's modulus is E = 205 000 MPa and the tensile limit is taken equal to 600 MPa, these parameters are evaluated from tensile tests. Computations are performed using the Finite Element code CAST3M.

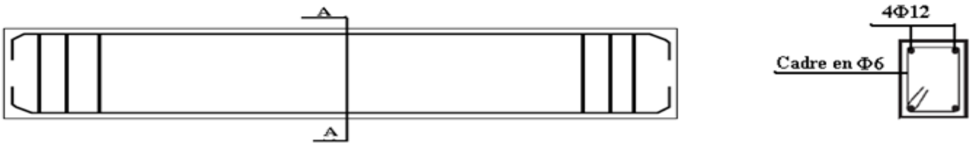


Figure 5.1 The longitudinal and transverse reinforcements of the LMT beam.

The loading history is given in Fig 5.2

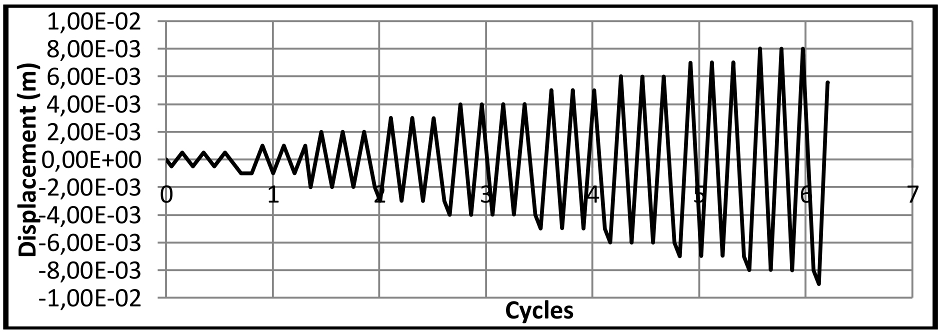


Figure 5.2 The loading history.

Table 5.1 Concrete Model Parameters

Parameters	Kconst (Pa)	ξ	G_f (N/m)	EPSR (m)	SIGF (Pa)
Values	$3x10^{+6}$	$4x10^{+4}$	100	$2x10^{-3}$	$1.5x10^{+6}$

5.2. Numerical Results under Monotonic Static Loadings

A first simulation is carried out under static monotonic loading with different discretizations in order to valid the fracture energy regularization approach. The beam has been discretized so that the finite element length is respectively l_c =10cm, l_c =5cm and l_c = 2cm.

The following figure (Fig. 5.3) represents the global Force-Displacement response for each discretization.

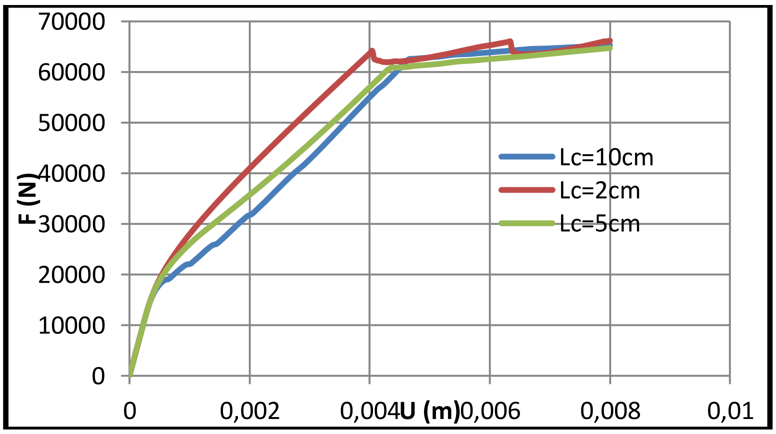


Figure 5.3 The Global response of the LMT beam under static monotonic loading.

5.3. Numerical Results under cyclic Loadings

Experimentally, two cyclic tests were performed on the same geometry of the beam permitting to appreciate the repeatability of the experimental protocol. The following figure shows the global behavior result Force-Displacement for the two tests (Lebon *et al*, 2010).

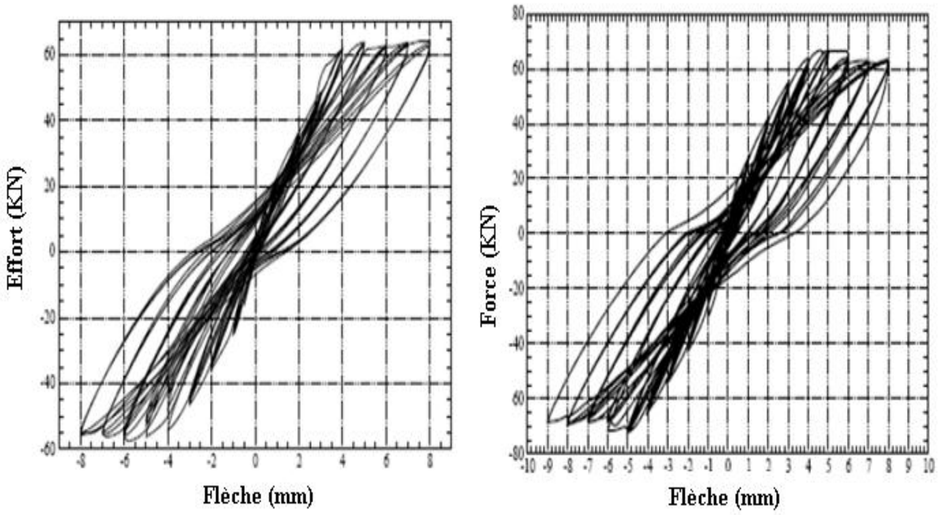


Figure 1.4 The experimental global behavior response a-Test1 b-Test2

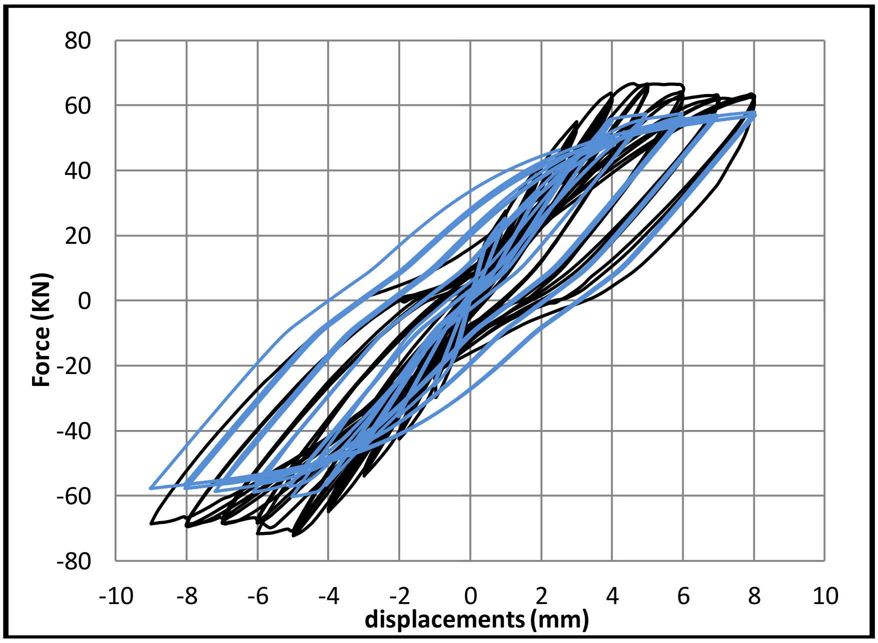


Figure 5.5 Comparison Model-Test responses (Test 2)

The global behavior is well reproduced as shown in Fig. 5.5. After the plastification of the longitudinal reinforcements, the form of cycles evolves according to steel behavior.

In order to evaluate the crack opening values, the procedure ‘’OUVFISS’’ is used. After a finite element computation using the damage model exposed above. This post-processing method gives the evolution of crack opening. Fig. 5.6 shows the evolution of crack opening under static monotonic loading. The maximum crack opening value is about 0.2 mm . This value is a good approximation compared with the experimental one (Lebon *et al*, 2010).

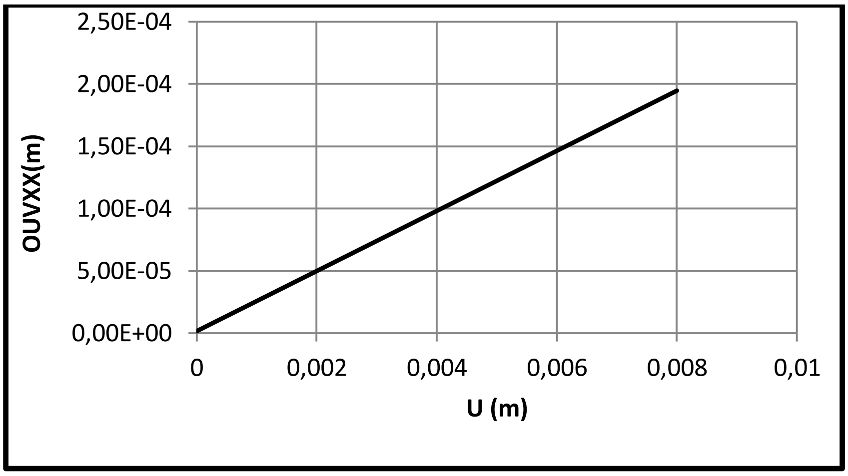


Figure 5.6 The evolution of the axial crack openings under monotonic static loadings (Crack openings Vs Imposed displacements)

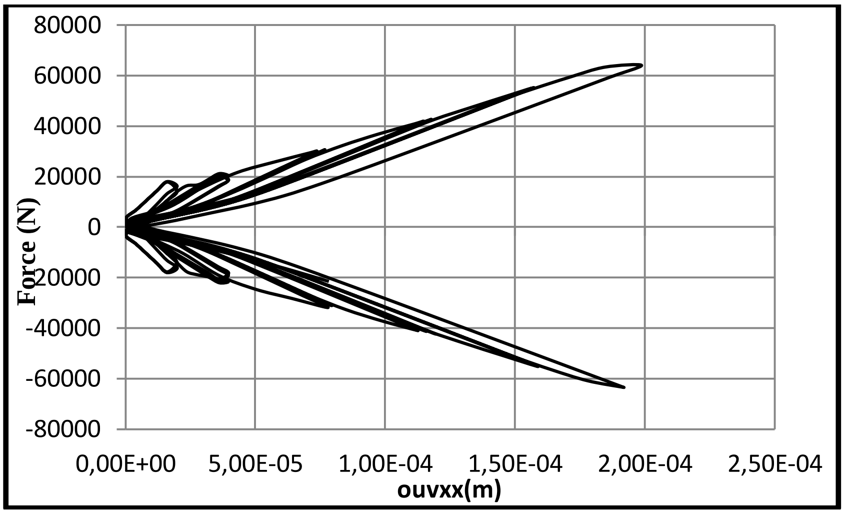


Figure 5.7 The evolution of the axial crack openings under cyclic loadings (Forces Vs Crack Openings)

Fig. 5.7 shows the evolution of the crack openings under cyclic loading. The crack closing process is well reproduced. When the load is canceled, the cracks are closed even if the material presents damage and inelastic strains.

6. CONCLUSIONS

In Earthquake Engineering, the multifiber beam modeling approach is commonly used. This approach presents many advantages (a reduced computation cost, a sufficient description of non linear constitutive behavior laws ..). However, the evaluation of crack openings is still a hard task. In this paper, a simplified modeling approach is proposed in order to evaluate crack openings under cyclic loading. The multifiber beams are used to reduce the computational cost. A practical method previously developed by the authors has been adapted to assess crack openings. The numerical results are in a good agreement compared with those given by the experimentation.

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