SEISMIC BEHAVIOR OF CHEVRON-BRACED RC FRAMED BUILDINGS

E. A. Godínez-Domínguez
Universidad Autónoma de Chiapas, Mexico

A. Tena-Colunga
Universidad Autónoma Metropolitana-Azcapotzalco, Mexico

L. E. Pérez-Rocha
Instituto de Investigaciones Eléctricas, Mexico

SUMMARY
The results of a study devoted to evaluate, using nonlinear dynamic analyses, the seismic behavior of six reinforced concrete moment resisting chevron braced framed buildings (RC-MRCBFs) are presented and discussed in this paper. Buildings were designed using a proposed capacity design methodology adapted to the seismic, reinforced concrete and steel guidelines of current Mexico's Federal District Code and the Manual of Civil Structures. Two-dimensional models that account for the interaction among frames were used for the nonlinear dynamic analyses of the capacity-designed buildings using RUAUMOKO software. Several artificial records corresponding to the maximum credible earthquake associated to the design spectra were used to carry out the nonlinear dynamic analysis. From the results obtained, it is possible to conclude that if capacity design principles and specific design parameters for the new design of RC-MRCBFs are used, suitable global ductility capacities and overstrength demands are obtained, and a satisfactory structural performance is achieved.

Keywords: ductile steel braced frames, reinforced concrete frames, capacity design, nonlinear dynamic analysis.

1. INTRODUCTION

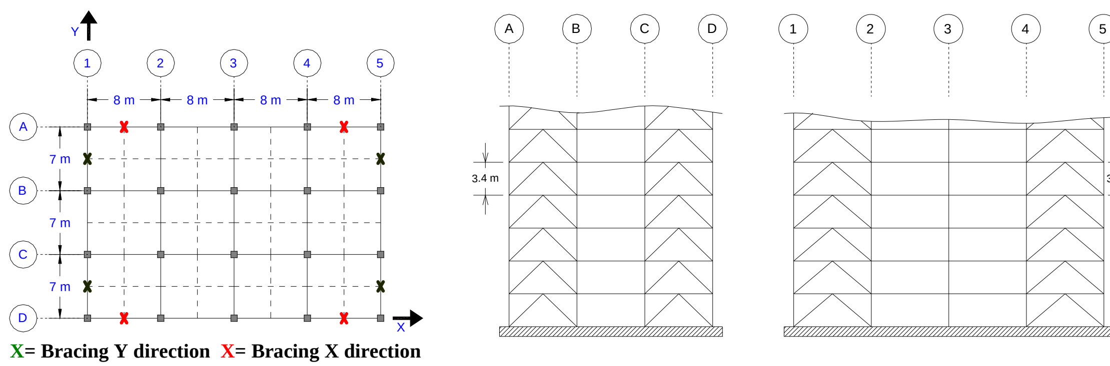
In the first stage of the research study conducted by the authors (Godínez-Domínguez and Tena Colunga 2010, Godínez-Domínguez 2010) the behavior of low to medium rise moment-resisting reinforced concrete concentric braced frames structures (ranging from four to 24 stories) subjected to lateral seismic loading was discussed. Also, some key design parameters were proposed, such as: (a) story drift at yielding, (b) ultimate drift capacities, (c) overstrength reduction factors and, (d) minimum required shear strength provided by the columns of the RC-MRCBFs to achieve good structural behavior. Some specific changes to the element design methodology available in Mexican codes were also presented.

In the second stage, in which nonlinear dynamic analyses of two-dimensional models that account for the interaction among frames were performed, it was important to complement and validate the proposed and updated capacity design methodology for this dual system for the following reason. Key design parameters proposed in the first stage of this study were assessed using 2D nonlinear static analysis (pushover) only. Then, the main objective of this second stage was to verify if new RC-MRCBFs buildings of different heights designed using a code-oriented capacity design procedure and the key design parameters obtained from pushover analyses, could achieve a satisfactory final collapse mechanism (strong column – weak beam – weaker brace) and develop suitable global and local: a) ductility capacities, b) story drifts at yielding, c) ultimate drift capacities and, d) overstrength demands. For this purpose, the design of buildings of three different heights (8, 15 and 24 stories) was assessed, using the general procedure described in greater detail by Godínez-Domínguez (2010) and briefly in following sections.

2. SUBJECT BUILDINGS

Six steel chevron-braced reinforced concrete framed buildings were designed for two different guidelines: the Manual of Civil Structures (MOC-2008, 2009) and Mexico's Federal District Code (MFDC-04, 2004). Buildings were assumed to be located in different soil conditions in Mexico: firm soils of the coastline of the state of Guerrero (according to MOC-2008) and soft soils corresponding to the lakebed of Mexico City (zones IIIa and IIIb of MFDC-04). A seismic response modification factor Q=4, the maximum allowed in Mexican codes for these structures, was used for the design.

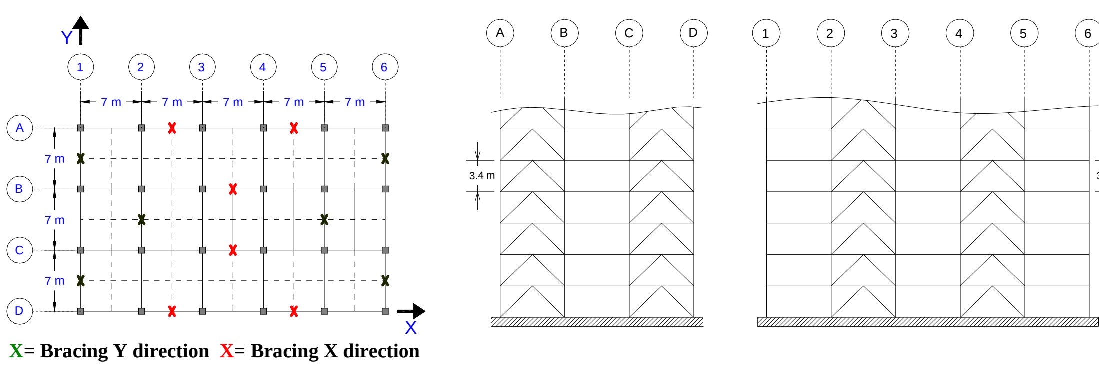
Buildings were eight, 15 and 24 stories in height and were assumed to be office buildings. For each height, buildings were designed for two different floor plans. The eight and fifteen story buildings are regular. Nevertheless, for the 24 story models, slenderness ratios are greater than 2.5 ($H/L > 2.5$) and, according to Mexican seismic codes (MOC-2008, MFDC-04), these buildings were designed as irregular structures. The typical floor plans considered in the study are shown in Figure 1 (which identifies T1 buildings) and in Figure 2 (which identifies T2 buildings). A typical story height of 3.4 m was assumed for all models. RC-MRCBFs buildings were designed for a specific shear strength ratio between the bracing system and the moment frame system depending on the slenderness ratio (H/L).



a) Floor plan

b) Elevation of perimeter frame models in *Y* direction (axes 1 and 5) and in *X* direction (axes A and D)

Figure 1. Floor plan and elevation view of the studied frames for T1 buildings



a) Floor plan

b) Elevation of perimeter frame models in *Y* direction (axes 1 and 6) and in *X* direction (axes A and D)

Figure 2. Floor plan and elevation view of the studied frames for T1 buildings

Following a common design practice by structural engineers in Mexico, the designed member sections for each RC-MRCBFs varied along the height of the frame. In order to prevent, as much as possible, important stiffness and strength irregularities along the height of the buildings, the cross sections of beams and columns change at stories different from those where the cross sections of the chevron braces change. Therefore, beams and columns change their cross section and/or steel reinforcement every four stories for eight story models and every five stories for the 15 and 24 story models. The box cross section of the steel bracing typically changes every three stories in building models of eight and 15 stories, but for the twenty-four building models, the cross section changes every four stories in order to achieve a design as optimum as possible.

3. DESIGN METHODOLOGY

A conceptual capacity design methodology has been used in this research for the design of RC-MRCBFs. The methodology, which is briefly described by Godínez-Domínguez and Tena-Colunga (2010) and commented in detail elsewhere (Godínez-Domínguez 2010), explicitly takes into account the sequence for designing resisting elements in order to warrant the expected collapse mechanism (strong column–weak beam–weaker brace): (1) bracing elements, (2) beams, (3) columns, (4) connections between the frame and the bracing system and, (5) panel zone (joint area). The axial force transmitted from the bracing system to connections, columns, as well as to the beams subjected to such forces because of the bracing configuration, is addressed in this design procedure.

Also, some design recommendations, based on the results of a previous study devoted to evaluate, using static nonlinear analyses, the behavior of low to medium rise ductile moment-resisting reinforced concrete concentric braced frames structures (models ranged from 4 to 24 stories) using chevron steel bracing were used. These results are described and commented in detail elsewhere (Godínez-Domínguez and Tena-Colunga 2010, Godínez-Domínguez 2010). The following design parameters were proposed: a) overstrength reduction factor (R, Eqn. 3.1), b) story drift limit for the serviceability limit state (Eqn. 3.2), c) story drift limit for the collapse prevention limit state (Eqs. 3.3 and 3.4), d) minimum required shear strength percentage provided by the columns of the RC-MRCBFs to resist earthquake loading (Eqn. 3.5). In Eqs. 3.1 to 3.5, the following notation is used: T_e is the natural period for the building, T_a is the control period that defines the starting point of the plateau for the design spectrum, R is the overstrength factor (Ω_0 in US codes), Δ_y is the story drift for serviceability limit state, Δ_{max} is the story drift for collapse prevention limit state, V_{RCol} is the minimum shear strength percentage provided by the columns, H is the height of the building and L is the smallest dimension in plan for the subject building.

$$R = \begin{cases} 1.7 + 2.3\left(1 - \sqrt{T_e / T_a}\right) & if \quad T_e \le T_a \\ 1.7; & if \quad T_e > T_a \end{cases} \tag{3.1}$$

$$\Delta_y = 0.002 \tag{3.2}$$

$$\Delta_{\text{max}-\text{proposed}} = 0.013 \tag{3.3}$$

$$\Delta_{\text{max}-\text{NTCS-04}} = 0.015 \tag{3.4}$$

$$V_{RCol} \ge 50 + 1.2\left(\frac{H}{L}\right)^2 \tag{3.5}$$

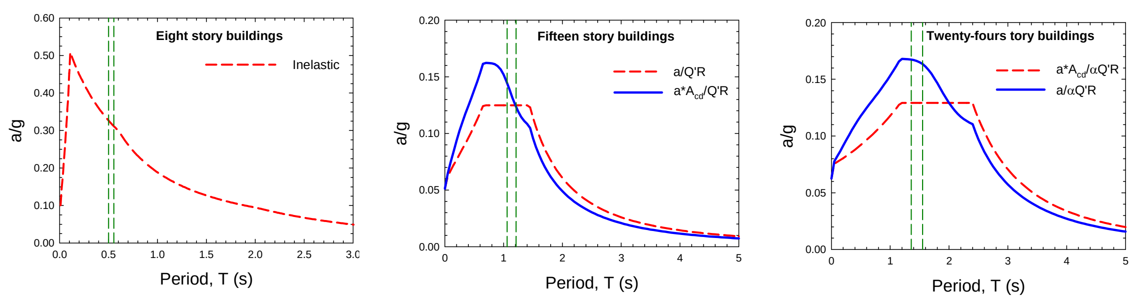
3.1. Soil Conditions and Inelastic Design Spectra for each Studied Building

Due to the dynamic characteristics of eight story buildings, they were assumed to be located in firm soils of the coastline of the state of Guerrero, which according to MOC-2008 (MOC-2008 2009, Tena-Colunga *et al.* 2009), it represents one of the zones with highest seismic hazard in the Mexican territory. The fundamental periods for the translational modes for the 8T1 building were T_{1X}=0.554s and T_{1Y}=0.542s, and for the 8T2 buildings were T_{1X}=0.528s and T_{1Y}=0.501s. Fifteen story buildings (15T1 and 15T2) were assumed to be located in the soft soils of lakebed zone IIIa of Mexico City according to the seismic guidelines of MFDC-04 (NTCS-04 2004), because in this zone dominant ground periods are near to the structural natural periods of the studied buildings and therefore, important inelastic demands are expected due to near resonant response. This fact would allow evaluating the effectiveness of the proposed design parameters and procedure to achieve a satisfactory structural performance under severe earthquake shaking. The fundamental periods for the translational modes for the 15T1 building were T_{1X}=1.212s and T_{1Y}=1.143s, and for the 15T2 buildings were T_{1X}=1.06s and T_{1Y}=1.061s. Finally, twenty-four story buildings (24T1 and 24T2) were assumed to be located in soft soils of the lakebed zone IIIb of Mexico City according to the seismic guidelines for MFDC-04 (NTCS-04 2004), which represent the highest seismic hazard zone in Mexico City. The structural natural periods for the translational modes for the 24T1 building were T_{1X}=1.357s and T_{1Y}=1.418s, and for the 24T2 buildings were T_{1X}=1.486s and T_{1Y}=1.533s.

Inelastic acceleration design spectra were computed, for each building, according to the seismic provisions considered in the design (Figure 3). Vertical lines depicted in Figure 3 indicate the range of structural natural periods for the eight, fifteen and Twenty-four story buildings reported above.

According to the results presented by Miranda and Ruiz (2002) and Terán (2005), to obtain reasonable lateral design strength estimations for structures with degrading hysteretic behavior (stiffness) located in soft soils, it is important to take into account the effect of the hysteretic behavior. Due to aforementioned, to obtain the inelastic design spectrum, the reduced spectral ordinates were affected by a degrading hysteretic behavior factor A_{cd} formerly proposed in MOC-2008 for the design of stiffness degrading structures in soft soils (15 and 24 story buildings, Figs. 3b and 3c). For comparison purposes, the inelastic design spectrum in which the effect of the degrading hysteretic behavior factor A_{cd} is neglected is presented in Figure 3.

It is worth noting that in twenty-four story models, the slenderness ratio in the two studied buildings is 3.89, greater than the limit H/L=2.5 allowed in NTCS-04 (NTCS-04 2004) for the design of regular structures. Therefore, according to NTCS-04, these structures were designed as irregular structures. Also, torsional effects were considered in the design process of all building models. In addition, all response quantities (displacements, internal forces, etc.) were obtained by combining each orthogonal response using the CQC combination procedure.



a) Eight story buildings b) Fifteen story buildings c) Twenty-four story buildings

Figure 3. Inelastic acceleration design spectra for eight, fifteen and twenty-four story buildings

4. NONLINEAR DYNAMIC ANALYSES

In order to assess the global and local seismic behavior of the capacity-designed buildings, nonlinear dynamic analyses of two-dimensional models that account for the interaction among frames were performed using RUAUMOKO software (Carr 2004). The bracing elements were modeled using the Remennikov model (Remennikov and Walpole 1997). The reinforced concrete elements were modeled using the modified Takeda model. Both hysteretic models were taken from RUAUMOKO library. Several artificial records associated to the design spectra were used to carry out the nonlinear dynamic analysis.

4.1. Modeling Assumptions

P-Δ effects were considered in all analyses. Soil-structure interaction was not included to avoid the introduction of other variables that may interfere with the interpretation of results.

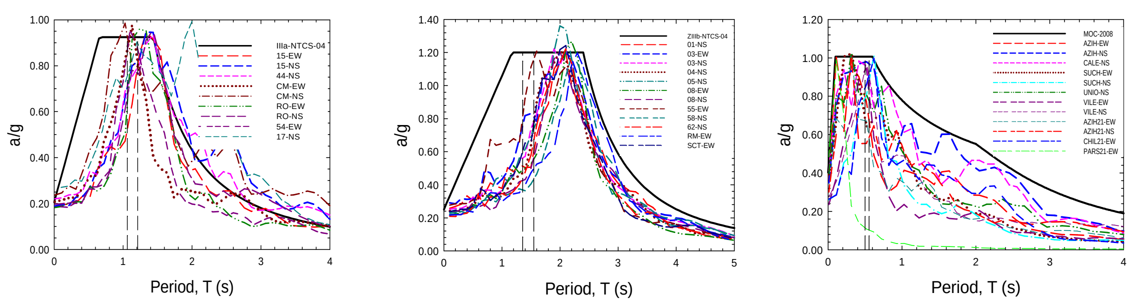
Overstrength due to concrete confinement, using the modified Kent-Park model (Park *et al.* 1982), and the stress-strain curve for the steel reinforcement proposed for rebars produced in Mexico F_y=450 MPa (4,577 kg/cm^2 or 65 ksi) was considered for RC beams and columns (Rodríguez and Botero 1995). The contribution of the slab reinforcement to the resisting bending moments of beams was also included in the assessment of overstrength capacities. For the assessment of overstrength in the steel bracing, F_y=360 MPa (3,670 kg/cm^2 or 52 ksi) was considered for A-36 steel for the determination of both tension and buckling loads (Bruneau *et al.* 1998).

In order to identify the different models, a cryptogram was defined: *Ndbn*, where *N* indicates the number of stories of the building, *d* indicates the direction of analysis (*X* or *Y*) and *bn* the floor plan used according to Figures 1 (T1) and 2 (T2).

4.2. Acceleration Records

Given that existing acceleration records corresponding to strong earthquakes are below the spectral accelerations considered in the design spectra specified in the building codes employed for the seismic design of all buildings (MOC-2008 for eight story models and NTCS-04 for fifteen and twenty-four story models), artificial records for a postulated M_s = 8.4 subduction earthquake for a selected number of stations located in each studied zone were obtained. The methodology used to generate the artificial records for the subduction earthquake scenario considered both in MOC-2008 and NTCS-2004 is described in detail elsewhere (Godínez-Domínguez and Tena-Colunga 2010, Godínez-Domínguez 2010).

Once identified and selected the accelerometric stations to be considered, numerical simulations were performed for each station in such a way that artificial acceleration records related to the seismic hazard of the studied zones were obtained, this is, artificial records which peak pseudo acceleration (S_a) and frequency content are comparable with the elastic design spectrum of the considered zone (Figure 4). It can be observed from Figure 4 that a good agreement between artificial elastic response spectrum for each station and the code-specified elastic design spectrum exists.



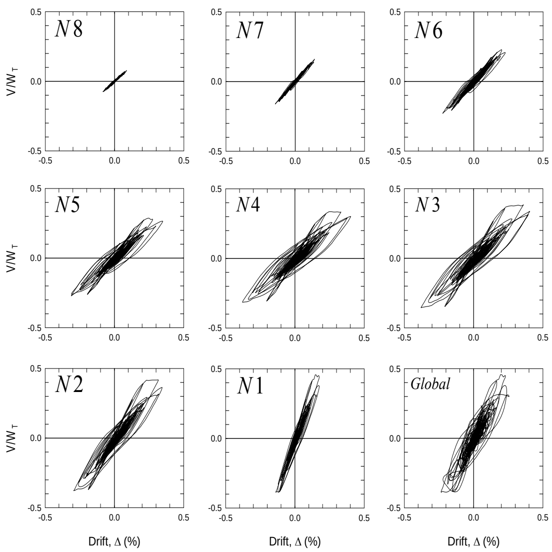
a) Zone III$_a$ of MFDC　　b) Zone III$_b$ of MFDC　　c) Coastline zone of the state of Guerrero

Figure 4. Elastic acceleration design spectra *vs* elastic response spectra obtained for each artificial record

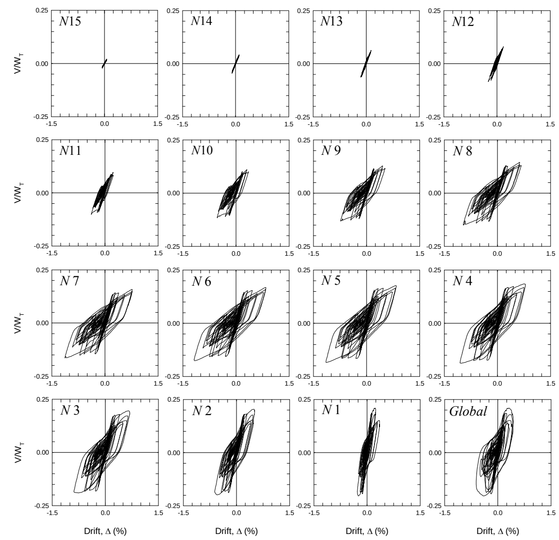
4.3. Processed Information

In order to save space, only selected results are shown in this section. The processed information is presented and discussed in much greater detail elsewhere (Godínez-Domínguez 2010).

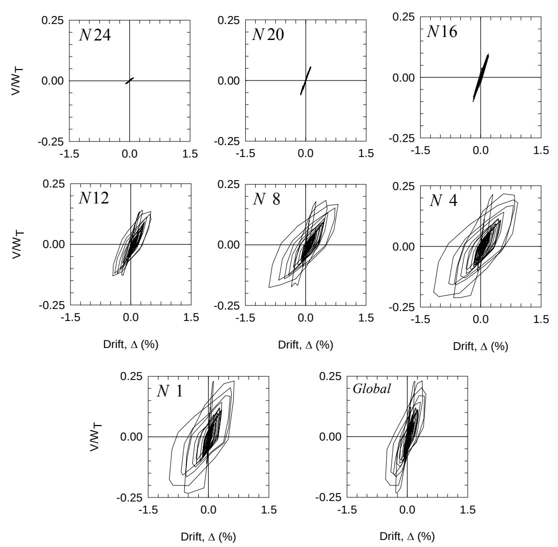
Normalized story and global hysteresis curves (V/W_T *vs* Δ) for the eight, fifteen and twenty-four story models (8XT2, 15XT2 and 24XT2) are shown in Figure 5. For the eight story models (Fig. 5a), it can be observed that, in spite of considering artificial acceleration records whose elastic response spectrum are comparable with the elastic design spectrum (Figure 4c, firm soil according to MOC-2008), a reduced inelastic behavior was developed in 8XT2 models (the same effect was observed in all eight story building models). Unlike eight story building models, a higher inelastic behavior was observed in all fifteen and twenty-four story building models, as it can be observed from story and global hysteresis curves (Figs. 5b and 5c), as well as from the normalized yielding mapping envelopes (Fig. 6).



a) 8XT2 model under the action of AZIH-NS

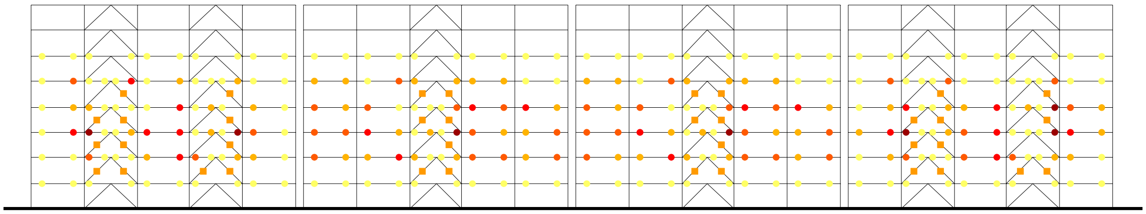


b) 15XT2 model under the action of 15-EW

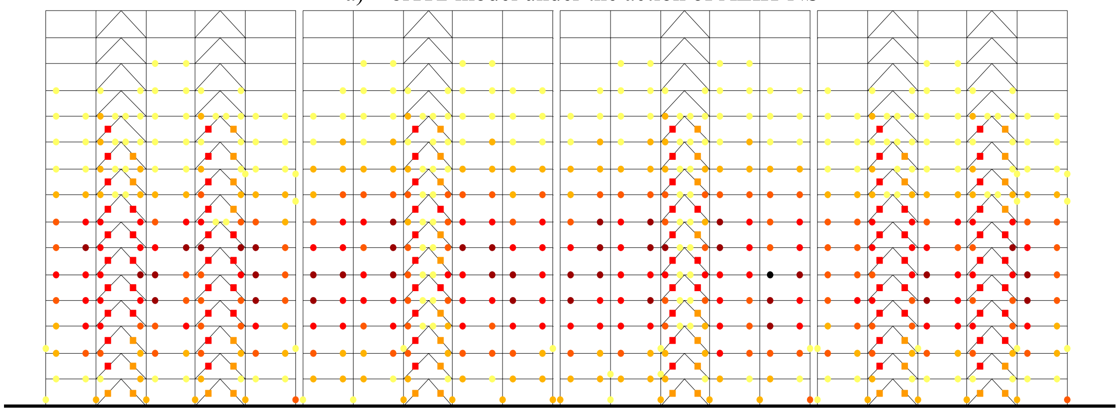


c) 24XT2 model under the action of 04-NS

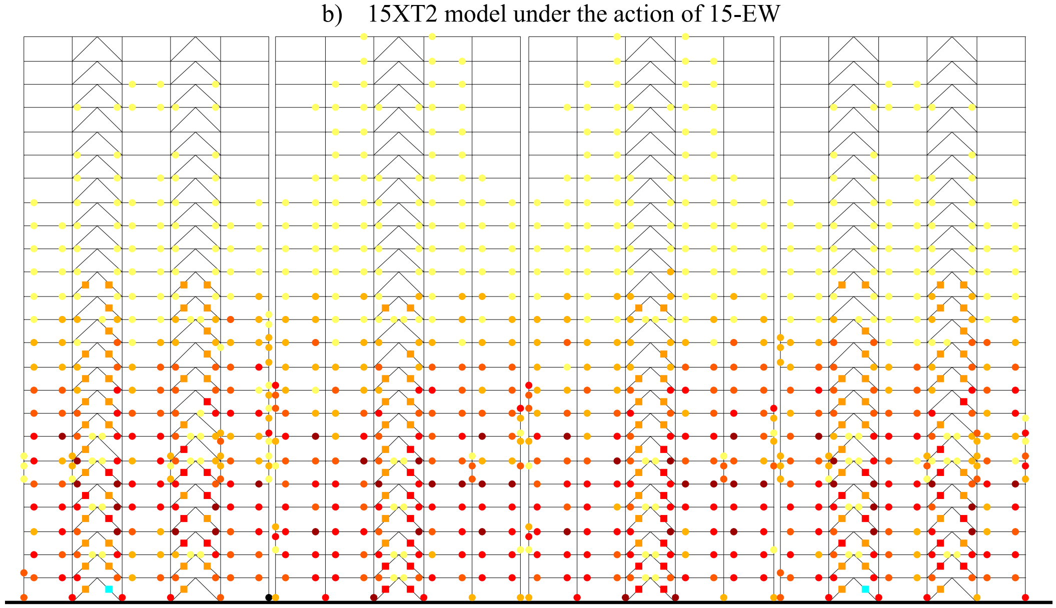
Figure 5. Normalized story and global hysteresis curves (V/WT *vs* Δ) for 8XT2, 15XT2 and 24XT2 models under the action of the acceleration record that generate response maxima.



a) 8XT2 model under the action of AZIH-NS



b) 15XT2 model under the action of 15-EW



c) 24XT2 model under the action of 04-NS

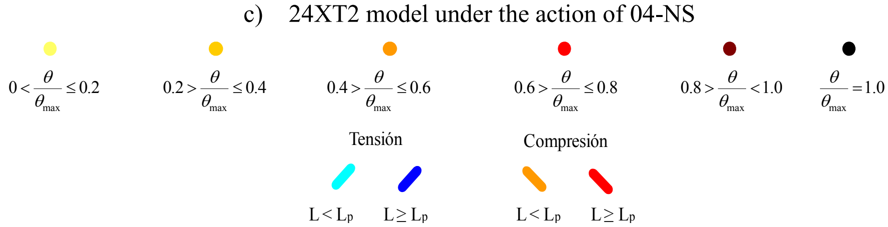


Figure 6. Mapping of accumulated plastic rotations for 8XT2, 15XT2 and 24XT2 models under the action of the acceleration record that generate the response maxima.

The following envelopes were also obtained from the story hysteresis curves: (a) story drift angles related to the first yielding of resisting elements, usually braces (Δ_{fe}), (b) equivalent story drift at yielding (Δ_y), corresponding to an elastic-perfectly plastic envelope of the hysteretic response defined according to what it is already proposed in the literature (Newmark and Hall 1982), (c) peak dynamic story drift angles ($\Delta = \Delta_i/H_i$), (d) average story secant stiffness of nonlinear half cycles (k_{ave}) normalized with respect to the elastic story stiffness (k_{el}), (e) equivalent total number of inelastic cycles (small and large amplitude) for each story, (f) peak story ductility demands (μ), and, (g) maximum dynamic story shear indexes (V/W_T). Some of the commented envelopes are presented in Figure 7 for 15XT2 model; one of the most demanded fifteen story models. Average responses were obtained using the results of at least nine artificial records.

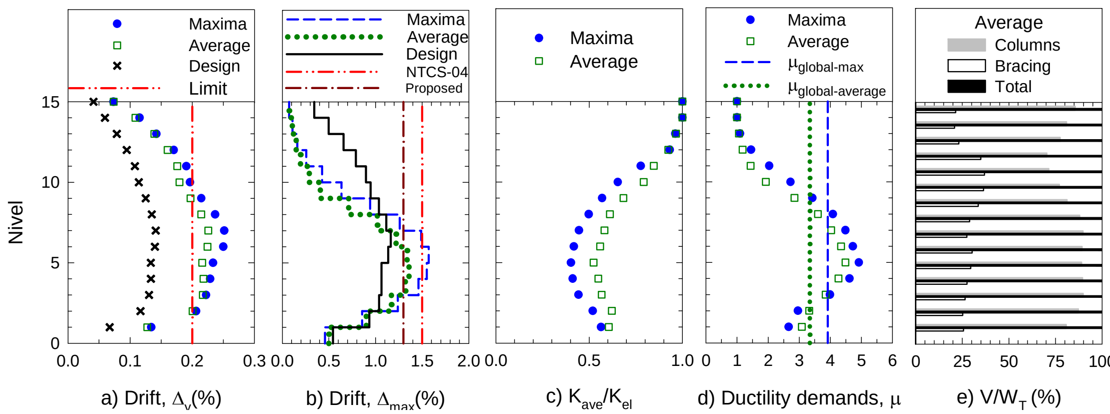


Figure 7. Envelopes of response maxima and average response for 15XT2 model

Half cycles were considered for assessing k_{ave} because of the important differences often observed in the amplitude of adjacent positive and negative half cycles due to the variation of the intensity of the ground motion. The equivalent number of nonlinear cycles that each story experienced during the dynamic response and the average story secant stiffness of nonlinear half cycles (k_{ave}) normalized with respect to the elastic story stiffness (k_{el}) are two parameters that are relatively easy to obtain and that can be very useful to give insight regarding cyclic characteristics.

It is worth noting that for simplicity, in Figure 7: a) detected elastic responses are identified with $\mu = 1$ and k_{ave}/k_{el}=1, b) the drift limit $\Delta_{ser} = 0.002$ (0.2%) proposed in the first stage of this research (Godínez-Domínguez 2010) for elastic response under the service earthquake is also marked for reference purposes, c) drift limits $\Delta = 0.013$ (proposed) and $\Delta = 0.015$ (NTCS-04, MOC-2008, ASCE-7-05), to check for collapse prevention limit state are also depicted for reference purposes.

Mexican seismic codes are moving towards design procedures where an overstrength factor *R* is directly used to reduce the elastic design spectrum. This is the philosophy in the procedure outlined in Appendix A of NTCS-04 (NTCS-04 2004) and in the new guidelines for the Manual of Civil Structures (MOC-2008 2009). Other modern international seismic codes have a similar design philosophy (i.e. UBC-97).

Therefore, it is of utmost interest to compare the proposed curve defined by Eqn. 3.1, which was proposed and used for the design of all buildings, with the demanded (or developed) overstrength for the RC-MRCBFs buildings when subjected to each consider artificial acceleration record. The maximum, minimum and average results are presented in Figure 8. The demanded overstrength was computed by dividing the obtained V/W_T ratio by the seismic coefficient used for their design. It is worth noting that all periods were normalized with respect to the control period T_a of their corresponding design spectrum, which varies in both NTCS-04 (for each considered zone) and MOC-2008. As commented before, the *R* factors used in the design (Eqn. 3.1) are based on the results of previous study (Godínez-Domínguez 2010, Godínez-Domínguez and Tena-Colunga 2010), where a series of nonlinear static analysis of RC-MRBFs were performed.

It can be observed from Figure 8 that for the range of considered periods, a reasonable agreement between the maximum and average developed overstrength and the proposed curve proposed in Eqn.

3.1. The developed overstrength for eight story models (stiffer models) is slightly smaller than for higher building models, as a consequence of the low inelastic behavior demanded in the eight story models.

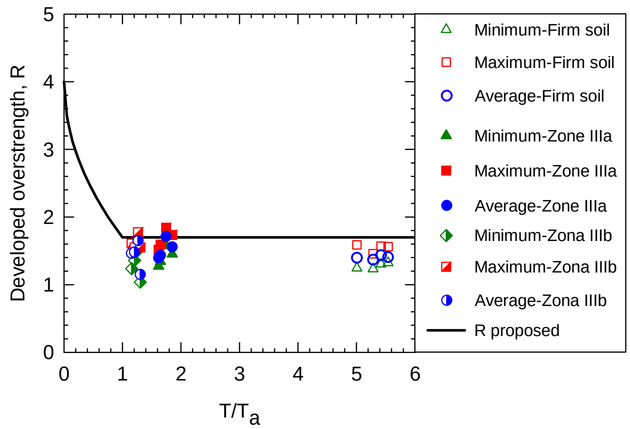


Figure 8. Demanded (developed) overstrength for the RC-MRCBFs building models

5. CONCLUDING REMARKS

The following general comments and observations can be made from the results presented in this paper:

(1) From average envelopes for the equivalent story drift at yielding (Δ_y) of all building models (eight, 15 and 24 stories), a reasonable correlation was observed between obtained drifts and the proposed limiting value. Therefore, for the design of new RC-MRCBFs buildings with slenderness ratios between $0.4 \leq H/L \leq 4$, the proposed story drift limit Δ_{ser} = 0.2% for the serviceability limit state seems to be adequate.

(2) From maxima and average response envelopes for the peak dynamic story drift angles (Δ_{max}), mainly for fifteen and twenty-four story models, it can be concluded that the story drift limit for collapse prevention state Δ = 0.015 currently proposed in NTCS-04 of MFDC-04, MOC-2008 and other international building codes (i.e., ASCE-7), is a better option for reviewing the collapse prevention limit state than the one proposed in a previous study (Δ = 0.013). These results give additional numerical support to the story drift limit for the collapse prevention performance level established in NTCS-04 (NTCS-04 2004), MOC-2008 (MOC-2008 2009) and ASCE-7 (ASCE 7-05 2005).

(3) From the mapping of accumulated plastic hinge rotations, it can be concluded that collapse mechanisms for medium-rise models (eight and fifteen stories) correlate reasonably well with the expected failure mechanism of strong column–weak beam–weaker brace. Nevertheless, as the height of the structure increases (twenty-four story models), the expected failure mechanism does not correlate well. Some incipient plastic rotations are formed at the column ends in the bottom third of the building's height, and inelastic behavior for the bracing elements is only observed at the bottom stories. In fact, beams located in the upper levels are weaker than the braces that behave elastically, which it is not consistent with the assumed collapse mechanism.

(4) From the assessed developed overstrength, it can be concluded that the proposed overstrength factor R for design (Eqn. 3.1) is reasonable for the design of ductile moment-resisting reinforced concrete concentric braced frames structures (RC-MRCBFs) using chevron steel bracing.

From the results obtained in the whole research study reported in detail by Godínez-Domínguez

(2010), it can be concluded that if the capacity design methodology and the key design parameters proposed by the authors are used, it is possible to design low and medium rise ductile RC-MRCBFs buildings (below 24 stories) located in the lakebed zone of Mexico City and coastline of the state of Guerrero. For such buildings the expected failure mechanism of strong column–weak beam–weaker brace do correlate reasonably, and suitable global ductility capacities and overstrength demands are developed when the columns of the moment frames resist at least 50% of the total seismic shear force. This conclusion is backed up with the results obtained from the nonlinear static analyses of RC-MRCBFS previously reported by Godínez-Domínguez and Tena-Colunga (2010), and from the results of the nonlinear dynamic analyses of the building models presented in this paper and elsewhere (Godínez-Domínguez 2010). Nevertheless, it was also found from the results of nonlinear static and nonlinear dynamic analyses that taller buildings (24 stories) located in the lakebed zone of Mexico City developed smaller global ductility capacities than those observed in low-rise and medium-rise models and the expected failure mechanism of strong column – weak beam – weaker brace did not correlate completely well.

ACKNOWLEDGEMENTS

The PhD fellowship granted to the first author by the National Science and Technology Council of Mexico (Conacyt) is gratefully acknowledged.

REFERENCES

ASCE 7-05 (2005). Minimum design loads for buildings and other structures. ASCE Standard ASCE/SEI 7-05, American Society of Civil Engineers, ISBN 0-7844-0809-2.

Godínez-Domínguez, E. and Tena-Colunga A. (2010). Nonlinear behavior of code-designed reinforced concrete concentric braced frames under lateral loading. *Engineering Structures*, **32**, 944-963.

Godínez-Domínguez, E. (2010). Estudio del comportamiento de marcos dúctiles de concreto reforzado con contraventeo chevrón. *PhD Thesis*, División de Ciencias Básicas e Ingeniería, Universidad Autónoma Metropolitana Azcapotzalco, July (in Spanish).

NTCS-04 (2004). Normas Técnicas Complementarias para Diseño por Sismo. Gaceta Oficial del Distrito Federal, Tomo II, No. 103-BIS, October (in Spanish).

NTCC-04 (2004). Normas Técnicas Complementarias para Diseño de Estructuras de Concreto. Gaceta Oficial del Distrito Federal, Tomo II, No. 103-BIS, October (in Spanish).

NTCM-04 (2004). Normas Técnicas Complementarias para Diseño de Estructuras Metálicas. Gaceta Oficial del Distrito Federal, Tomo II, No. 103-BIS, October (in Spanish).

MFDC-04 (2004). Reglamento de Construcciones para el Distrito Federal. Gaceta Oficial del Departamento del Distrito Federal (in Spanish).

MOC-2008 (2009). Manual de diseño de obras civiles. Diseño por sismo. Comisión Federal de Electricidad, November (in Spanish).

Tena-Colunga, A., Mena-Hernández U., Pérez-Rocha L., Avilés J., Ordaz M. and Vilar J. (2009). Updated seismic design guidelines for buildings of a model code of Mexico. *Earthquake Spectra*, **25:4**, 869-898, November.

Miranda, E. and Ruiz J. (2002). Influence of stiffness degradation on strength demands of structures built on soft soil sites. *Engineering Structures* **24**, 1271-1281.

Terán, A. (2005). Consideraciones para establecer la resistencia lateral de diseño de estructuras ubicadas en la zona del lago del D.F. *XV Congreso Nacional de Ingeniería Sísmica*. **VI-01.**

Carr, A. (2004). RUAUMOKO: computer program library. Department of Civil Engineering, University of Canterbury, New Zealand.

Remennikov, A. and Walpole W. (1997). Analytical prediction of seismic behaviour for concentrically-braced steel systems. *Earthquake Engineering and Structural Dynamics* **26**, 859-874.

Park, R., Priestley M. and Gill W. (1982). Ductility of square-confined concrete columns. *ASCE Journal of Structural Engineering*, **108:4**, 929-950.

Rodríguez, M. and Botero J. (1995). Comportamiento sísmico de estructuras considerando propiedades mecánicas de aceros de refuerzo mexicanos. *Revista de Ingeniería Sísmica*, SMIS, **49**, 39-50.

Bruneau, M., Uang C-M. and Whittaker A. (1998). Ductile design of steel structures, second edition, McGraw-Hill, New York.

Newmark, N. M. and Hall W. J. (1982). Earthquake spectra and design, Monograph series, Earthquake Engineering Research Institute. Oakland.