Comparative Study Between Centralized and Decentralized LQR Controls of a Building–Elevator System

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SUMMARY:
When both a building and its equipment have a vibration controller, the controller of the equipment should consider the behaviour of the building and vice versa. First, we classified control schemes according to the amount of state variables communicated between the controllers and the type of control objectives. Then, a high-rise building with an elevator system is analysed under seismic excitation to compare the performance of the classified control schemes that use a linear quadratic regulator. It is observed that the story drift and the response of the elevator rope are best reduced when all the state variables are communicated, but they can be sufficiently reduced even if not all the state variables are communicated.

Keywords: Seismic response control, Building-elevator system, Linear quadratic regulator

1. INTRODUCTION

In urban areas, a large number of high-rise buildings have been constructed and elevators are presently indispensable. An elevator system consists components such as a hoist (driving machine), suspension rope, car (cage). A high-rise building has a rope-sway problem in which vibration of the rope is amplified due to resonance because the natural periods of the building and the rope are approximately equal. Recent earthquakes, e.g. the Mid Niigata Prefecture Earthquake in 2004 and the 2011 off the Pacific coast of Tohoku Earthquake, caused damage and problems in the elevator systems.

Considering the rope sway problem, the Japan Building Equipment and Elevator Center Foundation and the Japan Elevator Association (2009) revised the technical standard for elevators. However, the standard does not aim at suppressing rope sway but protecting the elevator system; thus, countermeasures are expected to reduce the vibrations of the rope. Currently, the control devices for a building and an elevator system are developed independently. However, since the vibration of equipment in a building, such as an elevator, depends on the vibration of the building, a vibration controller of the building, e.g. an active mass damper (AMD), can also control the response of the elevator rope. In addition, when a building and its equipment have a vibration control actuator, it is more effective for the controller(s) to consider the dynamic behaviour of each.

There are several studies on a building–elevator system: Watanabe et al. (2007) studied on rope sway caused by a long-period ground motion, and Otsuki et al. (2006a) and Otsuki et al. (2006b) showed that active control systems can reduce the vibration of the rope effectively.

Based on this background, this study first classifies the control schemes for a coupled system of a building and its equipment both of which have a control device. Then, we compare the performance of the control schemes and clarify the effective control schemes. Specifically, we focus on a high-rise building, which has a vibration controller, such as an AMD, on the top floor and an elevator system, which has a vibration controller under the hoist, as shown in Fig. 1.1.

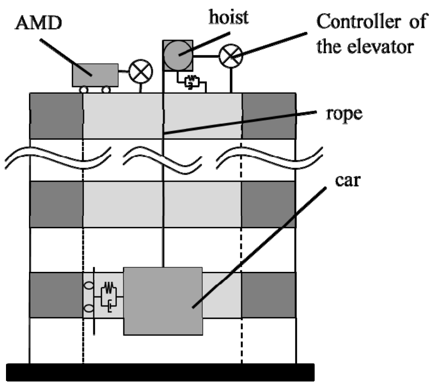


Figure 1.1. Model of the building–elevator system used in this study

2. CLASSIFICATION OF CONTROL SCHEMES FOR A BUILDING–EQUIPMENT SYSTEM

This study focuses on the coupled system of a building and its equipment that independently have a vibration control actuator. One actuator aims to reduce the vibration of the building, and the other aims to reduce the vibration of the equipment (here, elevator rope). In this section, the control schemes are classified according to the number of control units and control objectives. The basic concept of the classification is shown in Fig. 2.1.

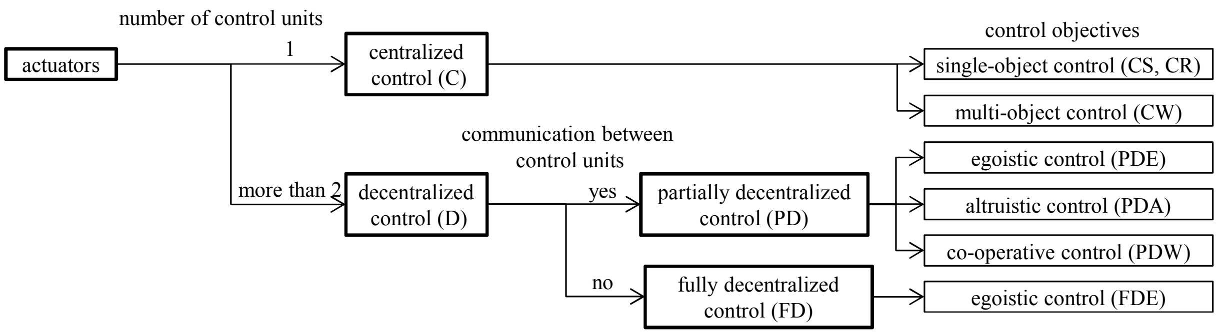


Figure 2.1. Basic concept of the classification of the control schemes for a building–equipment system

First, we define the term ‘corresponding actuator’ as the actuator controlled by an objective control unit, and ‘corresponding system’ as the system of the main control target of an objective control unit or actuator. It is assumed that the state variables of the corresponding system can be observed by the control unit.

As shown in Fig. 2.1, the control schemes can be classified on the basis of the number of control units. When there is only one control unit for the actuator(s), the scheme is called ‘centralized control’ (in formulas and figures in this paper, this scheme is represented by the index C), and if there are more than two control units, we name the scheme ‘decentralized control’ (D). With respect to the decentralized control, when the state variables are communicated between the control units, we name the scheme as ‘partially decentralized control’ (PD); when they are not communicated, we name it ‘fully decentralized control’ (FD).

Under centralized control, one control unit controls all the actuators, and thus, the building and its equipment fulfil the corresponding systems. This scheme is further classified according to the number of control objects. If the control object is only one system, the scheme is named ‘single-object control’ (when the control object is the building, CS, and when the control object is the rope, CR). If the control objects are multiple systems, the scheme is named as ‘multi-object control’ (CW). In centralized control, all the schemes consider all the state variables and manipulate all actuators, and thus the interference of the control force of multiple actuators can be avoided.

Under PD, the control schemes are further classified according to the control objectives. If a control unit operates to reduce the vibration of the original target, we name the scheme 'egoistic control' (PDE); if it works to reduce the vibration of an object other than the original target, the scheme is named 'altruistic control' (PDA); and we name a scheme that controls multiple objects 'weighted control (co-operative control)' (PDW). In partially decentralized control, a few control units have one or more corresponding actuators, and the state variables are communicated between the control units. Since the number of communication patterns rapidly increases as the number of the state variables increases, we have to choose the state variables to be communicated.

Under FD, there are multiple control units, each of which has a corresponding actuator and the state variables are not communicated between the control units. In this case, the altruistic control and weighted control are not defined because the control units cannot obtain the state variables of an object of a 'uncorresponding' system. Hence there exists only the 'egoistic control' (FDE). Effectiveness of this control scheme deteriorates occasionally because of the interference of the control force because a control unit does not consider the control force of an 'uncorresponding' actuator.

3. ANALYSIS MODEL

This study deals with the model of a building–elevator system that has two actuators on top of the building, as shown in Fig. 1.1. Hereafter, we name the actuator installed on a building, such as AMD, a building's actuator; it uses a part of the velocity components relative to the ground as states of the corresponding system, which are acquired by the sensors. The other actuator is called an elevator's actuator, which uses a part of the relative displacement and velocity with respect to the reference point of the rope as states of the corresponding system, so that the actuator can move the hoist and control the vibration of the rope.

3.1. Building model

First, we construct a linear elastic model of the building, onto which a control force u_m is applied. The building has N_m layers and mass, stiffness, damping coefficient and displacement relative to the ground of the i-th layer are $m_{\mathrm{m}i}$, $k_{\mathrm{m}i}$, $c_{\mathrm{m}i}$ and $x_{\mathrm{m}i}$, respectively. The equation of motion is given as follows:

$$\mathbf{M}_\mathrm{m}\ddot{\mathbf{x}}_\mathrm{m} + \mathbf{C}_\mathrm{m}\dot{\mathbf{x}}_\mathrm{m} + \mathbf{K}_\mathrm{m}\mathbf{x}_\mathrm{m} = -\mathbf{M}_\mathrm{m}\{1\}\ddot{z} + \mathbf{f}_\mathrm{m}u_\mathrm{m} \tag{3.1}$$

where $\mathbf{M}_\mathrm{m}$, $\mathbf{K}_\mathrm{m}$ and $\mathbf{C}_\mathrm{m}$ are the mass, stiffness and damping matrices, respectively, $\mathbf{x}_\mathrm{m}$ is the displacement vector of the building relative to the ground, $\mathbf{f}_\mathrm{m}$ is the distribution vector of the control force u_m, and z is the ground displacement.

3.2. Elevator model

Second, we construct the elevator model. The elevator model consists of a rope, hoist and car; the rope is modelled as one string composed of N_r finite elements. A hoist is modelled as a mass point with mass m_t, which is mounted on the building with a spring that has stiffness k_t and a damper with a damping coefficient c_t. The hoist is moved by the elevator's actuator with a force u_e. The car is modelled as a mass point with mass m_k, which is connected to a guide rail (here, the mass point of the building model) by a spring with stiffness k_k and a damper with a damping coefficient c_k. Following is the equation of motion of the elevator model:

$$\mathbf{M}_\mathrm{e}\ddot{\mathbf{x}}_\mathrm{e} + \mathbf{C}_\mathrm{e}\dot{\mathbf{x}}_\mathrm{e} + \mathbf{K}_\mathrm{e}\mathbf{x}_\mathrm{e} = -\mathbf{M}_\mathrm{e}\{\mathbf{1}\}\ddot{z} + \mathbf{f}_\mathrm{e}u_\mathrm{e} \tag{3.2}$$

where $\mathbf{M}_e$, $\mathbf{K}_e$ and $\mathbf{C}_e$ are respectively the mass, stiffness and damping matrices; $\mathbf{x}_e$ is the displacement vector relative to the ground; and $\mathbf{f}_e$ is the distribution vector of the control force. The top element of the rope is assigned to the first, which connects to the hoist, and the bottom of the rope connects to the car. The respective parameters, α, β and γ are given by $\alpha = \rho AL/6N_r$, $\beta = T_r N_r/L$ and $\gamma = c_r L/6N_r$ where ρ, A, L, T_r and c_r are the density, the cross sectional area, the entire length, the tension force and the damping coefficient of the rope, respectively.

3.3. Model of a building–elevator system

Finally, we construct a model of a building–elevator system by combining the building and elevator models. Using Eqns. 3.1 and 3.2, the equation of motion of a building–elevator system is given by

$$\mathbf{M}\mathbf{x}_c + \mathbf{C}\mathbf{x}_c + \mathbf{K}\mathbf{x}_c = -\mathbf{M}\{1\}\ddot{z} + \mathbf{f}\mathbf{u} \tag{3.3}$$

where $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ are the mass, stiffness and damping matrices, respectively; $\mathbf{x}_c$ is the displacement vector of the building and the rope relative to the ground; $\mathbf{f}$ is the distribution matrix of the control force and $\mathbf{u}$ is the control force vector of the building–elevator system.

The state equation of the system is derived by rewriting Eqn. 3.3:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\ddot{z}(t) \tag{3.4}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \mathbf{B} = \begin{Bmatrix} \mathbf{O} \\ \mathbf{M}^{-1}\mathbf{f} \end{Bmatrix}, \mathbf{G} = \begin{Bmatrix} \mathbf{0} \\ -\mathbf{1} \end{Bmatrix}, \mathbf{x} = \begin{Bmatrix} \mathbf{x}_c \\ \dot{\mathbf{x}}_c \end{Bmatrix}, \mathbf{1} = \begin{Bmatrix} 1 \\ \vdots \\ 1 \end{Bmatrix} \text{ (3.5), (3.6), (3.7), (3.8), (3.9)}$$

The state variable $\mathbf{x}$ consists of the displacement and the velocity, and the control output $\mathbf{y}$ is expressed by the following equation, in which the response in the control output is determined by assigning the matrices $\mathbf{H}$ and $\mathbf{D}$.

$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \tag{3.10}$$

4. CONTROL SYSTEM DESIGN

In this study, we adopt a state-feedback control using a linear quadratic regulator (LQR), which minimizes an evaluation function (functional) in quadratic form. To avert the accidental rope tangling, the maximum rope sway, i.e. the maximum distance between the mass point of the building and the node of the rope element, should be reduced. Although the LQR method minimizes an evaluation function, which is a time integral of quadratic terms with respect to the state variables and the control force. In other words, the method is not directly formulated to reduce the maximum response. The advantage of the method is the ease in formulation, and the fact that it is known to reduce the maximum response empirically.

Control system designs differ depending on the number of the communicated state variables and control inputs. Thus, we design a centralized control, a partially decentralized control and a fully decentralized control separately, and formulate the evaluation function of each control objective.

4.1. Centralized control

For centralized control, we use Eqn. 3.4 to model the control system design. For the control objectives to reduce the responses, we compare (1) the distance between the building and rope (hereafter, building-rope distance) (CR), (2) the story drift of the building (CS), and (3) the weighted sum of the building-rope distance and the story drift of the building (CW).

4.1.1 Control of rope vibration (CR)

When the control object is the building-rope distance, which is indicated by the vector $\mathbf{y}_{\mathrm{CR}}$ in terms of all the nodes of the rope, the vector $\mathbf{y}_{\mathrm{CR}}$ and the evaluation function J_{CR} are given by:

$$\mathbf{y}_{\mathrm{CR}}(t) = \begin{Bmatrix} y_{\mathrm{CR}1}(t) \\ \vdots \\ y_{\mathrm{CR}(N_r+1)}(t) \end{Bmatrix} = \begin{bmatrix} -1 & & \mathbf{O} & \mathbf{O} & & 1 \\ & \ddots & & & \iddots & \\ \mathbf{O} & & -1 & 1 & & \mathbf{O} \end{bmatrix} \mathbf{x}(t) = \mathbf{H}_{\mathrm{CR}}\mathbf{x}(t) \tag{4.1}$$

$$J_{\mathrm{CR}} = \int_0^{t_\infty} [\mathbf{y}_{\mathrm{CR}}{}^T(t)\mathbf{Q}_{\mathrm{CR}}\mathbf{y}_{\mathrm{CR}}(t) + \mathbf{u}^T(t)\mathbf{R}_{\mathrm{CR}}\mathbf{u}(t)]\mathrm{d}t \tag{4.2}$$

4.1.2 Control of building vibration (CS)

When the control object is the story drift of the building, which is indicated by the vector $\mathbf{y}_{\mathrm{CS}}$, the vector $\mathbf{y}_{\mathrm{CS}}$ and the evaluation function J_{CS} are given by:

$$\mathbf{y}_{\mathrm{CS}}(t) = \begin{Bmatrix} x_1(t)/\Delta h \\ \vdots \\ \{x_{N_m}(t) - x_{N_m-1}(t)\}/\Delta h \end{Bmatrix} = \frac{1}{\Delta h}\begin{bmatrix} 1 & & & \mathbf{O} & \\ -1 & 1 & & & \\ & \ddots & \ddots & & \mathbf{O} \\ \mathbf{O} & & -1 & 1 & \end{bmatrix} \mathbf{x}(t) = \mathbf{H}_{\mathrm{CS}}\mathbf{x}(t) \tag{4.3}$$

$$J_{\mathrm{CS}} = \int_0^{t_\infty} [\mathbf{y}_{\mathrm{CS}}{}^T(t)\mathbf{Q}_{\mathrm{CS}}\mathbf{y}_{\mathrm{CS}}(t) + \mathbf{u}^T(t)\mathbf{R}_{\mathrm{CS}}\mathbf{u}(t)]\mathrm{d}t \tag{4.4}$$

4.1.3 Multi-object control (CW)

When the control object is the sum of the building-rope distance and the story drift of the building, a weight coefficient matrix $\mathbf{Q}_{\mathrm{CW}}$ is given by the weighted sum of the two coefficient matrices in terms of $\mathbf{x}$ with the weights ϕ and ψ:

$$\mathbf{Q}_{\mathrm{CW}} = \phi\mathbf{H}_{\mathrm{CR}}^T\mathbf{Q}_{\mathrm{CR}}\mathbf{H}_{\mathrm{CR}} + \psi\mathbf{H}_{\mathrm{CS}}^T\mathbf{Q}_{\mathrm{CS}}\mathbf{H}_{\mathrm{CS}} \tag{4.9}$$

The evaluation function J_{CW} is given as follows:

$$J_{\mathrm{CW}} = \int_0^{t_\infty} [\mathbf{x}^T(t)\mathbf{Q}_{\mathrm{CW}}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}_{\mathrm{CW}}\mathbf{u}(t)]\mathrm{d}t \tag{4.10}$$

4.2. Partially decentralized control

Under partially decentralized control, the building control unit can obtain the state variables of the building and relative displacement and velocity of the rope from the top to the N_{ur}-th nodes. The rope-control unit can obtain the state variables of the displacement and velocity of the building from the top to the N_{um}-th mass points. All state variables are estimated by these state variables, and thus, the equations of motion used for the controls of the building and the rope are separately given by:

$$\mathbf{M}\ddot{\mathbf{x}}_{\mathrm{c}} + \mathbf{C}\dot{\mathbf{x}}_{\mathrm{c}} + \mathbf{K}\mathbf{x}_{\mathrm{c}} = -\mathbf{M}\mathbf{1}\ddot{z} + \mathbf{f}'_{\mathrm{m}}u_{\mathrm{m}}, \quad \mathbf{f}'_{\mathrm{m}} = [\mathbf{f}_{\mathrm{m}} \quad \mathbf{0}]^T \qquad (4.11), (4.12)$$

$$\mathbf{M}\ddot{\mathbf{x}}_{\mathrm{c}} + \mathbf{C}\dot{\mathbf{x}}_{\mathrm{c}} + \mathbf{K}\mathbf{x}_{\mathrm{c}} = -\mathbf{M}\mathbf{1}\ddot{z} + \mathbf{f}'_{\mathrm{e}}u_{\mathrm{e}}, \quad \mathbf{f}'_{\mathrm{e}} = [\mathbf{0} \quad \mathbf{f}_{\mathrm{e}}]^T \qquad (4.13), (4.14)$$

As described in these equations, the control force of the other actuator is not used in the partially decentralized control. Under partially decentralized control, a part of the state variables of the non-original target are communicated, and the control units estimate all the state variables using a Kalman filter. Therefore, we can consider noise influence.

With respect to the partially decentralized control, we compare the following three cases: (1) the actuator of the building controls the story drift of the building and the actuator of the rope controls the building-rope distance (PDE), (2) the actuator of the building controls the building-rope distance and the actuator of the rope controls the story drift of the building (PDA), and (3) both actuators control the building-rope distance and the story drift of the building (PDW).

4.2.1 Partially decentralized egoistic control (PDE)

Under PDE control, the actuator of the building controls its story drift, and the actuator of the rope controls the building-rope distance. Using the estimated state variable $\hat{\mathbf{x}}$, the respective story drift of the building $\hat{\mathbf{y}}_{\mathrm{PDS}}$ and the building-rope distance $\hat{\mathbf{y}}_{\mathrm{PDR}}$ are given as follows:

$$\hat{\mathbf{y}}_{\mathrm{PDS}}(t) = \mathbf{H}_{\mathrm{CS}}\hat{\mathbf{x}}(t), \quad \hat{\mathbf{y}}_{\mathrm{PDR}}(t) = \mathbf{H}_{\mathrm{CR}}\hat{\mathbf{x}}(t) \tag{4.15), (4.16}$$

The evaluation function of the building J_{PDEm} and that of the rope J_{PDEe} are given as follows:

$$J_{\mathrm{PDEm}} = \int_0^{t_\infty} [\hat{\mathbf{y}}_{\mathrm{PDS}}^T(t)\mathbf{Q}_{\mathrm{PDEm}}\,\hat{\mathbf{y}}_{\mathrm{PDS}}(t) + u_{\mathrm{m}}^T(t)\,R_{\mathrm{PDEm}}\,u_{\mathrm{m}}(t)]\mathrm{d}t\,, \quad \mathbf{Q}_{\mathrm{PDEm}} = \mathbf{I} \tag{4.17), (4.18}$$

$$J_{\mathrm{PDEe}} = \int_0^{t_\infty} [\hat{\mathbf{y}}_{\mathrm{PDR}}^T(t)\mathbf{Q}_{\mathrm{PDEe}}\,\hat{\mathbf{y}}_{\mathrm{PDR}}(t) + u_{\mathrm{e}}^T(t)\,R_{\mathrm{PDEe}}\,u_{\mathrm{e}}(t)]\mathrm{d}t\,, \quad \mathbf{Q}_{\mathrm{PDEe}} = \mathbf{I} \tag{4.19), (4.20}$$

4.2.2 Partially decentralized altruistic control (PDA)

On the other hand, under PDA control, the actuator of the building controls the building-rope distance, and the actuator of the rope controls the story drift of the building. The evaluation function of the building J_{PDAm} and the evaluation function of the rope J_{PDAe} are given by:

$$J_{\mathrm{PDAm}} = \int_0^{t_\infty} [\hat{\mathbf{y}}_{\mathrm{CR}}^T(t)\mathbf{Q}_{\mathrm{PDAm}}\,\hat{\mathbf{y}}_{\mathrm{CR}}(t) + u_{\mathrm{m}}^T(t)\,R_{\mathrm{PDAm}}\,u_{\mathrm{m}}(t)]\mathrm{d}t\,, \quad \mathbf{Q}_{\mathrm{PDAm}} = \mathbf{I} \tag{4.21), (4.22}$$

$$J_{\mathrm{PDAe}} = \int_0^{t_\infty} [\hat{\mathbf{y}}_{\mathrm{CS}}^T(t)\mathbf{Q}_{\mathrm{PDAe}}\,\hat{\mathbf{y}}_{\mathrm{CS}}(t) + u_{\mathrm{e}}^T(t)R_{\mathrm{PDAe}}u_{\mathrm{e}}(t)]\mathrm{d}t\,, \quad \mathbf{Q}_{\mathrm{PDAe}} = \mathbf{I} \tag{4.23), (4.24}$$

4.2.3 Partially decentralized weighted control (PDW)

Under PDW control, both actuators control the building-rope distance and the story drift of the building. The respective evaluation functions of the building and rope, J_{PDWm} and J_{PDWe}, are given by:

$$J_{\mathrm{PDWm}} = \int_0^{t_\infty} [\hat{\mathbf{x}}^T(t)\mathbf{Q}_{\mathrm{CW}}\,\hat{\mathbf{x}}(t) + u_{\mathrm{m}}^T(t)\,R_{\mathrm{PDWm}}\,u_{\mathrm{m}}(t)]\mathrm{d}t \tag{4.25}$$

$$J_{\mathrm{PDWe}} = \int_0^{t_\infty} [\hat{\mathbf{x}}^T(t)\mathbf{Q}_{\mathrm{CW}}\,\hat{\mathbf{x}}(t) + u_{\mathrm{e}}^T(t)\,R_{\mathrm{PDWe}}\,u_{\mathrm{e}}(t)]\mathrm{d}t \tag{4.26}$$

4.3. Fully decentralized control

Under fully decentralized control (FD), the state equations are derived from Eqns. 3.1 and 3.2:

$$\dot{\mathbf{x}}_{\mathrm{mss}}(t) = \mathbf{A}_{\mathrm{m}}\,\mathbf{x}_{\mathrm{mss}}(t) + \mathbf{B}_{\mathrm{m}}\,u_{\mathrm{m}}(t) + \mathbf{G}\,\ddot{z}(t) \tag{4.27}$$

$$\dot{\mathbf{x}}_{\mathrm{ess}}(t) = \mathbf{A}_{\mathrm{e}}\,\mathbf{x}_{\mathrm{ess}}(t) + \mathbf{B}_{\mathrm{e}}\,u_{\mathrm{e}}(t) + \mathbf{G}\,\ddot{z}(t) \tag{4.28}$$

where

$$\mathbf{A}_{\mathrm{m}} = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}_{\mathrm{m}}^{-1}\mathbf{K}_{\mathrm{m}} & -\mathbf{M}_{\mathrm{m}}^{-1}\mathbf{C}_{\mathrm{m}} \end{bmatrix}, \quad \mathbf{B}_{\mathrm{m}} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{M}_{\mathrm{m}}^{-1}\mathbf{f}_{\mathrm{m}} \end{Bmatrix}, \quad \mathbf{x}_{\mathrm{mss}} = \begin{Bmatrix} \mathbf{x}_{\mathrm{m}} \\ \dot{\mathbf{x}}_{\mathrm{m}} \end{Bmatrix} \tag{4.29), (4.30), (4.31}$$

$$\mathbf{A}_{\mathrm{e}}=\begin{bmatrix}\mathbf{O} & \mathbf{I}\\ -\mathbf{M}_{\mathrm{e}}^{-1}\mathbf{K}_{\mathrm{e}} & -\mathbf{M}_{\mathrm{e}}^{-1}\mathbf{C}_{\mathrm{e}}\end{bmatrix},\quad \mathbf{B}_{\mathrm{e}}=\begin{Bmatrix}\mathbf{0}\\ \mathbf{M}_{\mathrm{e}}^{-1}\mathbf{f}_{\mathrm{e}}\end{Bmatrix},\quad \mathbf{x}_{\mathrm{ess}}=\begin{Bmatrix}\mathbf{x}_{\mathrm{e}}\\ \dot{\mathbf{x}}_{\mathrm{e}}\end{Bmatrix} \qquad (4.32), (4.33), (4.34)$$

With respect to the fully decentralized control, we consider only the egoistic control (FDE). The control objective of the control unit of the building is the story drift of the building, and thus, the respective story drift of the building $\mathbf{y}_{\mathrm{FDEm}}$ and the evaluation function of the building J_{FDEm} are given by

$$\mathbf{y}_{\mathrm{FDEm}}(t)=\begin{Bmatrix} x_1(t)/\Delta h\\ \vdots\\ \{x_{N_{\mathrm{m}}}(t)-x_{N_{\mathrm{m}}-1}(t)\}/\Delta h\end{Bmatrix}=\frac{1}{\Delta h}\begin{bmatrix}1 & & & \mathbf{O}\\ -1 & 1 & & \\ & \ddots & \ddots & \mathbf{O}\\ \mathbf{O} & & -1 & 1\end{bmatrix}\mathbf{x}_{\mathrm{mss}}(t)=\mathbf{D}_{\mathrm{FDEm}}\mathbf{x}_{\mathrm{mss}}(t) \qquad (4.35)$$

$$J_{\mathrm{FDEm}}=\int_0^{t_\infty}[\mathbf{y}_{\mathrm{FDEm}}^T(t)\mathbf{Q}_{\mathrm{FDEm}}\,\mathbf{y}_{\mathrm{FDEm}}(t)+u_{\mathrm{m}}^T(t)R_{\mathrm{FDEm}}\,u_{\mathrm{m}}(t)]\mathrm{d}t,\quad \mathbf{Q}_{\mathrm{FDEm}}=\mathbf{I} \qquad (4.36), (4.37)$$

Because the control objective of the control unit of the rope is the building-rope distance, the respective vector of the building-rope distance $\mathbf{y}_{\mathrm{FDEe}}$ and the evaluation function of rope J_{FDEe} are given by

$$\mathbf{y}_{\mathrm{FDEe}}(t)=\begin{Bmatrix} y_{\mathrm{FDEe1}}(t)\\ \vdots\\ y_{\mathrm{FDEe}(N_r+1)}(t)\end{Bmatrix}=\left\{[\mathbf{I}\ \ \mathbf{O}]-\begin{bmatrix}\frac{N_r+1-1}{N_r} & & -\frac{1-1}{N_r} & \\ \vdots & \mathbf{O} & \vdots & \mathbf{O}\\ \frac{N_r+1-(N_r+1)}{N_r} & & -\frac{1-(N_r+1)}{N_r} & \end{bmatrix}\right\}\mathbf{x}_{\mathrm{ess}}(t)$$
$$=\mathbf{D}_{\mathrm{FDEe}}\mathbf{x}_{\mathrm{ess}}(t) \qquad (4.38)$$

$$J_{\mathrm{FDEe}}=\int_0^{t_\infty}[\mathbf{y}_{\mathrm{FDEe}}^T(t)\mathbf{Q}_{\mathrm{FDEe}}\,\mathbf{y}_{\mathrm{FDEe}}(t)+u_{\mathrm{e}}^T(t)R_{\mathrm{FDEe}}\,u_{\mathrm{e}}(t)]\mathrm{d}t,\quad \mathbf{Q}_{\mathrm{FDEe}}=\mathbf{I} \qquad (4.39), (4.40)$$

The location of the mass points of the building is estimated by linear interpolation using the displacement of the nodes at the top and the bottom of the rope.

5. DYNAMIC ANALYSIS

Table 5.1 shows the parameters of the building and the rope used in dynamic analysis. The story stiffness of the building is determined by the A_i distribution defined by the Building Standard Law of Japan with the first natural period T_{m1}. The damping matrix of the building $\mathbf{C}_{\mathrm{m}}$ is given by $\mathbf{C}_{\mathrm{m}}=2\zeta_{\mathrm{m1}}\mathbf{K}_{\mathrm{m}}/\omega_{\mathrm{m1}}$ where ζ_{m1} and ω_{m1} are the damping ratio and the natural circular frequency of the first mode of the building, respectively.

The weights ϕ and ψ in Eqn. 4.9 are given by the reciprocal numbers of the maximum building-rope distance and the maximum story drift of the building that are analysed under the condition of no control force, using a simulated input wave based on the design response spectrum. The 'Level-1' design response spectrum prescribed by the Notification No. 1461 of the Ministry of Construction, 31 May 2000 is used to create the simulated wave, with a return period of several decades. We assume that the car stops at the first layer and the length of the rope does not change.

We use the following input waves: 1) scaled records of El Centro 1940 NS, Taft 1952 EW and Hachinohe 1968 NS, which are scaled, such that the maximum velocities are 25 cm/s; 2) original records of K-NET Shinjuku 2004 EW and KiK-net Konohana 2011 NS, which were observed near a building suffering a rope sway accident and 3) the simulated wave based on the design response spectrum. The original records include long period components and the response spectra are relatively large around the fundamental period of the building, 5.8 s.

Table 5.1. Parameters of the Building and Elevator System

System	Parameter	Symbol	Value
Building	Number of mass points	N_{m}	60
	Number of communicated state components of the layers	N_{um}	10 layers from the top
	Fundamental natural period	T_{m1}	5.8 s
	Fundamental damping ratio	ζ_{m1}	0.01
	Story height	Δh	4 m
	Mass of each layer	m_i	1.0×10^6 kg
	Number of finite elements of the rope	N_{r}	59
	Number of communicated state components of the nodes	N_{ur}	15 nodes from the top
Elevator	Number of main ropes	n_{r}	5
	Line density of the rope	ρA	1.7 kg/m
	Total length of the rope	L	236 m
	Damping ratio of the rope	ζ_{r}	0.008
	Mass of the hoist	m_{t}	1.9×10^4 kg
	Stiffness between the hoist and the top of the building	k_{t}	3.0×10^6 N/m
	Damping between the hoist and the top of the building	c_{t}	9.0×10^3 Ns/m
	Mass of the car	m_{k}	7.5×10^3 kg
	Stiffness between the car and the top of the building	k_{k}	2.7×10^5 N/m
	Damping between the car and the top of the building	c_{k}	4.7×10^4 Ns/m

Actuators limit the maximum output, stroke and so on, but we consider only the limit of the maximum output as shown in Eqns. 5.1 and 5.2, for simplicity:

$$u_{\mathrm{m}}(t) = \begin{cases} u_{\mathrm{m}}^*(t) & \text{if } |u_{\mathrm{m}}^*(t)| \le u_{\mathrm{m\,max}} \\ u_{\mathrm{m\,max}} \operatorname{sgn}\left(u_{\mathrm{m}}^*(t)\right) & \text{if } |u_{\mathrm{m}}^*(t)| > u_{\mathrm{m\,max}} \end{cases} \tag{5.1}$$

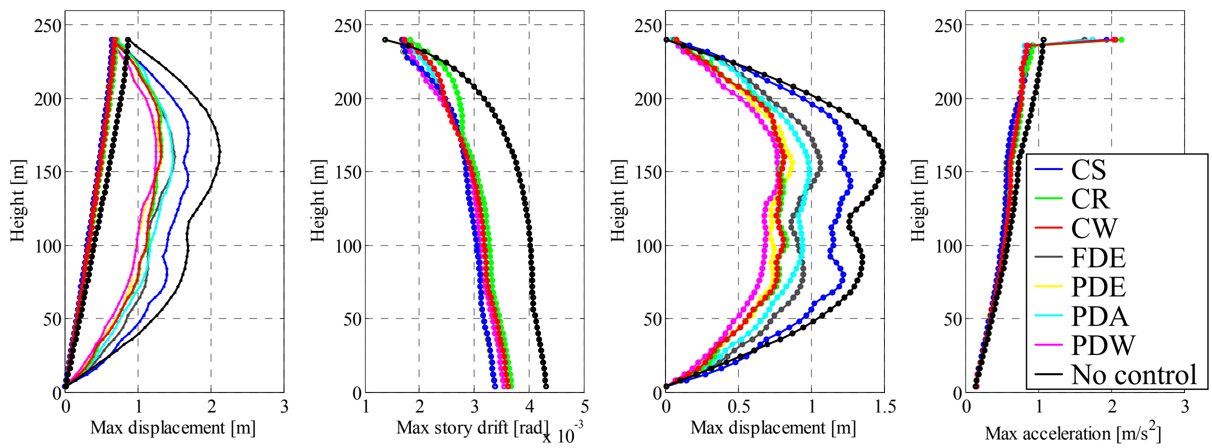
$$u_{\mathrm{e}}(t) = \begin{cases} u_{\mathrm{e}}^*(t) & \text{if } |u_{\mathrm{e}}^*(t)| \le u_{\mathrm{e\,max}} \\ u_{\mathrm{e\,max}} \operatorname{sgn}\left(u_{\mathrm{e}}^*(t)\right) & \text{if } |u_{\mathrm{e}}^*(t)| > u_{\mathrm{e\,max}} \end{cases} \tag{5.2}$$

where $u_{\mathrm{m}}^*(t)$, $u_{\mathrm{e}}^*(t)$ are the optimum control forces and $u_{\mathrm{m\,max}}$ and $u_{\mathrm{e\,max}}$ are the maximum control forces. In this study, $u_{\mathrm{m\,max}}$ and $u_{\mathrm{e\,max}}$ are set to 500 and 100 kN, respectively.

6. RESULTS AND DISCUSSION

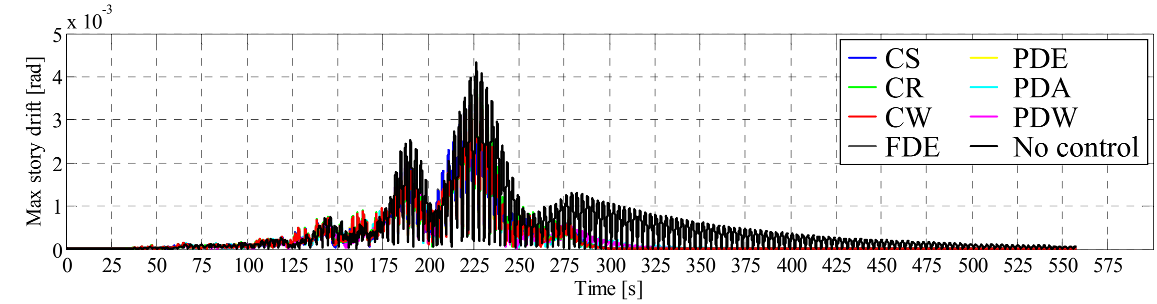
We compare the building-rope distance, the story drift of the building and the acceleration of the building. The results using the record of KiK-net Konohana 2011 NS are shown in Figs. 6.1 and 6.2. In Fig. 6.1(a), the response of the building and the rope are shown by the circles and lines, respectively. In dynamic analysis, we evaluate the response in a duration twice that of the input wave to observe the speed of reduction in the response after ground motion ceases. The weight coefficient of the control force in the evaluation function, R, is decided on the basis of repeated computations, such that the accumulated time for the control force exceeding the limit is within 10 ± 0.001 s.

In Fig. 6.1(a), in all the controlled cases, the maximum building-rope distance is smaller than that in the uncontrolled case. Hence, we confirm that the control does not deteriorate or cause divergence of the responses. The performance of each control scheme differs, especially in the response of the rope. Fig. 6.1(c) shows the difference in response reduction in the building-rope distance among the control schemes. The effectiveness of response reduction is small under the CS, PDA and FDE controls. The reasons are as follows: 1) the building-rope distance is not included in the objective of CS control, 2) under PDA control, the control force of the rope controller is too small to reduce the response of the building, and consequently, the rope response is not sufficiently reduced and 3) under FDE control, the building-rope distance is not included in the objective of the actuator of the building, which controls forces larger than that of the rope's actuator. The partially decentralized control uses the state variables, including estimation errors, but PDE control performs efficiently. In Figs. 6.1(b) and (d), the difference among the control schemes is not observed. The response is large around the top of the building in any control scheme because the actuators are installed at the top.

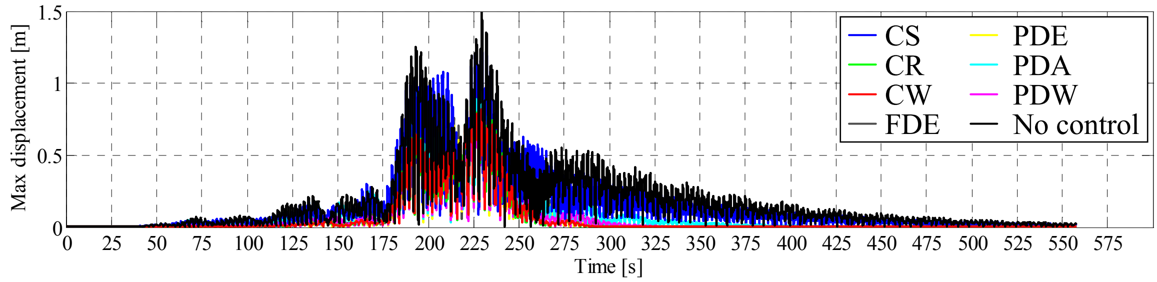


(a) Displacement (b) Story drift (c) Building-rope distance (d) Acceleration

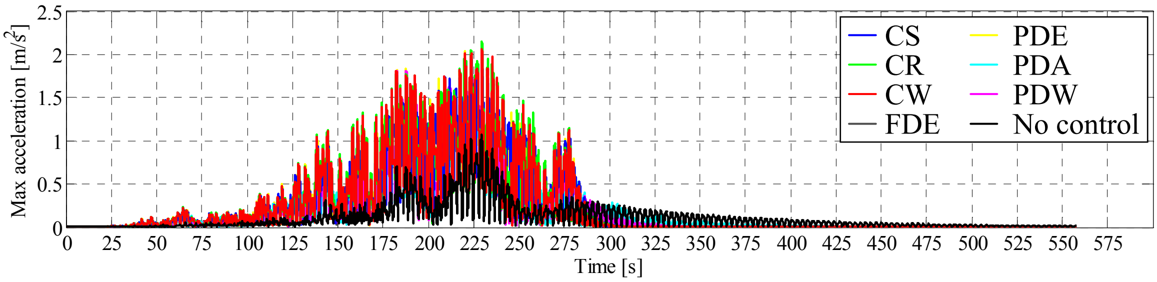
Figure 6.1. Maximum response of the mass points and nodes



(a) Maximum story drift of the building



(b) Maximum building-rope distance



(c) Maximum acceleration of the building

Figure 6.2. Time history of the maximum response

In Fig. 6.2(a), the story drift is almost reduced always under any control compared with the uncontrolled case, and the response is reduced faster; however, there is a difference among the control schemes. In Fig. 6.2(b), the maximum building-rope distance under the control is larger than that under no control in the beginning of the vibration, but the maximum distance is smaller. Although the time at the maximum response is almost the same, the control force reduces the response faster. Moreover in Fig. 6.2(c), the time at the maximum acceleration of the building is also almost the same. The maximum response under control is larger than that under no control because of the reaction force of the actuator; note that the response of acceleration can be effectively reduced by using a frequency-shaped linear quadratic Gaussian controller according to Kohiyama and Baba (2010). The control force reduces the acceleration response faster.

We observe almost the same trend using other waves. The maximum building-rope distance is smaller than that in the uncontrolled case, and the performance of each control scheme differs, especially in response of the rope. Moreover, the maximum story drift and acceleration are large around the top of the building in any control scheme and the response is reduced faster.

Damage in the building and accidents such as rope tangling occur, when the story drift of the building and the building-rope distance are large. In the analysis results, the maximum story drift under all input waves is 0.0037 ($\approx$ 1/270) rad, and thus, the building is supposed not to suffer any damage. On the other hand, under PDE control, the maximum building-rope distance is 0.6 m, which is smaller than that under no control while using the record of KiK-net Konohana 2011 NS. The threshold distance causing a rope-tangling accident differs depending on the elevator system, but the number is still large and not negligible.

7. CONCLUSIONS

The control schemes of a building–equipment system, such as a building–elevator system, are classified on the basis of the number of control units, control objectives and the presence of communication between the control units. The responses can be sufficiently reduced with actuators using an LQR controller and a Kalman filter, even if the communicated state variables are limited. In future, we will propose a method to select the appropriate state variables to be communicated in a partially decentralized control. In addition, we will further improve the decentralized control performance and consider strong ground motions that are likely to occur in larger earthquakes, in which performance of the control devices is more important.

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