Numerical simulations of the behavior of tuned liquid dampers subjected to seismic actions

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SUMMARY:
The use of energy dissipation devices is seen as an efficient way to protect structures against seismic actions. Recently, Tuned Liquid Dampers (TLD) had drawn the attention of the scientific community as simple but effective way to reduce the response of structures against dynamic loads. This paper describes numerical simulations on TLD isolated and included in simple structures. For this purpose, was used an open access software (CLAWPACK) with potential to simulate phenomena that occur within TLDs. The CLAWPACK corresponds to a set of Fortran routines developed to obtain numerical solutions of hyperbolic systems of partial differential equations, as is the case of the nonlinear phenomena underlying the shallow water wave theory. In the numerical simulations some key parameters which included water height, excitation amplitude and number of devices were varied. The main obtained results for TLD and TLDs + SDF are compared with the results of an experimental program.

Seismic protection, Passive devices, Tuned Liquid Dampers, Vibration mitigation, Numerical simulations

1. INTRODUCTION

For implementation of TLDs in real structures, either existing or to be constructed, numerical simulations are, if properly calibrated and validated, a good alternative to predict the dynamic behaviour of passive devices for seismic protection of structures as well as their performance as part of structural systems. TLDs are no exception; however, and due to their highly nonlinear behaviour when subject to increasing dynamic load amplitudes, approaches to their behaviour by means of numerical simulations should be done with discretion.

As has been shown and described [Falcão Silva, 2010], there are two types of approaches: i) a purely mathematical one and a ii) mechanical one, depending on the type of theoretical basis on which they rely. As it relies on equations used for many years in coastal engineering [Lamb, 1932], the first type of approach shows results closer to the actual behaviour of the fluid motion inside TLD devices.
With the mathematical numerical simulations developed it was intended to: i) propose a numerical calculation tool which allows the approach of the dynamic behaviour of TLDs when isolated or included in structural systems, considering increased dynamic load amplitudes, ii) present simulations of the tests made under the extensive experimental program developed [Falcão Silva, 2010], and iii) verify the suitability of the proposed models through direct comparisons between the results obtained from tests with TLDs when isolated or when included in simple structures of one (SDF) or more degrees of freedom (2DOF).

The results for simulations of the tests performed during an experimental program are presented in the paper. Also presented are the results both for isolated (shallow or deep) TLDs subject to unidirectional loads and for structural systems of one or several degrees of freedom (SDF and 2DOF) subject to unidirectional and bidirectional loads. It is also important to mention that the comparison between the results of numerical simulations and experimental results will be presented, thus allowing to calibrate the numerical models and to extend their scope of application.

2. TUNED LIQUID DAMPERS: UNIDIRECTIONAL LOAD

2.1. Elevation of the water surface

A way to assess the quality of numerical simulations for the different types of TLDs tested experimentally is, for example, by comparing the records obtained for the water surface elevation at points corresponding to the pressure transducers placed in the experimental program and the results of CLAWPACK 2D_LNEC [Falcão Silva, 2010] for the same points. In Fig. 2.1. to Fig. 2.4., is presented a representative selection of some results obtained for the sensors placed at the bottom of the TLD (shallow and deep) and considering variable excitation amplitudes.

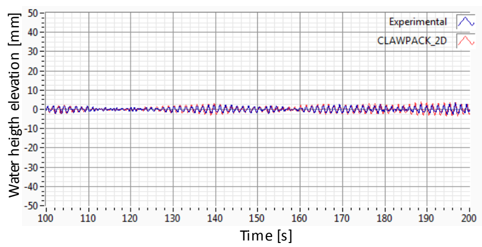

Figure 2.1. Water surface elevation, low amplitude, shallow TLD (Detail)

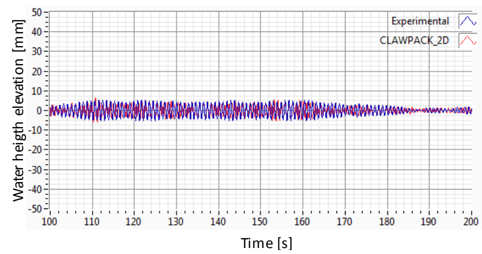

Figure 2.2. Water surface elevation, low amplitude, deep TLD (Detail)

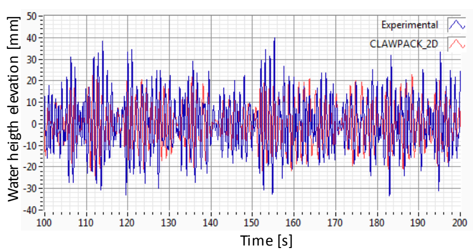

Figure 2.3. Water surface elevation, high amplitude, shallow TLD (Detail)

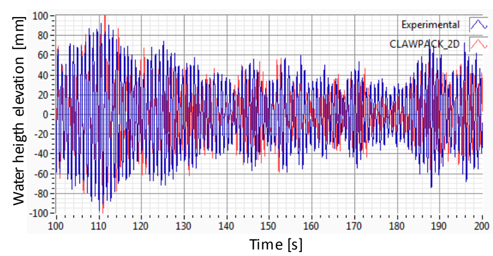

Figure 2.4. Water surface elevation, high amplitude, deep TLD (Detail)

As can be observed, for shallow TLDs, on average, the numerical simulations approach the shape and size of the wave originated, succeeding that, in certain cases, regardless of the water height at rest, the top of the wave does not reach values as high as those obtained experimentally for amplitudes of low to moderate load. On the contrary, when it comes to high load amplitudes, in some individual cases, the peaks of the waves obtained by numerical simulation are beyond the peaks of the corresponding waves obtained experimentally. A plausible explanation for what has happened, both for low to moderate amplitudes and for high amplitudes, is that, in the sensors placed very near the top walls of the TLDs, local turbulence and splash can occur, when the wave reaches the wall, occasionally creating unpredictable water elevations, which are impossible to simulate in CLAWPACK 2D_LNEC.
Moreover, even taking into account the wave theories underlying the formulation of CLAWPACK 2D_LNEC, it is very difficult that simulations can predict some small disorders on the surface of the water or the appearance of ripples at the top of the main waves, that may result from some splashes or not simulated three-dimensional movements. In certain circumstances, namely in relation to the fluid response to different random movements imposed, these instabilities can also be directly responsible for localized changes in fundamental vibration frequencies during the dynamic action, particularly when high load amplitudes are considered. In fact, as they are very particular situations and due to the local boundary conditions imposed, it is very difficult to account them using the proposed routines.
During the comparative analysis with the numerical results, some differences in the records of the

pressure sensors from the experimental program were confirmed, regarding the influence of local acceleration components, namely vertical accelerations of the fluid, which result from the instability created during the fluid motion along the walls of the device. Given the randomness and unpredictability of the motion, these components are difficult to account, either experimentally, due to inadequate instrumentation, or numerically, given the simplifications made by the finite volume methods used to solve the mathematical models proposed by the routines. In view of what was observed, it can be said that in shallow TLDs, numerical results appear to slightly underestimate the tops of the waves, for low to moderate amplitudes, especially in what concerns the fundamental vibration frequency, where there are more disturbances on the water surface.

As for deep TLDs, were identified a very evident change in the vibration frequency, confirmed by the number of wave crests for the same period of time, considering precisely the same height of fluid and dynamic solicitation amplitude in the numerical simulations and experimental tests. The fundamental frequency shows local changes which are based on the nonlinearities that arise from the movement on the water surface. These disturbances, similarly to what has been referred for other TLDs studied, were very objectively identified in the course of the experimental program [Falcão Silva, 2001]. The difference between wave crests resulting from numerical and experimental simulations became increasingly apparent, what is closely related to the fact that the wave theories used in numerical simulations are accurate only for shallow TLD. The observations seem to indicate that, for use in numerical simulations of real structures with deep TLDs included, it is necessary to slightly reduce the level of the fluid height at rest in an amount related to the observed relationship between frequencies, in order to obtain a compatible simulation for the actual behaviour of the fluid inside that device.

In addition to what has already been indicated, it is also referred that the difference observed in numerical simulations, based on the linear and non-linear wave coastal engineering theories [Le Méauté, 1976] [LeVeque, 2001], and physical simulations for the elevation of the water surface, is related to the fact that, when nonlinear phenomena of breaking wave and / or turbulence occur, which are more evident for moderate to high excitation amplitudes, the pressure sensors used in the instrumentation of the physical models may lose measure field, since its proper functioning presupposes the existence of a continuous column of water over them. Now, in the presence of nonlinear phenomena, these columns of water unsettle more easily, thus emerging swirls, unevenness and splashes, and the records obtained may contain very significant errors that can bias the conclusions.

In addition, it is also referred that the models underlying the routines used in numerical simulations are exact for shallow TLDs and approximate for deep TLD, being the approximation worse as the devices are deeper, i.e. the higher is the ratio between the fluid height at rest and the length of the device in the preferred direction of the dynamic load [Falcão Silva, 2010]. The wave theories used in the routines proposed for the numerical simulations do not allow the direct simulation of the turbulence, which appears associated with nonlinear phenomena of breaking wave, although the corresponding dissipation energy is approximate in the front of the waves broken. The proper simulation of turbulence, which appears associated to the said nonlinear phenomena of breaking wave, can be made by resolution of the Navier-Stokes equations [Gardarsson, 1996].

2.2. Dissipative force

Given its direct influence in the field of vibration mitigation, the shear force resulting in the device, also called dissipative force, arises either as an alternative to the variable water surface elevation, shown and described above, and as a great element to measure, assess and calibrate results. It is known that the inclusion of TLDs in structures intends to balance the displacement and / or accelerations of the structural system in which they are included, in order to dampen or mitigate sloshing/vibrations that occur as a result of any dynamic action (wind or earthquake) imposed upon them. The water sloshing motion, linear or nonlinear, inside the device, creates a force, when there is a difference in the water surface elevation at each top wall of the tank as well as when the wave formed hits the wall. This force, if the TLD is properly adjusted to the frequency the vibrations of which is intended to mitigate, will oppose the motion of the structure in which it is included, damping its motion and, in some cases, even completely stopping the structure where it is included. .

The determination of acting shear forces or dissipative forces arising from numerical simulations include the term resulting from the hydrostatic pressure, F_{hidro}, of the water on both walls of the top of the TLD in the sloshing direction, but with opposite directions, and the term resulting from the vertical acceleration F_{acci} of the water surface near the top walls of the TLD [Falcão Silva, 2010].The variable force, which expresses the overall response of the device against the action of a specific dynamic action, emerges as the best control parameter. Given the above, and taking into account that the resulting forces are estimated on the basis of records of the water surface elevation at points along the top faces of the TLD, it is very likely that disorders identified within the moving fluid become null or at least are considerably reduced, when considering a global measure, such as the net force on the device or dissipative force.

Based on this formulation, it was possible to numerically calculate the forces for the different situations simulated (CLAWPACK 2D_LNEC) and compare the results with the values obtained during the experimental program. The above results allow to obtain an overall view of what has happened and draw conclusions regarding the use of the routines developed [Falcão Silva, 2010], for simulation of the dynamic behaviour of TLD devices when subject to seismic actions. On this point, is important to emphasize that the numerical results obtained for deep TLDs are approximate, according to what has been previously mentioned concerning the wave theories underlying the formulation of CLAWPACK2D_LNEC routines, and consequently in what concerns dissipative forces this difference is more evident. In Fig. 2.5. to Fig. 2.8. is presented a representative selection of the numerical and experimental dissipative obtained in shallow and deep TLDs, considering variable excitation amplitudes.

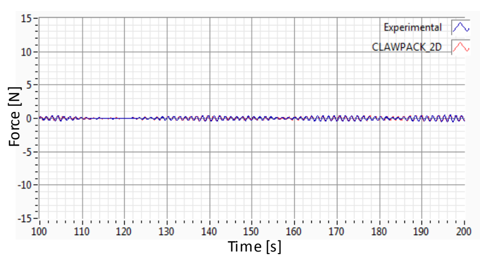

Figure 2.5. Dissipative force, low amplitude, shallow TLD (Detail)

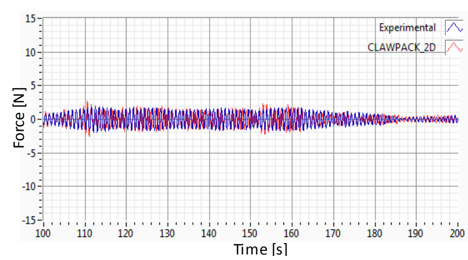

Figure 2.6. Dissipative force, low amplitude, deep TLD (Detail)

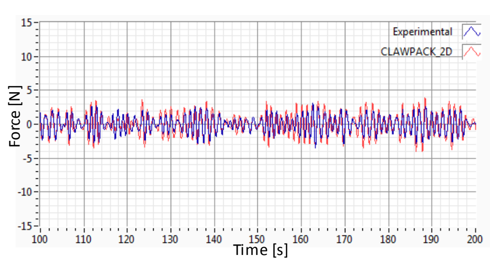

Figure 2.7. Dissipative force, high amplitude, shallow TLD (Detail)

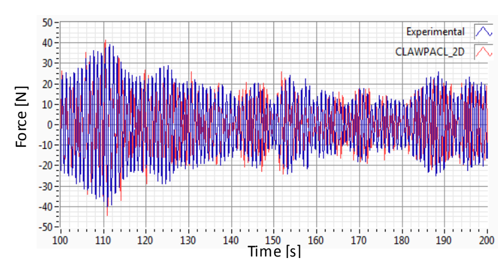

Figure 2.8. Dissipative force, high amplitude, deep TLD (Detail)

On average, the dissipative forces obtained for shallow TLDs in the numerical simulations carried out, can adequately be close to the corresponding experimental values. However, similarly to what has been observed for the water surface elevation, the peak values of the numerical dissipative forces are lower than the peak forces obtained experimentally, for low to moderate amplitudes. When it comes to high solicitation amplitudes, in some specific situations, the peak forces obtained numerically exceed the maximum corresponding forces obtained experimentally. In fact, the changes and the reasons mentioned for the variable water surface elevation still remain in what concerns forces, although in a considerably lighter way. The profiles of the forces obtained for deep TLDs allow confirming that there is indeed a slight discrepancy between the number of cycles for the same period of time, which will be confirmed later, when determining the frequency response functions. A difference between forces resulting from numerical and experimental simulations has been observed, what is closely connected with the fact that the wave theories used in the numerical simulations express exact and approximate results, when they relate to shallow and deep TLDs, respectively. The best approach of the numerical results for deep TLDs may be obtained by a slight adjustment of the water height at rest, as indicated in the following section.

2.3. FRF estimation

Another way to identify the suitability of the numerical simulations is the comparison of frequency response functions (FRF) obtained experimentally and numerically. These can be determined by the relationship between an input signal and an output signal; for example, the relationship between the displacements imposed (input) and the water surface elevation (output) or by the relation displacements / accelerations (input) imposed and the dissipative forces involved (output). As shown in previous works [Falcão Silva, 2010] this second hypothesis seems to lead to less disruption results and with a better level of consistency. The use of the force minimizes errors associated with the recording of the water heights on the top walls of the TLD. When determining FRF, it was decided to work on average, joining together the various tests made for each water height at rest and dynamic load amplitude and to determine an average value of the results, so that any eventual deviations could be minimized. In Fig. 2.9 to Fig. 2.12, are presented the FRF obtained for the average of the several series simulated numerically, considering different water heights at rest and excitation amplitudes, as well as their comparison with FRF obtained experimentally. The irregularities observed in experimental FRFs can be justified by the local disturbances corresponding to splashing or nonlinear turbulence of the water surface during the tests, which can hardly be simulated, especially when you follow the wave theories underlying the proposed CLAWPACK 2D_LNEC routines.

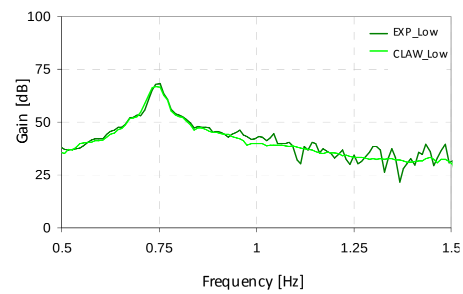

Figure 2.9. FRFs, low amplitude, shallow TLD

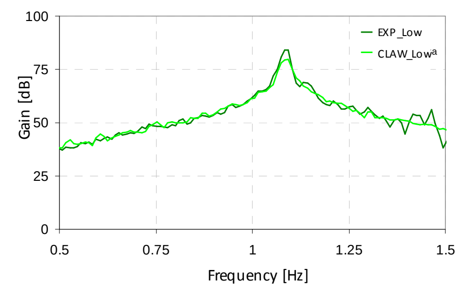

Figure 2.10. FRFs, low amplitude, deep TLD

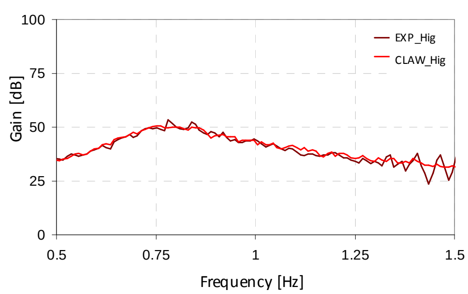

Figure 2.11. FRFs, high amplitude, shallow TLD

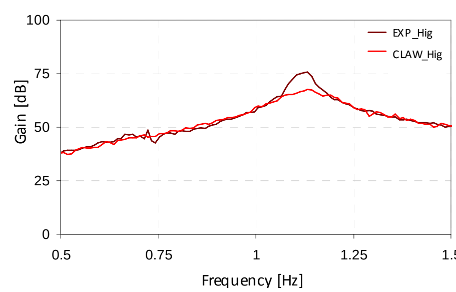

Figure 2.12. FRFs, high amplitude, deep TLD

When observing the graphs presented both for deep and shallow TLDs, it turns out that there is indeed a very good matching in terms of frequency, for dynamic load amplitudes ranging from low to high, as evidenced in the analyses of average FRFs shown. This observation confirms what was originally predicted for the routines proposed for simulation, since the wave theories underlying the formulation used express exact results for shallow TLDs and approximate results for deep TLDs.

3. SDF + TUNED LIQUID DAMPERS – UNIDIRECTIONAL LOAD

It was developed and proposed a set of routines that allow the simulation of the dynamic behaviour of a SDF structural system with a set of TLDs included, when subject to a specific dynamic action [Falcão Silva, 2010]. SDF structures were simulated with varying frequencies subject to random dynamic loads generated using the software LNEC-SPA [Mendes and Costa, 2007], having been obtained its components of displacement, speed and acceleration. The assessment of the quality of numerical simulations for SDF + TLD systems of two degrees of freedom is made, not only by comparing the records of the water surface elevation at points corresponding to the various pressure transducers placed in the experimental program [Falcão Silva, 2010] with the CLAWPACK2D_LNEC results for the same points, as presented in the previous section, but also and mainly by comparing records of displacements, speeds, accelerations and dissipative forces of the structural system. These variables will, no doubt, allow the assessment of the influence of the inclusion of TLDs on the dynamic behaviour of SDF systems and their efficiency in the mitigation of dynamic vibrations imposed. Also presented are the results obtained for a structural system with an estimated frequency of about 0.8Hz, tested during the experimental test program [Falcão Silva, 2010], and subject to a series that is closer, on average, to the average value of all the series tested, considering varying load amplitudes and the corresponding comparisons with the experimental results for the same dynamic load amplitudes. To facilitate the observation, are only presented representative details of the records concerning displacements and dissipative forces (Fig. 3.1. to Fig. 3.4). As mentioned in the previous section it was found, by direct observation, that about one third of the total test duration is relevant to the phenomena that arise. Based on that observation it was decided to present in the present paper only sections of the corresponding duration for analysis.

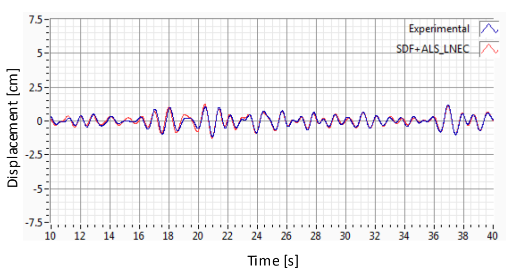

Figure 3.1. Displacements for SDF+TLD, low amplitude (Detail)

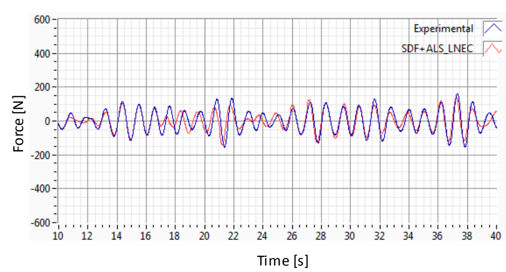

Figure 3.2. Dissipative forces for SDF+TLD, low amplitude (Detail)

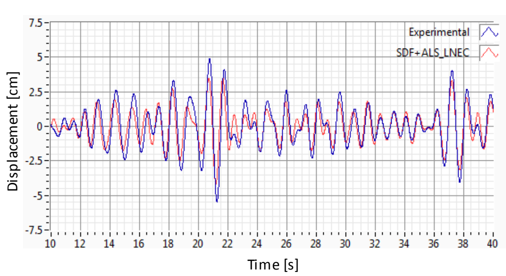

Figure 3.3. Displacements for SDF+ TLD, high amplitude (Detail)

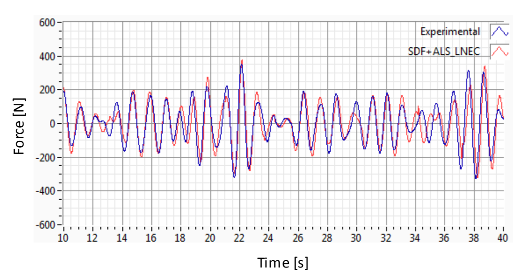

Figure 3.4. Dissipative forces for SDF+ TLD, high amplitude (Detail)

As can be seen by consulting the above figures, there is a very good matching between the results of numerical simulations obtained by the proposed set of routines, TLD_LNEC + SDF, and the results obtained experimentally for low to high amplitudes. In determining FRFs were joined together the numerical simulations developed for all experimental tests carried out for each water height at rest and excitation amplitude, having an average value been determined, so that any deviations could be reduced this way. It is presented below a figure resuming some representative examples of the type of FRFs obtained for the numerical simulations carried out on a SDF structural system with TLD included (SDF+TLD_LNEC) and compared with the corresponding experimental frequency response functions. The remaining results obtained in terms of time series of displacement, velocity, acceleration and dissipative forces, as well as the frequency response functions obtained for different transmission structures, admitting TLDs with different heights of water (ranging from shallow to deep), excitation and amplitudes tested during the experimental program [Falcão Silva, 2010].

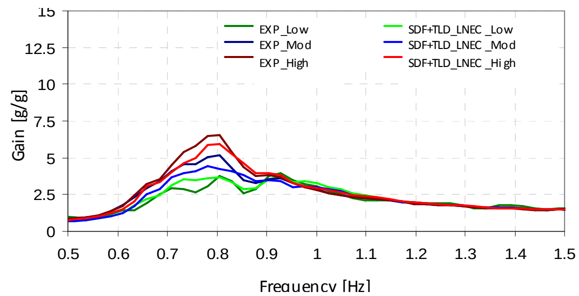

Figure 3.5. FRFs for SDF + TLD system (SDF approximate frequency of 0.81Hz)

When observing the graphs presented, it appears that in fact there is a very good matching of the frequencies of two modes that arise: i) one associated with the structure itself and ii) another associated with the TLD device. It can also be verified that the numerical methods proposed are very suitable to approach the behaviour of structural systems with TLDs included, and that it is not necessary to perform experimental tests, which in many circumstances may delay the studies, making it difficult to draw quick conclusions and expedite implementation in real structures. However, similarly to what has been observed for the simulations performed on the TLDs devices when isolated [Falcão Silva, 2010], we can verify that, in fact, the numerical simulations reflect less disturbed results, and the frequency response function is undoubtedly smoother and slightly below at the peak area, of 10 to 15%.

The differences observed for experimental and analytical FRFs, can clearly be justified by eventual local disturbances in the instrumentation used during the experimental program and / or nonlinear phenomena, which, as has been widely mentioned in previous sections, are sometimes difficult to account, even by the mathematical models based on the wave theories implemented in the calculation routines developed and proposed. The numerical simulations for the proposed unidirectional loads provide a good approach of the behaviour of these structures. The direct comparison of displacements, velocities and accelerations recorded for the same structural system and for each type of simulation enables to obtain interesting conclusions about the levels of vibration mitigation [Falcão Silva, 2010].

4. MDF + TUNED LIQUID DAMPERS – BIDIRECTIONAL LOAD

The structures tested were subject to random bidirectional dynamic loads and frequency contents between 0.1 and 3Hz, having been obtained the records of their displacement, speed and acceleration components, which subsequently were compared with experimental results obtained for the same type of loads. In Fig. 4.1 to Fig. 4.4, are presented the results of some numerical simulations carried out and the corresponding comparisons with experimental results for two diametrically opposed situations, in which it was found, in the first case, a correlation between input signals of 1%, and in the second, a correlation of 100 %.

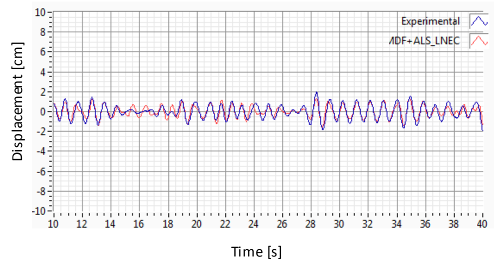

Figure 4.1. Displacement in x, 2DOF+TLD, correlation between signals 1% (Detail)

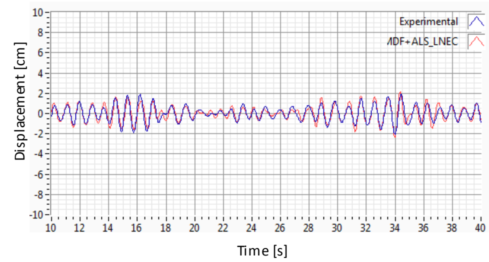

Figure 4.2. Displacement in x, 2DOF+TLD, correlation between signals 100% (Detail)

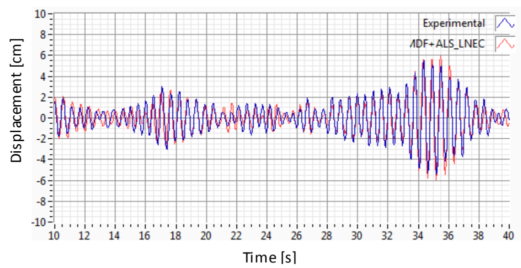

Figure 4.3. Displacement in y, 2DOF+TLD, correlation between entre signals 1% (Detail)

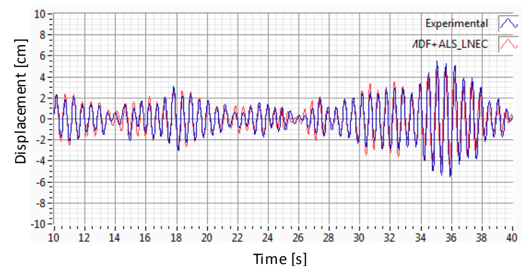

Figure 4.4. Displacement in y, 2DOF+TLD, correlation between signals 100% (Detail)

As can be seen, the progress of the displacements, obtained by means of numerical simulations carried out in structural system sets 2DOF + TLD, both for the x-direction or transverse direction of the seismic platform, as for the y direction or longitudinal direction of the platform seismic, shows an excellent matching with what was obtained in the experimental tests conducted in the same structure. However, very slight differences in peak values can be identified, most likely resulting from adjustments in damping. Since the embedding on the basis of the structural system was achieved using a bolted connection with steel plates, the properties of which are not accurately known, it is believed that, during loads, occasional damping changes can arise which cannot be completely reproduced in numerical simulations. The changes observed in damping will influence not only the displacement in the structure in both directions, but also speeds and accelerations [Falcão Silva, 2010]. In fact, the MDF+ ALS _LNEC routines, which correspond to an extension of SDF + ALS_LNEC routines for systems with more than one degree of freedom, allow with great adequacy the simulation of structural systems with more than one mode of vibration. Similarly to what has been done for systems with one degree of freedom, the numerical FRF for each simulated case and load direction were also determined, being the numerical results compared with the experimental results.

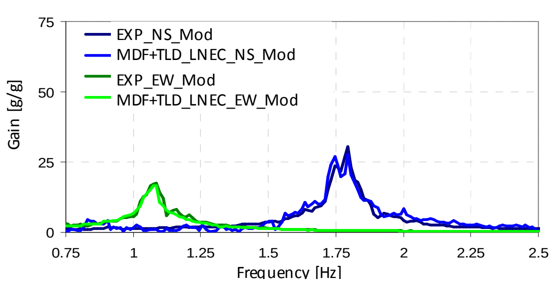

Figure 4.5. FRFs for 2DOF+TLD, correlation between signals 1%

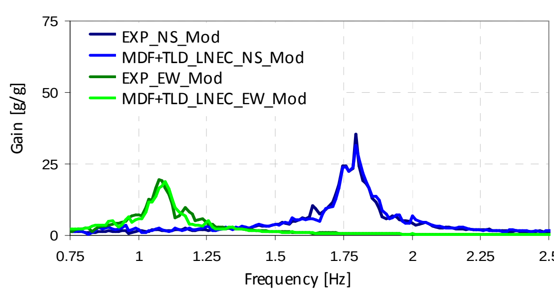

Figure 4.6. FRFs for 2DOF+TLD, correlation between signals 100%

Observing the representations shown in each of the previous figures, it appears that, as identified for the series of displacements, there is, in fact, a very good matching of the frequencies of two modes

that arise for each direction tested: one associated with the structure itself and the other associated with the TLD devices tested. In addition to that, and similarly to what has been observed for the structural system sets SDF + TLD, was verified that the numerical simulations correspond to peak values slightly lower than the corresponding experimental values and, at the limit, a difference ranging from 5 to 10% can be observed. It has been identified that the emergence of two peaks, associated respectively with the modes of the main structural system and TLD, is more evident, as the adjustment between frequencies of the main structure and the TLD devices is better [Falcão Silva, 2010].
Similarly to the observed in the simulations performed on TLD devices when isolated and on SDF + TLD, it is still verified that the numerical simulations effectively show less disturbed FRF. The irregularities present in the experimental results are, as already mentioned, the result of eventual faults in the instrumentation used.

5. CONCLUSIONS

The numerical simulations program carried on TLDs and on transmission structures (SDF and 2DOF) with the referred devices coupled, made possible to obtain different conclusions as to its suitability to predict the behaviour of the above mentioned structural systems when subject to unidirectional and bidirectional dynamic loads. So, for isolated TLD, when not included in any type of structural system:

i. There is a good matching between the records of the water surface elevation obtained experimentally and numerically. The matching is much better in the range of low to moderate amplitudes. As it was possible to conclude when observing the graphical representations of time series and wave profiles presented, the adaptations proposed in the original routines of the CLAWPACK program allow an adequate simulation of the dynamic behaviour of the fluid inside the device when subject to random dynamic loads;
ii. There is a good matching between the records of the forces on the walls of the device obtained experimentally and numerically. In fact, in the studied cases, the shear force in the tank is not affected by a significant hydrodynamic component, being very reliable an approach based on the hydrostatic force, obviously resulting from the hydrostatic pressure on the container walls;
iii. The experimental frequencies were approximated by numerical simulations with an error considered low. Differences registered are mainly due to local disturbances identified in measuring sensors (for experimental results), to the appearance of nonlinear phenomena (breaking waves) not covered by the mathematical modelling underlying the numerical simulations, which begin to be evident with increasing amplitude of excitation, and also, in some circumstances, where bidirectional loads are imposed, to the influence of motion in non-collinear directions.

In the case of the transmission systems with TLDs included for mitigation of dynamic vibration:

i. There is a good matching between the displacements records whatever the range of excitation amplitudes imposed;
ii. Frequencies and vibration modes obtained by means of numerical simulations approached the experimental results with a negligible error. The small differences identified resulted primarily of local disruptions in the air springs responsible for the introduction of stiffness to the SDF transmission structure (unidirectional loads) and in the fixing zone of the tri-axial seismic platform for 2DOF transmission structures (bidirectional loads). However, as it was not conceptualize a precise way to estimate the referred local disturbances, to include in numerical simulations for increasing dynamic load amplitudes, differences in relation to the results obtained experimentally will, most certainly, arise.

The results obtained numerically were analyzed, hence enabling to complement information already available in previous works [Sun, 1991] [Gardarsson, 1996] [Yu, 1991] [Yalla, 2001]. Thus, it was possible to obtain a good approach of the dynamic behaviour of ALS when subject to dynamic loads. It should also be noted that an analysis of the numerical results obtained, as mentioned for experimental results [Falcão Silva, 2010], is not exhausted, and therefore it is possible to determine new parameters and optimal relations between them. Results presented in previous sections allow illustrating with high approximation the dynamic behaviour of TLDs, observed during the dynamic tests conducted in the uniaxial and triaxial seismic platforms LNEC. The results indicate that the numerical simulations proposed can be extended to more complex structural systems representative of real structures existing in the Portuguese housing park [Falcão Silva, 2010].

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