Combination of the IDA-based Bayesian Probability Network and the Response Surface Method in System Reliability Assessment of Steel Frames

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ABSTRACT
A new enhanced probabilistic methodology representing systematic seismic structural collapse is presented in this paper. The systematic collapse occurs when adequate number of component failures in terms of plastic hinge form. After a failure modes identification process using pushover analysis, a systematic representation of influential uncertainty sources in seismic collapse, including modeling random variables, likely failure modes and collapse limit-states are implemented by means of Bayesian Probability Network (BPN). As a new approach, the Incremental Dynamic Analysis procedure is folded in the BPN. In order to make the methodology more time-effective, Response Surface procedure is incorporated.
First, the same 1-bay, 2-storey special steel moment frame as the one being used by the authors (2011) is considered and the accuracy and the efficiency of the improved methodology are evaluated. Then, the enhanced methodology is implemented on a larger frame and the collapse reliability index is extracted.

Keywords: Seismic Collapse Reliability Assessment, Bayesian Probability Network, IDA, Response Surface, Failure Mode.

1. INTRODOCTION

Trend towards reliability analysis in the collapse limit state has newly increased considerably in the concept of Performance-Based Earthquake Engineering. Nonlinearity in the structural behavior plays an important role in the collapse limit state, while a robust reliability analysis requires a comprehensive incorporation of aleatory and epistemic uncertainties as well as appropriate model class selection and prediction. Record To Record (RTR) variability imposing inherent randomness in different levels of seismic excitation intensities are often implicitly considered as aleatory Random Variable (RV) in models through Incremental Dynamic Analysis (IDA). Yet consideration of the modeling (epistemic) uncertainty has been only compromised in some codes upon a judgmental implicit manner (FEMA P695, 2009). Recently several approaches have been introduced in order to incorporate uncertainty by a set of modeling RV`s in structural seismic collapse assessment such as Monte Carlo, FOSM with mean estimate and FORM/SORM. Although combined with some reductive approaches such as Latin Hyper cubic Sampling, Monte Carlo technique, due to its straightforward concept, requires numerous simulation analyses to incorporate uncertainties in reliability evaluation. FOSM method tends to approximate uncertainty assessments with non negligible errors when limit state functions are highly nonlinear (The case we often confront when analyses are performed in collapse limit state) (Liel et al, 2009). FORM and SORM methods, when evaluating a fragility curve, not only require abundant reliability analyses, but also their evaluation of the parts of the fragility curve where probabilities are high, is not accurate enough (Liel et al, 2009). Furthermore, the two latter approaches do not involve aleatory and modeling RV`s in a coupled manner, which adds extra error to the computations when both sources of uncertainty, i.e. modeling and aleatory, are considered together in a model.

To partially overcome the aforementioned deficiencies in the collapse threshold, a new method was proposed by Liel et.al (2009), ASOSM, in which response surface-based method was incorporated within Monte Carlo framework. They used regression technique to create a response surface, so as to predict median collapse capacity as well as the fragility curve of a set of case study concrete frames through Monte Carlo simulations.

One way to considerably reduce the number of simulations in a reliability analysis, while keeping the accuracy and practicality in an acceptable range, is to benefit from the concept of Bayes rule. In the course of this concept, probabilities are directly calculated conditioned to the events upon which they are dependent, via the total probability theorem integrand. An effective tool to involve this procedure is Bayesian Probability Network (BPN). Banazadeh and Fereshtehnejad (2011) developed a methodology through which epistemic uncertainties were included in the reliability process by using BPN in a condition that RTR variability was also considered via IDA in a coupled manner. In this methodology, they also embedded a new approach for the collapse limit state as the instant at which a Failure Mechanism (FM) is occurred in the structure when analyzing in every incremented steps of IDA.

Although "collapse limit state" is simply regarded as (Ibarra and Krawinkler, 2005) "The structural inability to sustain gravity loads while being under seismic excitation", in numerical calculations with deteriorative models, it is a vague subject. An approach proposed by Vamvatsikos and Cornell (2002), well-known as Incremental Dynamic Analysis (IDA), made it possible to predict global collapse with a simplified assumption. Global collapse is simply defined as the onset the structural maximum inter-story drift is either in the vicinity of the threshold of 10% or when the slope of the IDA curve lessens than 20% of the starting one in elastic range. However, not only the proposed criteria is based on an engineering judgment, but also relating the collapse to a global Engineering Demand Parameter (EDP) would cost inaccuracy due to disregarding the local collapse modes. Different collapse scenario in terms of local collapse is likely to happen. Various combinations of local collapses could lead the structure to a global collapse. The significant amount of inherent and epistemic uncertainty could be a reason that any of these collapse scenario occurs in a seismic event. In a moment frame which is seismically designed to have only a ductile collapse, component failures are only permitted to be flexural ones also known as Plastic Hinges (PH`s) at element ends. Evidently, Having the knowledge of a step by step PH`s formation until collapse, could make the structural response more controllable (e.g. averting brittle FM`s) and paves the way for optimum design of buildings.

Yet, due to both aleatory and epistemic uncertainties existing in a probabilistic seismic hazard assessment followed by a probabilistic seismic demand assessment, a variety of collapse mechanisms, presenting different structural lateral resistances, is likely to happen and thus needs to be checked. In the previous methodology proposed by the authors (2011), this imposed extra time in every step of the nonlinear time history analysis and could be also more tangible when it comes to structures with more FM`s likely to form. Hence, in order to benefit from the advantages provided by the methodology, as well as making it a time-effective one, authors combined response surface–based method into BPN. This technique is first applied to the same 1-bay, 2-storey Special Steel Moment Resisting Frame (SSMRF), as the one being used by the authors (2011), is considered and the accuracy and the efficiency of the improved methodology are evaluated. Then, the newly proposed approach is implemented on a 2-bay, 4-storey Intermediate Steel Moment Resisting Frame (ISMRF) and the results of the systematic reliability analyses are discussed.

2. CASE STUDY STRUCTURES

For the purpose of comparison, first, the same 1-bay 2-storey SSMRF was evaluated (Banazadeh and Fereshtehnejad, 2011). In the following, reliability analysis with the enhanced methodology was performed on a 2-bay 4-storey ISMRF (See Fig. 1). For both structures, concentrated PH model is utilized in a way that each beam and column is constituted of an elastic element in the middle, ending by two nonlinear rotational springs, known as element`s components. Damage can only be realized in element components. A deteriorative hysteric model based on the proposal of Ibarra et al (2006) was adopted (See Fig. 2). The calibration of the model has been taken from the suggestion of Lignos (2008). Shear panel nonlinearity was also included in the models (Gupta and Krawinkler, 1999).

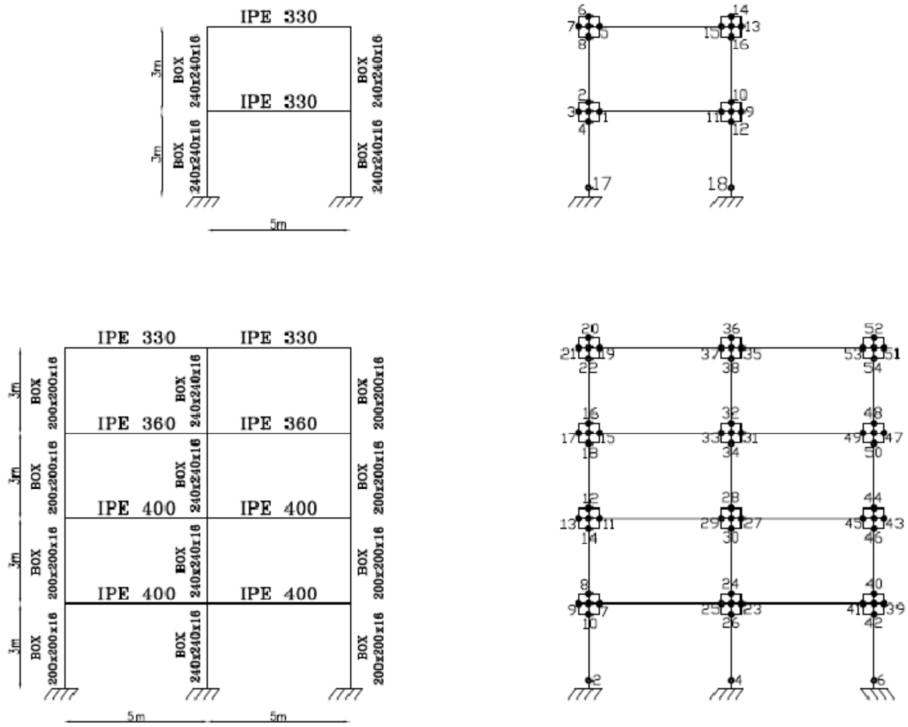

Figure 1. Modeling details of the sample frame structures.

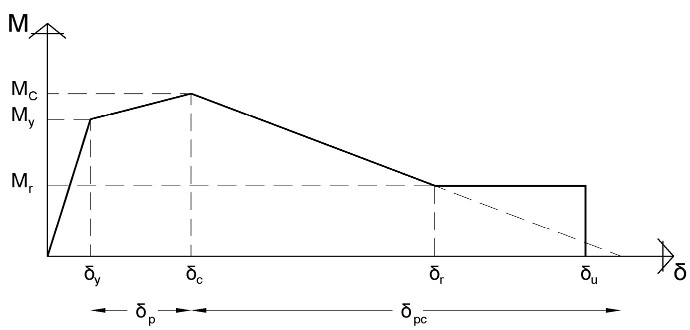

Figure 2. Backbone curve of the modified Ibarra-Krawinkler material for element ends.

3. ALEATORY AND EPISTEMIC UNCERTAINTIES

RTR variability, including 44 far-field ground motion records suggested by FEMA P695 (2009) was judged as an aleatory RV, since by indirectly considering its effect in the BPN, updating is not performed for this source of uncertainty (Der Kiureghian and Ditlevsen, 2008). Epistemic RV`s are categorized in 3 groups:

3.1. Hazard variability

First mode period ($S_{a(T1)}$, ξ=5%) of the case study structures was selected as a proper Intensity Measure (IM) for Probabilistic Seismic Hazard Analyses (PSHA) and consequently structural performance assessment within the IDA framework. PSHA curves, originated from the work by Mahdavi (2005), for the location of the structures, in terms of annual rate of exceedance versus $S_{a(T1)}$, is calculated and shown in Fig. 3.

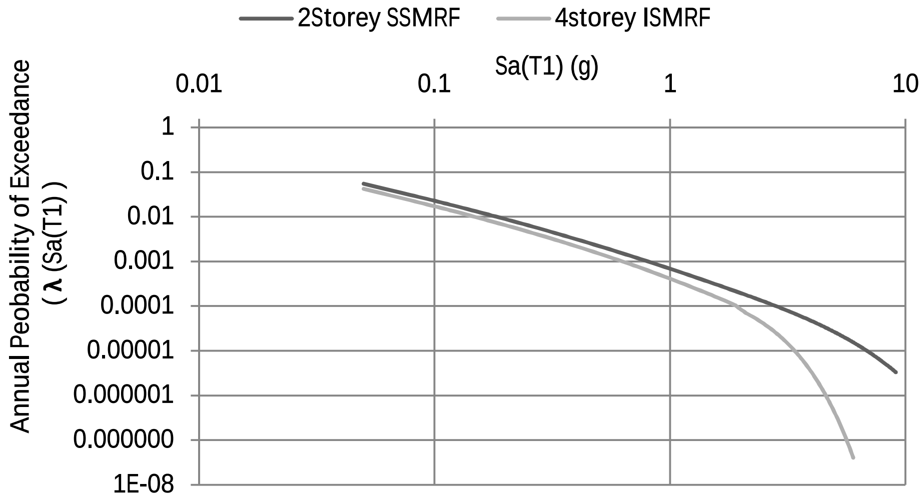

Figure 3. Hazard curves for the case study frames.

3.2. Modeling Variability

Through a literature review, sensitive modeling RV`s were recognized. These RV`s are specified merely to beam hinges and are shown in table 1. In the reliability analysis, beam element statistical characteristics are assumed fully correlated. This should also be noted that for comparing the enhanced proposed methodology with the previous approach, when assessing the 2-storey SSMRF, only the first three RV`s were considered in the developed BPN. However, all the 4 modeling sources of uncertainty were involved for reliability evaluation of the 4-storey ISMRF.

Table 1.Statistical specifications of the modeling RV`s.

Modeling RV	Probability Distribution Function	Mean	Standard Deviation	Reference
M_y/M_p	Lognormal	1.17	0.21	(Lignos, 2008)
δ_P	Lognormal	Variable with section	0.24	(Lignos et al, 2010)
δ_{PC}	Lognormal	Variable with section	0.26	(Lignos et al, 2010)
Λ	Lognormal	Variable with section	0.35	(Lignos et al, 2010)

In this table, M_y, δ_P and δ_{PC} are according to Fig. 2, M_p is the beam plastic moment capacity and Λ is the ratio of the beam total energy capacity to the effective yield moment.

3.3. Systematic Variability

Despite the very limit states in pushover analyses which were solely based on deformation, in IDA due to extra hysteretic deterioration in components, criterions need to incorporate both deformation and energy deteriorations. Among various Damage Indices (DI`s) proposed, *Park&Ang* (1987) (PA) and *Benavent-Climent* (2007) (BC) were selected to inspect formation of a PH. This RV is defined as "DI" in the developed BPN`s. Through each of these DI`s, three ultimate rotation capacity corresponding to $\delta_c$, $(\delta_c+10\%)/2$ and 10% were deliberated and independently considered as a systematic RV in the developed BPN`s. This RV is also named "DI-Deformation" in the created BPN`s.

4. OVERVIEW OF THE "SYSTEMATIC IDA-BASED BAYESIAN PROBABILITY NETWORK" METHODOLOGY (SIBPN)

The SIBPN methodology presented by Banazadeh and Fereshtehnejad (2011) is a novel, yet comprehensive, and systematic representation of structural collapse using combined IDA and BPN, in which likely collapse scenarios at the presence of all sources of uncertainty are investigated.

The idea is primarily originated from the methodology suggested by Mahadevan et al (2001). They conducted system reliability assessment on a simple 1-bay, 1-storey moment frame with an elastic behavior subjected to two uncertain point loads, through BPN. Later, this method was merged with IDA framework (Banazadeh and Fereshtehnejad, 2011), through which RTR variability is considered through the IM. The methodology is briefly conducted in two phases (Banazadeh and Fereshtehnejad, 2011):

1- Performing a series of pushover analyses with multiple sensitive RV`s so as to recognize FM`s prevailing over the structural response in the forthcoming nonlinear dynamic analyses.
2- Developing a BPN including the probable FM`s (resulted from the first phase), dependent upon the most influential RV`s, through a set of IDA`s for each realization.

4.1. Detection of Likely Failure Modes

An FM is a sequential chain of PH`s that collectively violate the overall stability by making a frame mechanism. Since taking all ideally possible FM`s into consideration is neither numerically possible nor practically necessary, in advance of the SIBPN, an identification procedure to detect the most likely FM`s using pushover analysis has been suggested. A reliable codified design methodology intends to reduce the likelihood of undesirable FM`s while there is always some level of possibility that the system experiences collapse due to unforeseen FM`s. This is mostly due to existence of different sources of uncertainty during life cycle of a moment frames building.
In this phase, first, through a parametric study, sensitive modeling RV`s (which will be then incorporated in the BPN in the next phase) are detected. To attain comprehensive probable FM`s within pushover analyses as well as sensitivity assessment, a wide range of realizations corresponding to the distribution function of each of the opted RV`s, is considered. Following this goal, lateral load pattern in pushover analyses is also recommended to be taken as another RV based on the most effective mode shapes.

PH`s in pushover analyses are defined on the basis of rotation limit state. A plastic hinge occurs as soon as the rotation at element ends exceeds the value δ_{pl}. In elastic perfect-plastic elements δ_{pl} is where the PH reaches to its residual strength with zero stiffness. Yet when it comes to multi linear backbone curves, this performance function contains rigorous complexities. Thus, to cover a wide range of PH formation characteristics, induced by ground motion records with various excitation energy, intensity and frequency contents, four states of plastic deformation thresholds are assumed for δ_{pl}: δ_y, δ_c, $(\delta_c+10\%)/2$ and rotation equivalent to 10% (See Fig. 2).

The concept of detecting the likely collapse scenarios is comprehensive and straightforward. That is, during the push stage of a frame with selected model parameters and lateral load distributions, and based on the preselected values for δ_{pl} a progressive chain of PH's are formed. Pushover analysis, rationally and comprehensively is determined to be stopped when either the lateral strength becomes 50% of the maximum value after reaching the apex or the structure becomes unstable due to singularity of the tangent stiffness matrix. At the end of the pushover analyses, likely FM`s with sequential PH`s are detected.
Through this stage, unlikely FM`s are eliminated so as to make the methodology feasible for large structures with numerous degrees of freedom.

4.2. Developing an IDA-Based BPN Including the Likely FM`s

A BPN is a Directed Acyclic Graph (DAG) which contains: Basic (parent nodes) and dependent (child nodes) RV`s, Directed acyclic arches representing the dependency among nodes and conditional probabilities between dependent nodes (Faber, 2007). By discrete calculation of the total probability theorem integrand, a BPN computes mean annual rate of collapse ($\lambda_{collapse}$) of a system including uncertainty sources (see Eq.1).

$$l_{Collapse} = \int_{IM}\int_{RVs} P[Collapse \mid im, RVs] \times dF(RVs) \times \mid d\,l\,[im] \mid \tag{1}$$

Where *P[collapse|im,RVs]* is the probability of collapse conditioned on specific value of IM and modeling *RVs*, *dF(RVs)* is the differential of the cumulative joint probability function of the vector of modeling RVs and *|d[λ (im)]|* is the absolute differentiation of the PSHA curve.

For the 1-bay, 2-storey SSMRF considered in the paper by Banazadeh and Fereshtehnejad (2011), the developed BPN is depicted in Fig. 4. In this graph, Basic RV`s are placed in the first layer, while the collapse reliability index node ($\lambda_{collapse}$) is designated in the second layer.

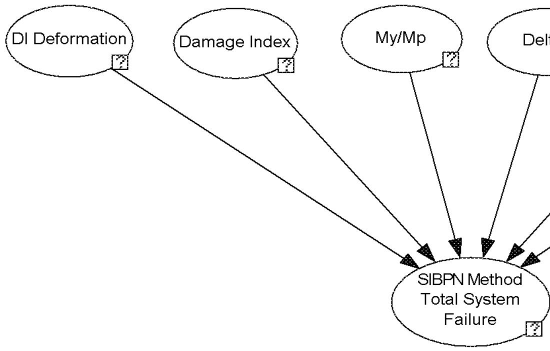

Figure 4. The developed BPN for the 1-bay, 2-storey SSMRF using GeNIe software.

Generally, in the IDA framework, a set of nonlinear dynamic analyses are carried out at incremental levels of the IM and a desired EDP is evaluated at these levels. In order to define conditional probability of the ($\lambda_{collapse}$) chance node, using IDA approach, the appraised EDP would be the numbers (and if necessary sequences) of the PH`s. In each intensity level (a specific realization of Sa (T1) chance node), a set of 44 IDA`s corresponding to the 44 time history records, for a particular state of the other basic RV`s, are performed and plasticization of each node is investigated. As a result, numbers and sequences of PH`s are recorded at the end of the Nonlinear Response History Analysis (NRHA). Following that, it is evaluated whether or not any of the FM`s extracted from pushover analysis, has taken place. If the answer for a record is positive, the conditional probability of the related FM for the specific arrangement of RV`s (which is defined by "$RV_i=rv_i$" in Fig. 5), adds up with a value of 1/44 (i.e. the Probability Mass Function (PMF) of each record). Therefore, conditional probability values corresponding to every realization, for a series of incremental IM`s, is a fragility function. This approach is also illustrated schematically in Fig. 5 for the evaluated structure in the previous work by authors (2011). It is worthy to note that resurrection is ignored and for the sake of simplicity, it is assumed after the instant at which an FM have formed, the structure collapses with the same FM at the first place, for higher levels of IM.

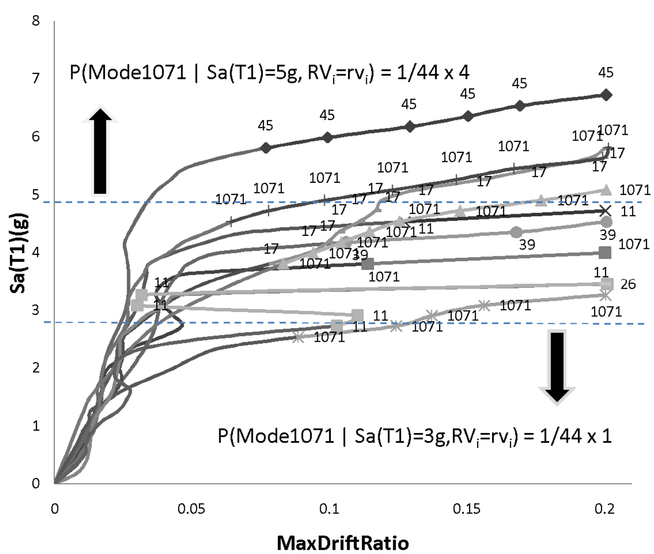

Figure 5. Illustration of conditional probability extraction for the $\lambda_{collapse}$ chance node using the SIBPN method.

At the end by numerically solving the total probability theorem via BPN, $\lambda_{collapse}$ and consequently system reliability index ($\beta_{collapse}$) of the structure will be derived.

5. INCORPORATION OF THE RESPONSE SURFACE PROCEDURE IN SIBPN METHODOLOGY (RSIBPN)

In order to complete the BPN, NRHA should be performed for every realization of the basic RV`s. Furthermore, when detecting FM`s in every incremented steps of IDA curve, every likely FM`s derived from pushover analysis needs to be checked whether is formed or not. Hence, when the number of basic RV`s increases and/or the number of degrees of freedom in the structure rises to the extent that its probable FM`s become too much in number, the proposed SIBPN approach needs to be time-efficiently enhanced. An approach by which the number of NRHA`s could be significantly decreased, is the Response Surface (RS) procedure.

5.1. A Quick Review on the Response Surface Procedure

Box-Wilson Central Composite Design (CCD) (NIST/SEMATECH, 2012) approach is utilized in order to build a response surface that expediently predicts the conditional fragility curves in the reliability analyses. In the CCD technique, the target variables are calculated for a set of star and factorial points of the basic RV`s (See Fig. 6). By perturbing only one RV from its mean on the condition that other RV`s have their mean values, star points are produced. Factorial points are generated when all RV`s are deviated from their mean by having a desired distance from their expected values. The distances for both star and factorial points depend on the experimental properties desired for design and the number of RV`s (NIST/SEMATECH, 2012). On this basis, star values for both the 1-bay 2-storey SSMRF and the 2-bay 4-storey ISMRF were generated by the distance of ±1 Standard Deviation (SD) from the mean values, while factorial values, due to the diversity in the number of the basic RV`s, were different for the assumed structures: The distance considered for the 3 basic modeling RV`s in the SSMRF was ±1.7 times the SD from the mean values, while it was defined as ±2 times the SD from the mean values in the ISMRF with 4 basic RV`s.

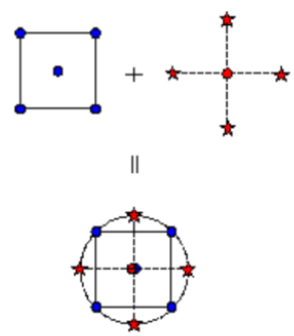

Figure 6. Star and factorial points in the Box-Wilson CCD approach (NIST/SEMATECH, 2012)

5.2. Predicted Target Variables (TV`s) within the RSIBPN method

As previously explained, the conditional probabilities required to fulfill the BPN in the SIBPN methodology is in fact values of multiple fragility curves. These curves strongly tend towards lognormal CDF shape. As a result, lognormal mean and SD of the fragility curves specified for each state of systematic and modeling RV`s were considered as the TV`s to be predicted via RS approach. The same TV`s were also considered by Jalali et al (2011). They combined RS method with Monte Carlo simulations to perform reliability assessment for steel moment frames with generic locally reinforced connections.

This is noteworthy that due to the discrete and explanatory nature of the aforementioned systematic RV`s, particular response surfaces were regressed for each realization of these RV`s.
By combining RS approach with the proposed SIBPN methodology, required time for the 1-bay, 2-storey SSMRF reduced to almost less than 10% of the time used in SIBPN technique. This ratio could even decrease for higher number of modeling RV`s.

In the following, the accuracy of the enhanced methodology is evaluated and the results are compared with the ones taken from the SIBPN approach.

6. COMPARING THE RESULTS TAKEN FROM RSIBPN WITH SIBPN PROCEDURES

In order to gain the response surface of the 1-bay, 2-storey SSMRF, distinct regression functions were extracted in two hierarchical steps. In the first step, fragility functions corresponding to each combination of star and factorial points were regressed via a lognormal distribution. As a result, lognormal mean and SD for each of the CCD realizations were extracted (R-Square for all the realizations in this step were higher than 0.98). In the second step, for each state of the systematic RV`s, lognormal mean and SD as the two TV`s of the RS were regressed linearly based on the 15 combinations of the star and factorial points. Linear regression (which is shown in its general form in Eq.2 and Eq.3) presented high-accuracy for the predicted lognormal Mean while the R-Square values for the predicted lognormal SD were moderate.

$$LN_{Mean} = a_0 + a_1.C_{M_y} + a_2.C_{d_P} + a_2.C_{d_{PC}} \tag{2}$$

$$LN_{SD} = b_0 + b_1.C_{M_y} + b_2.C_{d_P} + b_2.C_{d_{PC}} \tag{3}$$

After the RS functions were extracted, BPN conditional probabilities were computed for each of the random states of the basic RV`s and via the forward propagation process of the BPN, $\lambda_{Collapse}$ of the 1-bay 2-storey SSMRF was calculated. Results are compared in table 2. It is obvious that the collapse reliability index extracted from both RSIBPN and SIBPN are almost identical.

Table 2.Comparing reliability indices via RSIBPN and SIBPN methodologies.

SIBPN Method		RSIBPN Method	
$\lambda_{Collapse}$	$\beta_{Collapse}$	$\lambda_{Collapse}$	$\beta_{Collapse}$
4.56990E-05	*3.912*	*5.81326E-05*	*3.854*

The fragility curve drawn for the structure incorporating all RV`s, on the basis of RSIBPN and SIBPN approaches are depicted in Fig. 7. This graphs also demonstrate that the RS approach has predicted the BPN conditional probabilities for the 1-bay 2storey SSMRF with negligible error.

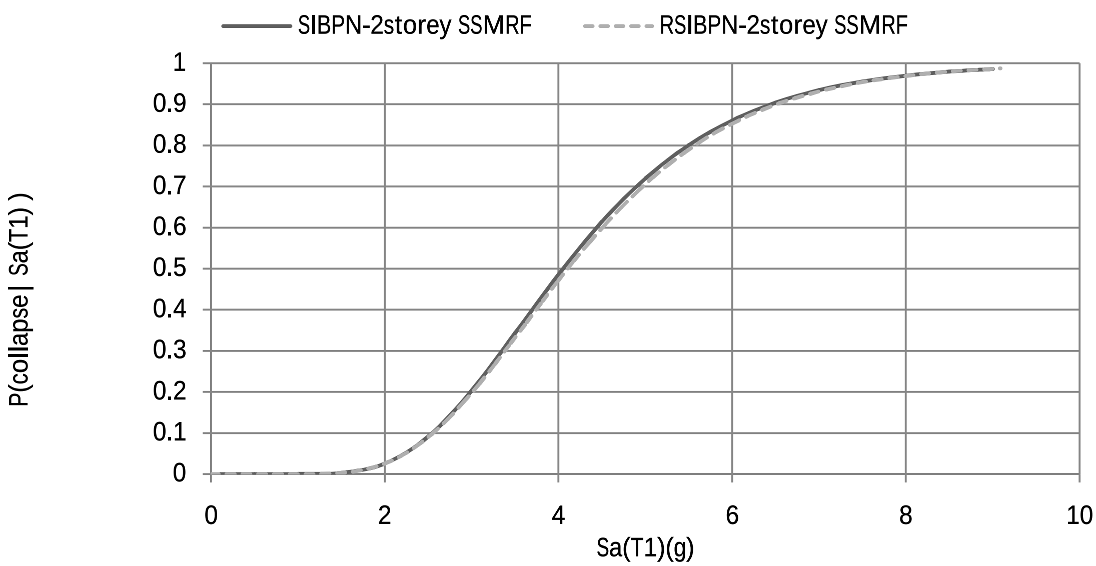

Figure 7. Fragility Curves for the case study SSMRF using SIBPN and RSIBPN approaches

7. IMPLEMENTING RSIBPN METHODOLOGY ON THE CASE STUDY ISMRF

Since the RSIBPN approach is much more time-efficient than the previously proposed SIBPN procedure, another source of uncertainty, Λ, as an indicator of the total energy capacity of a beam/column element in NRHA, has been added to the basic modeling RV`s.

In the first phase of the methodology, after performing multiple pushover analyses for every realization of modeling RV`s and including 3 effective spectral mode shapes, total number of 1204 likely FM`s were detected. By disregarding PH formation sequences, this number reduced to 42. After investigating these FM`s, it was realized that all these FM`s could be categorized into 5 basic failure mechanisms which are shown graphically in Fig. 8. These results show that failure is mainly expected to occur in the first and second floor for the intermediate design of the 4-storey frame structure.

By implementing the proposed RSIBPN methodology and performing 1100 IDA analyses, it was concluded that only 0.7% of the FM`s occurring in the dynamic analyses are not recognized. This could be an evidence for the accuracy of the proposed approach in determining likely FM`s in NRHA`s.

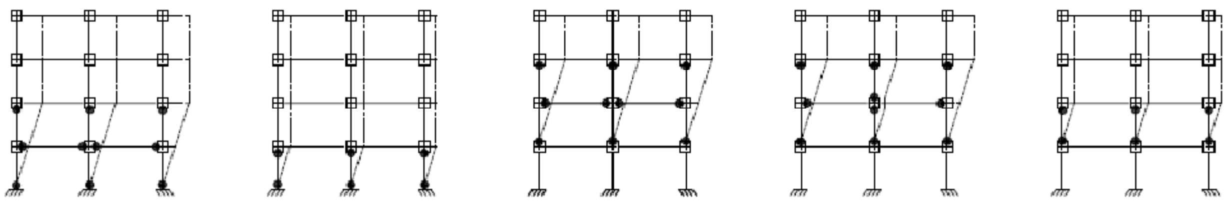

Figure 8. Basic collapse mechanisms of the case study ISMRF

In the following, reliability assessment for the 2-bay 4-story ISMRF was performed utilizing RSIBPN approach and collapse reliability index was computed as 3.362. This indicates less safety of ISMRF compared to the 2-story SSMRF evaluated before. Hence, it is obvious that via this approach, not only the collapse reliability analysis of structures could be computed in a time-efficient manner, but also structural response in the collapse limit state could be investigated deliberately and comprehensively in a probabilistic framework.

Finally it is recommended that the new enhanced methodology be investigated on more complex structures and more verification be gained for the RS approach combining with the SIBPN methodology.

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