Damage Localization of Output-Only Systems by DLV Method

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15 WCEE
LISBOA 2012

SUMMARY:
In this study, the feasibility of DLV method for damage detection of output-only frame systems is experimentally explored based on their dynamic responses with the SSI algorithm for system identification. Conditions of both single and multiple damages at various locations have been considered. Both a white noise scenario and the El Centro earthquake are considered as the seismic inputs with full or partial observation on the acceleration responses of the floors. The scheme proves effective and robust at a single damage condition under both stationary and non-stationary input excitations. The scheme, however, is insufficient for multiple damage conditions when the structure is partially observed. In such circumstances, the input excitation is needed for a more precise damage assessment as illustrated in a previous work (Wang, Hsieh & Wang, 2011). This study gives further insights on the scheme in terms of effectiveness, robustness, and limitation for damage localization of frame systems from seismic response data.

*Keywords: Stochastic Subspace Identification, Output-Only, Damage Locating Vectors, Flexibili*ty

1. INTRODUCTION

In recent years, structural health monitoring (SHM) has become a very active field of research in civil engineering. Initiated in the early 1960s by the military and aerospace industries, the concept and technical development of damage detection has found widespread applications in many other areas. A SHM system is pragmatic only if integrated with reliable measures in dynamic testing, system identification and damage detection, along with appropriate indices for damage assessment. The natural forces such as earthquakes, winds, floods or sea waves are favorable over any artificial measures as they provide considerable input energy to effectively excite the dynamic characteristics of the target structure at no cost. System identification schemes that make a direct use of the recorded data (say, acceleration) at limited locations are of great interest for practical reasons. Moreover, diagnosis techniques that are sensitive to moderate structural damages even in lack of knowledge of high-frequency modes are desired.

Stiffness is intuitively the most direct physical parameter related to structural damages. However, the sensitivity analysis of stiffness-based approaches requires an accurate analytical model of the intact structure for reference. Synthesis of the stiffness matrix requires contributions of higher modes that are practically difficult to identify with fidelity, let alone the task to obtain an accurate analytical model of real-life structure. On the contrary, the flexibility matrix can be sufficiently synthesized with a limited number of low-frequency modes as the modal contribution decreases in proportion to the square of the corresponding natural frequency. Flexibility-based techniques therefore have been considered of great potential in damage localization of structures. The pioneering work of Pandey and Biswas (Pandey & Biswas, 1994) demonstrated that the damage locations of a wide-flange steel beam could be identified by interrogating the change in the flexibility matrix. This technique has been further extended for damage detection of plane trusses (Pandey & Biswas, 1995). The method of damage locating vectors (DLV) proposed by Bernal (Bernal, 2002) in 2002 indeed is a break-through for the flexibility-based approaches in structural damage detection. The concept of the DLV method is to identify the members with zero stress under some specific loading patterns, namely the DLVs that span the null space of the change in flexibility matrix of the structure before and after the damage state. Structural elements resulting with zero stresses (or internal forces) under the static loads of the DLVs are considered potentially damaged. The DLV technique is capable of identifying multiple damages in the structure via a truncated modal basis without a predetermined reference model. This methodology has also been adopted for damage localization of space trusses or plates by Gao et al. (Gao, Ruiz-Sandoval & Spencer, 2002; Gao, Spencer & Bernal, 2007) and Huynh et al. (Huynh, He & Tran, 2005). Duan et al (Duan et al, 2005) adopted the DLV method for damage assessment of both multiple mass-spring systems and plane trusses. It was concluded that, with only the first two or three modes considered, the damages at two different places could be identified. Huang et al. (Huang, Wang & Lee, 2012) explored the damage localization of frame structures from seismic acceleration responses using the DLV technique with ARX model for system identification. The potential of the DLV method in the detection of local damages from global seismic structural responses for frame systems was confirmed.

The measurement of the natural or operating forces on real-life structures is generally formidable. Dynamic characteristics of the structures have to be extracted from the available output signals only. In such occasions, the stochastic subspace identification (SSI) technique (Van Overschee, & De Moor, 1996; Alicioglu & Lus, 2008) can be adopted to identify structural parameters of the discrete-time state equation from the covariance functions of the measured output signals. Nevertheless, the flexibility matrix required by the DLV analysis cannot be explicitly extracted from output signals only. To overcome this obstacle, Bernal (Bernal & Gunes, 2006) revised the flexibility-based DLV method to comply with stochastic realization results in the context of a state-space representation noting that the null space of the flexibility change implicitly contains the damage localization information and that the DLVs in this null space can be estimated without the flexibility matrix. Wang et al. (Wang, Hsieh, & Wang, 2011) experimentally verified this revised scheme of damage localization analysis for frame structures using the seismic acceleration responses with the SRIM technique (Juang, 1997) for system identification. The input ground motion, however, was taken into account in compliance with the SRIM technique.

Feasibility of the DLV method in association with the SSI technique for damage detection of planar frames has been explored numerically by Wang et al (Wang, Lin & Huang, 2012). As a further step to assess this technique under a more realistic condition, a series of shaking table tests has been conducted. Without loss of generality, a five-storey shear frame with diagonal bracings is considered. The damage conditions of the frame are simulated by partially removing some of the bracings. Conditions of both single and multiple damages at various locations have been considered. Both a white noise scenario and the 1940 El Centro earthquake are considered as the seismic inputs with full or partial observation on the acceleration responses of the floors. The white noise represents a stationary input disturbance, the assumption behind theoretical development of the stochastic analysis. While the case of El Centro earthquake is meant to verify the adaptability of the scheme when the input excitation is not stringently stationary. To comply with the output-only scenario, the information of ground motion is discarded in the stage of SSI analysis.

2. STOCHASTIC SUBSPACE IDENTIFICATION

A stochastic linear-invariant system is commonly represented in a discrete-time state-space model as (Van Overschee, & De Moor, 1996; Alicioglu & Lus, 2008):

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{w}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k) \end{aligned} \tag{2.1}$$

where $\mathbf{x}(k) \in R^{2n\times 1}$ and $\mathbf{y}(k) \in R^{m\times 1}$ are respectively the state and output vectors at time instant k.

$\mathbf{A} \in R^{2n\times 2n}$ is the system matrix and $\mathbf{C} \in R^{m\times 2n}$ is the observation matrix. $\mathbf{w}(k) \in R^{2n\times 1}$ and $\mathbf{v}(k) \in R^{m\times 1}$ are the un-measurable vector signals assumed to be zero-mean, stationary white noise vector sequences with the covariance matrices given by

$$E[\begin{pmatrix} \mathbf{w}(k) \\ \mathbf{v}(k) \end{pmatrix} \begin{pmatrix} \mathbf{w}^T(s) & \mathbf{v}^T(s) \end{pmatrix}] = \begin{bmatrix} \mathbf{R}_{ww} & \mathbf{R}_{wv} \\ \mathbf{R}^T{}_{wv} & \mathbf{R}_{vv} \end{bmatrix} \tag{2.2}$$

where $\mathbf{R}_{ww}$, $\mathbf{R}_{wv}$ and $\mathbf{R}_{vv}$ are the covariance matrices of the noise sequences $\mathbf{w}(k)$ and $\mathbf{v}(k)$. It is assumed that the stochastic process is stationary (Pandey & Biswas, 1994). Since $\mathbf{w}(k)$ and $\mathbf{v}(k)$ are independent of $\mathbf{x}(k)$, we have

$$\begin{aligned} E[\mathbf{x}(k)\mathbf{w}^T(k)] = 0 \\ E[\mathbf{x}(k)\mathbf{v}^T(k)] = 0 \end{aligned} \tag{2.3}$$

The output covariance matrices can be defined as:

$$\mathbf{R}_i = E[\mathbf{y}(k+i)\mathbf{y}^T(k)] = \mathbf{CA}^{\mathbf{i-1}}\mathbf{G} \tag{2.4}$$

where $\mathbf{R}_i \in R^{m\times m}$ with i being an arbitrary time lag, and

$$\mathbf{G} \;= E[\mathbf{x}(k+1)\mathbf{y}^T(k)] \;= \mathbf{A}\sum\mathbf{C}^T + \mathbf{R}_{wv} \tag{2.5}$$

in which $\mathbf{\Sigma} = E[\mathbf{x}(k)\mathbf{x}^T(k)]$ is the state covariance matrix.

By considering a sequence of N data for the output , $\mathbf{y}$, and reorganizing the data in the output block Hankel matrix as

$$\mathbf{Y}_{0|2i-1} = \frac{1}{\sqrt{j}} \begin{bmatrix} \mathbf{y}(0) & \mathbf{y}(1) & \cdots & \mathbf{y}(j-1) \\ \mathbf{y}(1) & \mathbf{y}(2) & \cdots & \mathbf{y}(j) \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}(i-1) & \mathbf{y}(i) & \cdots & \mathbf{y}(i+j-2) \\ \hline \mathbf{y}(i) & \mathbf{y}(i+1) & \cdots & \mathbf{y}(i+j-1) \\ \mathbf{y}(i+1) & \mathbf{y}(i+2) & \cdots & \mathbf{y}(i+j) \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}(2i-1) & \mathbf{y}(2i) & \cdots & \mathbf{y}(2i+j-2) \end{bmatrix}_{2mi\times j} = \begin{bmatrix} \mathbf{Y}_p \\ \hline \mathbf{Y}_f \end{bmatrix} \tag{2.6}$$

where the number of columns (j) is typically equal to $N-2i+2$. The Toeplitz matrix $\mathbf{T}_{i|1} \in R^{mi\times mi}$ may be expressed as

$$\mathbf{T}_{i|1} = \mathbf{Y}_f \mathbf{Y}_p{}^T = \mathbf{O}_i \mathbf{\Gamma}_i \tag{2.7}$$

where the controllability matrix $\mathbf{\Gamma}_i = \left(\mathbf{A}^{i-1}\mathbf{G} \quad \mathbf{A}^{i-2}\mathbf{G} \quad \mathbf{A}^{i-3}\mathbf{G} \quad \cdots \quad \mathbf{G}\right) \in R^{2n \times mi}$ and the observability matrix $\mathbf{O}_i = \begin{pmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{i-1} \end{pmatrix} \in R^{mi \times 2n}$. The Toeplitz matrix can be decomposed by singular value decomposition as

$$\mathbf{T}_{i|1} = \mathbf{USV}^T = \left[\mathbf{U}_1 \quad \mathbf{U}_2\right] \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix} = \mathbf{U}_1 \mathbf{S}_1 \mathbf{V}_1^T \tag{2.8}$$

where $\mathbf{S}_1$ is a diagonal matrix of the non-zero eigenvalues and $\mathbf{U}_1$, $\mathbf{V}_1$ the corresponding left and right eigenvectors. By comparing Equ. (2.7) and (2.8), the observability matrix and controllability matrix can respectively be derived as

$$\begin{aligned} \mathbf{O}_i &= \mathbf{U}_1 \mathbf{S}_1^{1/2} \\ \mathbf{\Gamma}_i &= \mathbf{S}_1^{1/2} \mathbf{V}_1^T \end{aligned} \tag{2.9}$$

The observation matrix, $\mathbf{C}$, and system matrix, $\mathbf{A}$, can be extracted from $\mathbf{O}_i$ without difficulty.

3. THE METHOD OF DAMAGE LOCATING VECTORS

Bernal (Bernal, 2002; Bernal & Gunes, 2006) proposed that the structure subjected to the damage locating vectors, $\mathbf{L}$, would undergo the same deformation before and after the damaged state. This statement immediately leads to

$$\mathbf{D}_F \mathbf{L} = \mathbf{0} \tag{3.1}$$

where $\mathbf{D}_F$ is the flexibility differential of the structure before and after damaged. When $rank(\mathbf{D}_F) < n$ (n is the degree of freedom of the structure), the basis corresponds to the null space of $\mathbf{D}_F$ is the damage locating vectors, $\mathbf{L}$, which can be derived from singular value decomposition of the flexibility differential of the structure before and after the damage state. Members with nearly zero stress under the loadings of DLVs are considered potentially damaged.

The flexibility matrix of the structure can be expressed with the system matrices of the continuous-time state-space representation as

$$\mathbf{F} = -\mathbf{C}_0 \mathbf{A}_c^{-1} \mathbf{H}^{-1} \mathbf{C}_0{}^T \tilde{\mathbf{D}} = \mathbf{Q}\tilde{\mathbf{D}} \tag{3.2}$$

where $\mathbf{A}_c = \dfrac{\ln(\mathbf{A})}{\Delta t} \in R^{2n \times 2n}$ is the continuous-time system matrix; $\mathbf{C}_0 = \left[\mathbf{I} \quad \mathbf{0}\right] \in R^{n \times 2n}$; $\mathbf{H} = \begin{bmatrix} \mathbf{C}_0 \\ \mathbf{C}_0 \mathbf{A}_c \end{bmatrix} \in R^{2n \times 2n}$; $\tilde{\mathbf{D}} = \mathbf{C}_0 \mathbf{A}_c \mathbf{B}_c = -\mathbf{M}^{-1}$ ($\mathbf{M}$ being the mass matrix of the system). With Equ. (3.1), the flexibility differential $\mathbf{D}_F$ can be expressed as

$$\mathbf{D}_F = \Delta \mathbf{Q} \tilde{\mathbf{D}}^i + \mathbf{Q}^d \Delta \tilde{\mathbf{D}} \;\; = \Delta \mathbf{Q} \tilde{\mathbf{D}}^i \tag{3.3}$$

where $\Delta\mathbf{Q} = \mathbf{Q}^d - \mathbf{Q}^i$ and $\Delta\tilde{\mathbf{D}} = \tilde{\mathbf{D}}^d - \tilde{\mathbf{D}}^i = \mathbf{0}$ since the mass matrix is unchanged. By taking the singular value decomposition of $\Delta\mathbf{Q}$, the eigen-vectors $\mathbf{V}_0^{\Delta\mathbf{Q}}$ correspond to the singular eigen-values is the damage locating vector $\mathbf{L} \in R^{n \times q}$. Due to noise and numerical errors, the ideal singular eigen-values are practically not existing. Therefore, Bernal suggests that the number of DLVs, q, is screened by

$$q = 0.5\left[\text{No. of } \gamma_i \le 0.1 \;,\; \gamma_i = \sqrt{\frac{s_i^{\Delta\mathbf{Q}}}{\max(s_i^{\Delta\mathbf{Q}})}}\right] \tag{3.4}$$

to the closest integer. Where $s_i^{\Delta\mathbf{Q}}$ is the i-th eigenvalue of $\Delta\mathbf{Q}$. Moreover, the normalized stress index $nsi_{j,\,i}$ of the j-th member or d.o.f. subjected to the i-th DLV, $\mathbf{L}_i$, is defined as

$$nsi_{j,\,i} = \left|\frac{\sigma_{j,\,i}}{\left|\sigma_{j,\,i}\right|_{\max}}\right| \tag{3.5}$$

where $\sigma_{j,\,i}$ is the stress or internal force of the j-th d.o.f. corresponding to $\mathbf{L}_i$. In addition, the weighted stress index WSI_j is defined as

$$\mathrm{WSI}_j = \sum_{i=1}^{q} nsi_{j,\,i} \tag{3.6}$$

When $\mathrm{WSI}_j \le 0.1(\mathrm{WSI}_j)_{\max}$, member j (or storey j) is considered potentially damaged.

4. EXPERIMENTAL VERIFICATIONS

A five-storey steel frame with diagonal bracings is devised for the shaking table tests. The damage conditions of the frame are simulated by removing some of the bracings. Conditions of both single and multiple damages have been considered. Both a white noise scenario and the 1940 El Centro earthquake are considered as the seismic inputs with full or partial observation on the acceleration responses of the floors in the testing program. The tests are arranged in the following manner:

A. Full observation
  Single storey damaged – Case *Ax* (*x=2,3,4* denoting the damaged storey);
  Multiple stories damaged –A15 (1F & 5F damaged), A135 (1F, 3F & 5F damaged)
B. Partial observation (Observing only Floors 1,3 and 5)
  Single storey damaged – Case *B1-135* (1F damaged), *B3-135* (3F damaged);
  Multiple stories damaged – Case *B15-135* (1F & 5F damaged)

4.1. Test Results

I. White Noise
A. Full observation
The stress indices for cases with single-storey damage under the White Noise input from DLV analysis are illustrated in Fig. 1. In all the cases (A2, A3, A4), each of the damaged stories are

identified without exception in accordance with the criterion suggested by Bernal.

Fig. 2 shows the stress indices for cases (A15 and A135) with multiple-storey damage. The damaged stories in both cases are successfully identified.

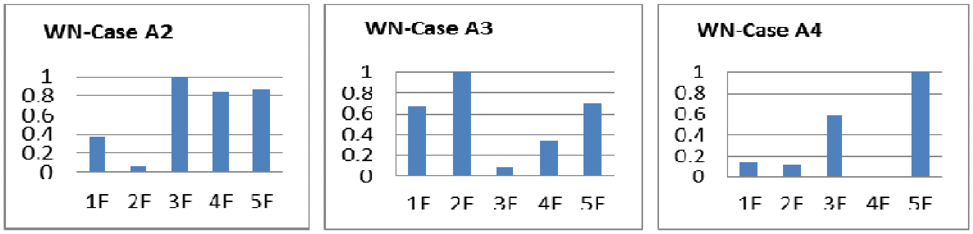

Figure 1. Stress Index under Full Observation (Single Storey Damaged, Input: White Noise)

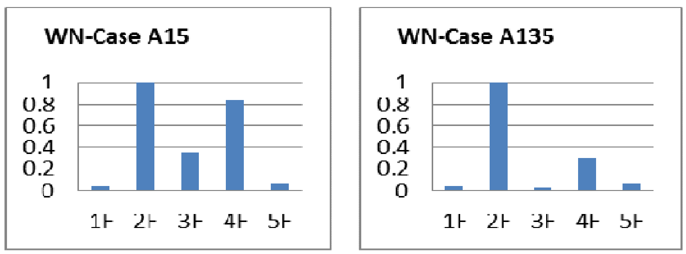

Figure 2. Stress Index under Full Observation (Multiple Stories Damaged, Input: White Noise)

B. Partial observation (Observing only Floors 1, 3 and 5)

To comply with the condition of partial observation, the accelerations of the 2nd and 4th floors are neglected in the system identification stage of SSI analysis. The damaged stories are set to be co-located with part of the observed floors. The stress indices with single or multiple damages are illustrated in Fig. 3. In the cases (B1-135 and B3-135) with single-storey damage, the damaged locations are again identified as in the full observation. The scheme fails, however, to identify the damaged locations in case B15-135 where two stories are designated as damaged.

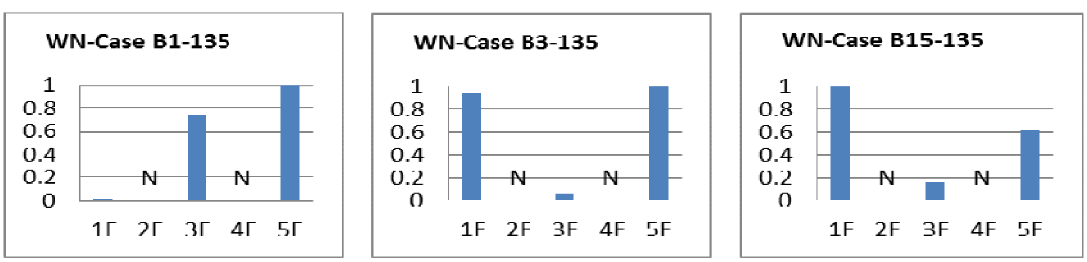

Figure 3. Stress Index under Partial Observation (Observing 1, 3 & 5 F, Input: White Noise)

II. El Centro Earthquake

A. Full observation

The stress indices from DLV analysis for the cases with single-storey damage under the El Centro earthquake are illustrated in Fig. 4. The damaged storey is correctly identified in all cases (A2, A3, A4). It is noted, however, that in case A2 not only the 2nd storey is determined to be damaged but also the 3rd storey where actually no damage is assigned. Nevertheless, the scheme still shows robustness to a certain extent even if the non-stationary nature of the earthquake is against the assumption behind the theoretical development of the SSI algorithm.

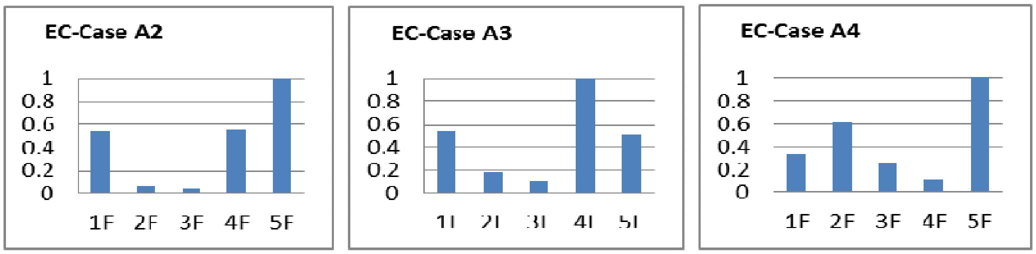

Figure 4. Stress Index under Full Observation (Single Storey Damaged, Input: El Centro)

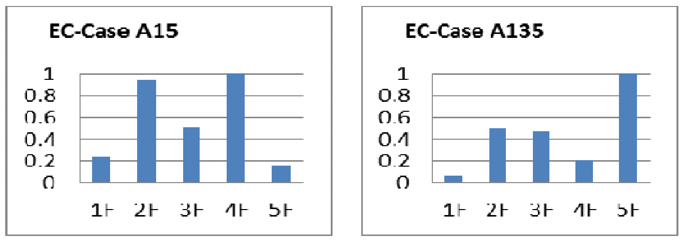

Figure 5. Stress Index under Full Observation (Multiple Stories Damaged, Input: El Centro)

B. Partial observation (Observing only Floors 1, 3 and 5)

The damaged stories are again set to be co-located with part of the observed floors. The stress indices with single or multiple stories damaged are illustrated in Fig. 7. In the cases (B1-135 and B3-135) with single-storey damage, the damaged locations are again identified as in the full observation. The scheme fails also to identify the damaged locations in case B15-135 as for the White Noise excitation.

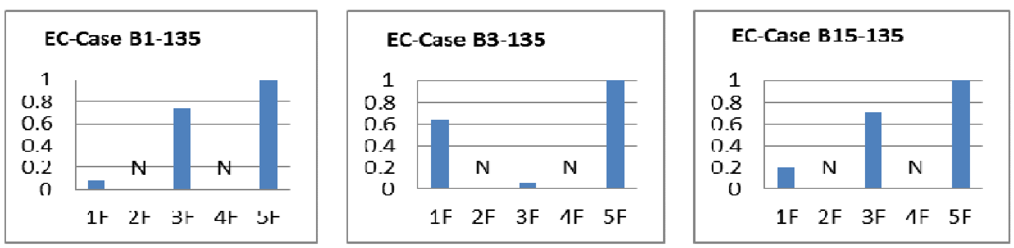

Figure 7. Stress Index under Partial Observation (Observing 1, 3 & 5 F, Input: El Centro)

ACKNOWLEDGEMENT

This work is supported by the National Science Council of Republic of China under contracts NSC 100-2625-M-009-003.

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