The Research of Lithospheric Plates Tectonics of the Earth on the Base of Data of Seismostations, GPS Systems, the Solutions of Problems of Elasticity Theory and the Earthquakes Prediction

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SUMMARY:
Modern science connects the rise of strong earthquakes with the Lithospheric Plates tectonics of the Earth (≈95% of the earthquakes). The existence of dense net of seismostations and modern satellite GPS systems permits us to follow the behaviour of the Earth Lithospheric Plates and its separate parts permanently, particularly, to measure the displacements of points on the facial surface of the Lithospheric Plates and its separate parts and follow their change in time. It, in its turn, permits us to determine the stress-strain state of the plate and follow its change at the given time interval.

The corresponding nonclassical three-dimensional problem of elasticity theory is solved. It is proved, that the solution becomes mathematically exact, when the functions entering the boundary conditions relative to displacements are polynomials.

Following the dynamics of the stress-strain states change of Lithospheric Plates, it is possible to single out more seismodangerous zones of the Earth at the given moment.

Keywords:Tectonic of Lithospheric Plates, GPS systems, prediction of earthquakes.

1. INTRODUCTION

The modern science connects the rise of strong earthquakes with the tectonics of Lithospheric plates of the Earth (≈95% of earthquakes) (Pichon et al (1973), Kasahara (1981)). And what are the Lithospheric plates of the Earth?

It is known that the planet Earth is not homogeneous (the Earth radius = 6378 km) and consists of the earth crust, upper and lower mantles; outer and inner kernels, established on the base of seismic, geological and other investigations.

As a rule, the essentially different speeds of propagation in the layers of longitudinal (Vp) and cross (shear) waves (Vs) are the distinctive feature of difference of these layers.

The power (thickness) of the earth crust on the continents changes from 20 to 70 km, in the oceans from 5 to 15km.

The earth crust is isolated from the upper mantle with Mokhorovichich (Mokho) interface, i.e. seismic border, on which the speed of the longitudinal elastic waves increases in jump-like way up to the value over 8km/sec., whereas in the earth crust it is usually 6-7 km/sec. (max Vp=7,4km/sec.) In the bounds of the earth crust three basic layeres are isolated by seismic characteristics: I sedimental layer ($2{,}0 \le Vp \le 5{,}0$ km/sec., $h_1=10\div25$ km), II granitic layer ($5{,}5 \le Vp \le 6{,}0$ km/sec., $h_2=30\div40$ km),III basalt layer ($6{,}5 \le Vp \le 7{,}4$ km/sec., $h_3=15\div20$ km).

The earth crust and part of the upper mantle up to the border on Aston sphere is called Lithosphere. The Lithosphere is split into some big pieces, which are called plates. The measures of the plates change from a hundred up to some thousand km. The biggest Lithospheric plates of the Earth are:

Antarktida, Eurasian, Indian-Australian, African, South-American, North-American, Pacific, Filippine, Naska, Anatolian, Arabian and other Lithospheric plates.

The geography of the seismic activity of the terrestrial globe points to the fact that the overwhelming majority of the earthquakes are grouped into relatively slender zones ("seismic zones"), seismic and tectonic activities of which are mainly ascribed to inter actions of adherent to each other Lithospheric plates, which are subjected to relative displacements along their contacting surfaces. Two types of tectonic movements are distinguished: 1) slow (century) and 2) quick (jump-like) connected with the earthquakes.

In the base of seismic activity the process of accumulations in rock deformations, which when reaching at critical value 10^{-4} and by Rikitake data $4{,}7 \times 10^{-5}$, brings to global destruction, the main part of the accumulations of the great potential of energy is isolated in the form of volume P (longitudinal or primary) and S (secondary or shear) waves, which spread with speed Vp and Vs.

$$V_p = \sqrt{\frac{(1-\nu)}{(1+\nu)(1-2\nu)}\frac{E}{\rho}}, \qquad V_s = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{E}{2(1+\nu)\rho}}$$

at $\nu = 0.25$, $V_p = \sqrt{3}V_s \approx 1.73V_s$, always $V_p > V_s$.

With the help of the speeds Vp, Vs the epicenter of the earthquake focus is determined.

Having the data for three seismic stations, not lying in the same plane, the epicenter of the earthquake is determined with great exactness.

From the stated above the underlined importance of determination of stress-strain states of the Earth Lithospheric plates and the monitoring of its change in time follows. For that it is possible to use measuring data of the rather dense net of the existing seismostations and satellite GPS systems, which, particularly, measure the values of the points displacements of the Lithospheric plates surface.

In the paper the solutions of the corresponding three-dimensional problem of elasticity theory for one-layered plate and multilayered packet from plates, permitting to find stress-strain states of the plate or the packet on the base of the data of seismostations and GPS systems are found. The corresponding problem is nonclassical boundary-value problem, as the conditions (they are six) are only given on the facial surface of the plate or the packet (the corresponding stresses tensor components are equal to zero, but the values of the points of this surface – as the data of seismostations and GPS systems are known). By the found solutions we have opportunity to follow the change of the stress-strain state and reveal the critical states as by time, as well as by place.

2. THE BASIC EQUATIONS AND FORMULATION OF THE BOUNDARY-VALUE PROBLEM

Let us have a packet from N orthotropic layers, occupying area $D = \{(x, y, z) : 0 \le x \le a,$ $0 \le y \le b,\ 0 \le z \le h, h = \sum_{i=1}^{N} h_i,\ \min(a,b) = \ell, \quad h << \ell\}$ (Fig. 1).

It is required to find the solution of the equations and correlations of three-dimensional problem of elasticity theory taking into account the volume forces (particularly the weight of the layers) and temperature actions by Duhamel-Neyman model (Aghalovyan (1997), Lexhnitsky (1981)):

$$
\begin{aligned}
&\frac{\partial \sigma_{xx}^{(k)}}{\partial x}+\frac{\partial \sigma_{xy}^{(k)}}{\partial y}+\frac{\partial \sigma_{xz}^{(k)}}{\partial z}+F_x^{(k)}=0, \quad (x,y,z), \quad k=1,2,...,N \\
&\frac{\partial u_x^{(k)}}{\partial x}=e_1^{(k)}+\alpha_{11}^{(k)}\theta^{(k)}, \quad (1,2,3;x,y,z) \\
&\frac{\partial u_y^{(k)}}{\partial z}+\frac{\partial u_z^{(k)}}{\partial y}=a_{44}^{(k)}\sigma_{yz}^{(k)}, \ \frac{\partial u_x^{(k)}}{\partial z}+\frac{\partial u_z^{(k)}}{\partial x}=a_{55}^{(k)}\sigma_{xz}^{(k)} \\
&\frac{\partial u_y^{(k)}}{\partial x}+\frac{\partial u_x^{(k)}}{\partial y}=a_{66}^{(k)}\sigma_{xy}^{(k)} \\
&e_m^{(k)}=a_{1m}^{(k)}\sigma_{xx}^{(k)}+a_{2m}^{(k)}\sigma_{yy}^{(k)}+a_{3m}^{(k)}\sigma_{zz}^{(k)}, \ m=1,2,3
\end{aligned} \tag{2.1}
$$

under the boundary conditions at $z=0$

$$
\begin{aligned}
&\sigma_{jz}(x,y,0,t)=0, \quad j=x,y,z \\
&u_j(x,y,0,t)=u_j^{+}(x,y,t)
\end{aligned} \tag{2.2}
$$

and under the conditions of full contact between the layers, which for k layer are written in the form of

$$
\begin{aligned}
&\sigma_{jz}^{(k)}(z=H_k)=\sigma_{jz}^{(k+1)}(z=H_k), \ \ H_k=\sum_{i=1}^{k}h_i, \ \ k=1,2,...,N-1 \\
&u_j^{(k)}(z=H_k)=u_j^{(k+1)}(z=H_k), \quad j=x,y,z
\end{aligned} \tag{2.3}
$$

where σ_{ij} are the stresses tensor components, u_x, u_y, u_z are the displacement vector components, h_i are the thicknesses of the layers, a_{ij} are the constants of elasticity, α_{ij} are the coefficients of the temperature extension, time t enters the conditions Eqn. 2.2 as parameter characterizing the stresses and displacements values for beforehand given time $t=t_n$, i.e. the moment of time, when the dimensions of seismic stations and GPS systems were conducted.

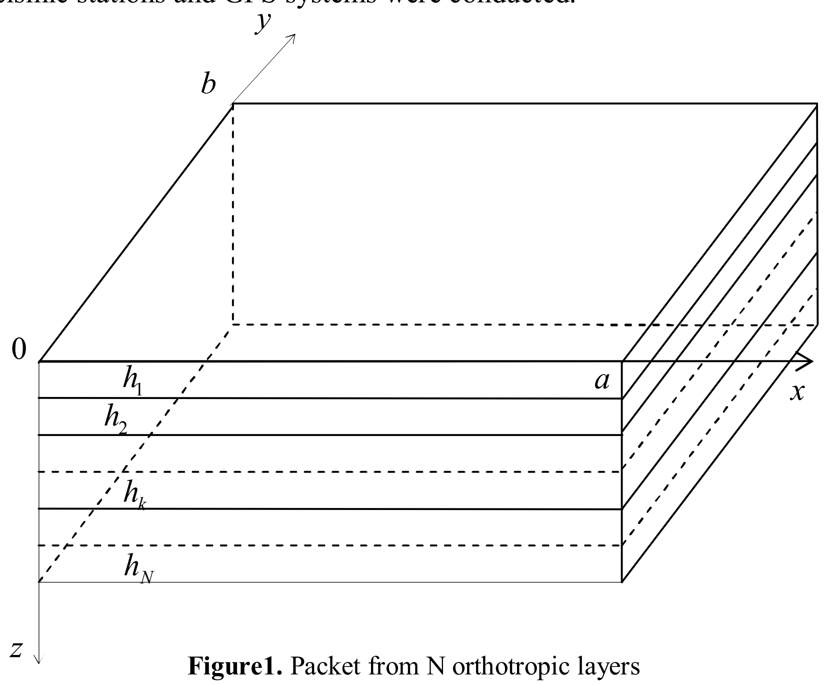

Figure1. Packet from N orthotropic layers

The conditions on the lateral surface of the packet are not defined concretely, as later it will be shown, that the solution of boundary layer type corresponds to them, i.e. such solution, which decreases quickly (exponentially) when removing from the lateral surface into the inside the packet. In practical applications the boundary layer is usually neglected.

3. THE ASYMPTOTIC SOLUTION OF THE PROBLEM

In order to solve the set up boundary-value problem, in the equations and correlations Eqn. 2.1 we pass to dimensionless variables and displacements

$$\begin{aligned} &\xi = x/\ell, \quad \eta = y/\ell, \quad \varsigma = z/h = \varepsilon^{-1} z/\ell \\ &u = u_x/\ell, \quad \mathrm{v} = u_y/\ell, \quad w = u_z/\ell, \quad \varepsilon = h/\ell \end{aligned} \tag{3.1}$$

as a result a singularly perturbed system relative to small parameter is obtained. The solution of such systems Eqn. 2.1 is combined of the solutions of the inner problem (I^{int}) and the boundary layer (I_b) (Aghalovyan (1997), Naife. (1976)). The solution of the inner problem is sought in the form of (Aghalovyan (1997), Aghalovyan (2008))

$$\sigma_{ij}^{(k)\text{int}} = \varepsilon^{-1+s} \sigma_{ij}^{(k,s)}, \quad (i, j = x, y, z), \; u^{(k)\text{int}} = \varepsilon^{s} u^{(k,s)} \;\; (u, \mathrm{v}, w) \;\;\; s = \overline{0, M} \tag{3.2}$$

here and later the notation $s = \overline{0, M}$ means summing by the repeating index s by integer values from zero to the number of approximations M.

Substituting Eqn. 3.2 into the transformed according Eqn. 3.1 system Eqn. 2.1 and equalizing in each equation the coefficients under the same degrees ε, for determination $\sigma_{ij}^{(k,s)}, u^{(k,s)}, \mathrm{v}^{(k,s)}, w^{(k,s)}$ we get the system

$$\begin{aligned} &\frac{\partial \sigma_{xx}^{(k,s-1)}}{\partial \xi} + \frac{\partial \sigma_{xy}^{(k,s-1)}}{\partial \eta} + \frac{\partial \sigma_{xz}^{(k,s)}}{\partial \zeta} + F_x^{(k,s)} = 0, \quad (x, y; \xi, \eta), \quad k = 1,2,\ldots,N \\ &\frac{\partial \sigma_{xz}^{(k,s-1)}}{\partial \xi} + \frac{\partial \sigma_{yz}^{(k,s-1)}}{\partial \eta} + \frac{\partial \sigma_{zz}^{(k,s)}}{\partial \zeta} + F_z^{(k,s)} = 0, \\ &F_j^{(k,0)} = \varepsilon^2 \ell F_j, \quad F_j^{(k,s)} = 0, \; s \neq 0 \\ &\frac{\partial u^{(k,s-1)}}{\partial \xi} = e_1^{(k,s)} + \alpha_{11}^{(k,s)} \theta^{(k,s)}, \quad (1,2,3; u, \mathrm{v}, w; \xi, \eta, \zeta) \\ &\frac{\partial w^{(k,s-1)}}{\partial \eta} + \frac{\partial \mathrm{v}^{(k,s)}}{\partial \varsigma} = a_{44}^{(k)} \sigma_{yz}^{(k,s)}, \frac{\partial w^{(k,s-1)}}{\partial \xi} + \frac{\partial u^{(k,s)}}{\partial \varsigma} = a_{55}^{(k)} \sigma_{xz}^{(k,s)} \\ &\frac{\partial \mathrm{v}^{(k,s-1)}}{\partial \xi} + \frac{\partial u^{(k,s-1)}}{\partial \eta} = a_{66}^{(k)} \sigma_{xy}^{(k,s)} \\ &\theta^{(k,0)} = \varepsilon \theta^{(k)}, \quad \theta^{(k,s)} = 0, \text{ at } s \neq 0, \; \sigma_{ij}^{(k,m)} \equiv 0 \\ &u^{(k,m)} = \mathrm{v}^{(k,m)} = w^{(k,m)} \equiv 0 \text{ at } m < 0 \end{aligned} \tag{3.3}$$

System Eqn. 3.3 permits integration by ζ , a result we have

$$
\begin{aligned}
&\sigma_{jz}^{(k,s)} = \sigma_{jz0}^{(k,s)}(\xi,\eta) + \sigma_{jz*}^{(k,s)}(\xi,\eta,\varsigma), \quad j = x,y,z \\
&\sigma_{xx}^{(k,s)} = -\frac{A_{23}^{(k)}}{A_{11}^{(k)}}\sigma_{zz0}^{(k,s)} - \frac{\gamma_{11}^{(k)}}{A_{11}^{(k)}}\theta^{(k,s)} + \sigma_{xx*}^{(k,s)}(\xi,\eta,\varsigma) \\
&\sigma_{yy}^{(k,s)} = -\frac{A_{13}^{(k)}}{A_{11}^{(k)}}\sigma_{zz0}^{(s)} - \frac{\gamma_{22}^{(k)}}{A_{11}^{(k)}}\theta^{(k,s)} + \sigma_{yy*}^{(s)}(\xi,\eta,\varsigma) \\
&\sigma_{xy}^{(k,s)} = \frac{1}{a_{66}^{(k)}}\left[\frac{\partial \mathrm{v}^{(k,s-1)}}{\partial \xi} + \frac{\partial u^{(k,s-1)}}{\partial \eta}\right] \\
&\mathrm{v}^{(k,s)} = a_{44}^{(k)}\varsigma\sigma_{yz0}^{(k,s)} + \mathrm{v}_0^{(k,s)}(\xi,\eta) + \mathrm{v}_*^{(k,s)}(\xi,\eta,\varsigma) \\
&u^{(k,s)} = a_{55}^{(k)}\varsigma\sigma_{xz0}^{(k,s)} + u_0^{(k,s)}(\xi,\eta) + u_*^{(k,s)}(\xi,\eta,\varsigma) \\
&w^{(k,s)} = \frac{A_{33}^{(k)}}{A_{11}^{(k)}}\varsigma\sigma_{zz0}^{(k,s)} + w_0^{(k,s)}(\xi,\eta) + w_*^{(k,s)}(\xi,\eta,\varsigma), \quad k = 1,2,...,N
\end{aligned}
\tag{3.4}
$$

where

$$
\begin{aligned}
&\sigma_{jz*}^{(k,s)} = -\int_0^{\varsigma}\left[F_j^{(k,s)} + \frac{\partial \sigma_{jx}^{(k,s-1)}}{\partial \xi} + \frac{\partial \sigma_{jy}^{(k,s-1)}}{\partial \eta}\right]d\varsigma, \quad j = x,y,z \\
&\sigma_{xx*}^{(k,s)} = \frac{1}{A_{11}^{(k)}}\left[a_{22}^{(k)}\frac{\partial u^{(k,s-1)}}{\partial \xi} - a_{12}^{(k)}\frac{\partial \mathrm{v}^{(k,s-1)}}{\partial \eta} - A_{23}^{(k)}\sigma_{zz*}^{(k,s)}\right] \\
&\sigma_{yy*}^{(k,s)} = \frac{1}{A_{11}^{(k)}}\left[a_{11}^{(k)}\frac{\partial \mathrm{v}^{(k,s-1)}}{\partial \eta} - a_{12}^{(k)}\frac{\partial u^{(k,s-1)}}{\partial \xi} - A_{13}^{(k)}\sigma_{zz*}^{(k,s)}\right] \\
&u_*^{(k,s)} = \int_0^{\varsigma}\left[a_{55}^{(k)}\sigma_{xz*}^{(k,s)} - \frac{\partial w^{(k,s-1)}}{\partial \xi}\right]d\varsigma, \quad \mathrm{v}_*^{(k,s)} = \int_0^{\varsigma}\left[a_{44}^{(k)}\sigma_{yz*}^{(k,s)} - \frac{\partial w^{(k,s-1)}}{\partial \eta}\right]d\varsigma \\
&w_*^{(k,s)} = \int_0^{\varsigma}\left[a_{13}^{(k)}\sigma_{xx*}^{(k,s)} + a_{23}^{(k)}\sigma_{yy*}^{(k,s)} + a_{33}^{(k)}\sigma_{zz*}^{(k,s)}\right]d\varsigma \\
&A_{11}^{(k)} = a_{11}^{(k)}a_{22}^{(k)} - \left(a_{12}^{(k)}\right)^2, \quad A_{13}^{(k)} = a_{11}^{(k)}a_{23}^{(k)} - a_{12}^{(k)}a_{13}^{(k)} \\
&A_{23}^{(k)} = a_{22}^{(k)}a_{13}^{(k)} - a_{12}^{(k)}a_{23}^{(k)}, \quad A_{33}^{(k)} = a_{33}^{(k)}A_{11}^{(k)} - a_{13}^{(k)}A_{23}^{(k)} - a_{23}^{(k)}A_{13}^{(k)} \\
&\gamma_{11}^{(k)} = \alpha_{11}^{(k)}a_{22}^{(k)} - \alpha_{22}^{(k)}a_{12}^{(k)} \\
&\gamma_{22}^{(k)} = \alpha_{22}^{(k)}a_{11}^{(k)} - \alpha_{11}^{(k)}a_{12}^{(k)}, \quad k = 1,2,...,N, \quad Q^{(k,m)} \equiv 0 \text{ at } m < 0
\end{aligned}
\tag{3.5}
$$

In general case the solution Eqn. 3.4, Eqn. 3.5 contains *6N* unknown functions $\sigma_{xz0}^{(k,s)}, \sigma_{yz0}^{(k,s)}$, $\sigma_{zz0}^{(k,s)}, u_0^{(k,s)}, \mathrm{v}_0^{(k,s)}, w_0^{(k,s)}$, $k = 1,2,..,N$, which are uniquely determined from six boundary conditions Eqn. 2.2 and *6(N-1)* conditions of the contact Eqn. 2.3. We consider, that the process of accumulation of critical deformations is quasistatic, that is why the equations of equilibrium have been used Eqn.2.1. The method permits us to consider dynamic problems as well. We describe the procedure of satisfaction of conditions Eqn. 2.2, Eqn. 2.3. At first the values of the first layer are determined by means of the satisfaction of the conditions Eqn. 2.2 (Aghalovyan (2011)). Using the formulae Eqn.3.2, Eqn. 3.4, Eqn.3.5 and satisfying the conditions Eqn. 2.2, we have

$$
\sigma_{jz0}^{(1,s)}(\xi,\eta) = 0, \ \sigma_{jz}^{(1,s)}(\xi,\eta,\zeta) = \sigma_{jz*}^{(1,s)}(\xi,\eta,\zeta), \quad j = x,y,z
$$

$$u_0^{(1,s)}(\xi,\eta)=u^{+(s)},\ u^{+(0)}=u^{+}/\ell,\ u^{+(s)}=0,\ s\neq 0,\quad (u,\mathrm{v},w)$$
$$u^{(1,s)}=u^{+(s)}+u_*^{(1,s)}(\xi,\eta,\zeta),\qquad (u,\mathrm{v},w) \tag{3.6}$$

Having the solution Eqn. 3.6, by the corresponding formulae Eqn. 3.4, Eqn. 3.5 the stresses $\sigma_{xx}^{(1,s)}$, $\sigma_{yy}^{(1,s)}$, $\sigma_{xy}^{(1,s)}$ are determined. So, the values of the first layer were completely determined after the satisfaction of the conditions Eqn. 2.2. Having the values of the stresses and displacements of the first layer, by means of satisfaction of the contact conditions Eqn. 2.3 between the first and second layers, all the desired values of the second layer are determined. Then using the contact conditions Eqn. 2.3 between the second and third layers, the stresses and displacements of the third layer are determined and later in the same way the values of the rest of the layers are determined. In general case the satisfaction of the conditions Eqn. 2.3 for an arbitrary k layer brings to the solution of the following recurrent equations

$$\begin{aligned}
&\sigma_{jz0}^{(k,s)}(\xi,\eta)+\sigma_{jz*}^{(k,s)}(\xi,\eta,\zeta_k)=\sigma_{jz0}^{(k+1,s)}(\xi,\eta)+\sigma_{jz*}^{(k+1,s)}(\xi,\eta,\zeta_k)\\
&a_{55}^{(k)}\zeta_k\sigma_{xz0}^{(k,s)}+u_0^{(k,s)}(\xi,\eta)+u_*^{(k,s)}(\xi,\eta,\zeta_k)=a_{55}^{(k+1)}\zeta_k\sigma_{xz0}^{(k+1,s)}+\\
&+u_0^{(k+1,s)}(\xi,\eta)+u_*^{(k+1,s)}(\xi,\eta,\zeta_k);\quad (u,\mathrm{v};x,y;a_{55},a_{44})\\
&\frac{A_{33}^{(k)}}{A_{11}^{(k)}}\zeta_k\sigma_{zz0}^{(k,s)}+w_0^{(k,s)}(\xi,\eta)+w_*^{(k,s)}(\xi,\eta,\zeta_k)=\\
&=\frac{A_{33}^{(k+1)}}{A_{11}^{(k+1)}}\zeta_k\sigma_{zz0}^{(k+1,s)}+w_0^{(k+1,s)}(\xi,\eta)+w_*^{(k+1,s)}(\xi,\eta,\zeta_k)\\
&\zeta_k=H_k/h,\quad H_k=\sum_{i=1}^{k}h_i
\end{aligned} \tag{3.7}$$

from where and from the formulae Eqn. 3.4 follows

$$\begin{aligned}
&\sigma_{jz0}^{(k+1,s)}(\xi,\eta)=\sigma_{jz0}^{(k,s)}(\xi,\eta)+\sigma_{jz*}^{(k,s)}(\xi,\eta,\zeta_k)-\sigma_{jz*}^{(k+1,s)}(\xi,\eta,\zeta_k)\\
&\sigma_{jz}^{(k+1,s)}(\xi,\eta)=\sigma_{jz0}^{(k,s)}(\xi,\eta)+\sigma_{jz*}^{(k,s)}(\xi,\eta,\zeta_k)-\\
&-\sigma_{jz*}^{(k+1,s)}(\xi,\eta,\zeta_k)+\sigma_{jz*}^{(k+1,s)}(\xi,\eta,\zeta)\\
&u_0^{(k+1,s)}(\xi,\eta)=a_{55}^{(k)}\zeta_k\sigma_{xz0}^{(k,s)}+u_0^{(k,s)}(\xi,\eta)+\\
&+u_*^{(k,s)}(\xi,\eta,\zeta_k)-a_{55}^{(k+1)}\zeta_k\sigma_{xz0}^{(k+1,s)}-u_*^{(k+1,s)}(\xi,\eta,\zeta_k)\\
&u^{(k+1,s)}(\xi,\eta,\zeta)=a_{55}^{(k+1)}(\zeta-\zeta_k)\sigma_{xz0}^{(k+1,s)}+a_{55}^{(k)}\zeta_k\sigma_{xz0}^{(k,s)}+u_0^{(k,s)}(\xi,\eta)+\\
&+u_*^{(k,s)}(\xi,\eta,\zeta_k)+u_*^{(k+1,s)}(\xi,\eta,\zeta)-u_*^{(k+1,s)}(\xi,\eta,\zeta_k)\qquad (u,\mathrm{v};x,y;a_{55},a_{44})\\
&w_0^{(k+1,s)}(\xi,\eta)=\frac{A_{33}^{(k)}}{A_{11}^{(k)}}\zeta_k\sigma_{zz0}^{(k,s)}+w_0^{(k,s)}(\xi,\eta)+w_*^{(k,s)}(\xi,\eta,\zeta_k)-\\
&-\frac{A_{33}^{(k+1)}}{A_{11}^{(k+1)}}\zeta_k\sigma_{zz0}^{(k+1,s)}-w_*^{(k+1,s)}(\xi,\eta,\zeta_k)\\
&w^{(k+1,s)}(\xi,\eta,\zeta)=\frac{A_{33}^{(k+1)}}{A_{11}^{(k+1)}}(\zeta-\zeta_k)\sigma_{zz0}^{(k+1,s)}+\frac{A_{33}^{(k)}}{A_{11}^{(k)}}\zeta_k\sigma_{zz0}^{(k,s)}+w_0^{(k,s)}(\xi,\eta)+\\
&+w_*^{(k,s)}(\xi,\eta,\zeta_k)+w_*^{(k+1,s)}(\xi,\eta,\zeta)-w_*^{(k+1,s)}(\xi,\eta,\zeta_k),\quad (k=1,2,\ldots,N-1)
\end{aligned} \tag{3.8}$$

For the first layer we have the solution Eqn. 3.6, then from Eqn. 3.8 at $k=1$ the solution for the second layer will be determined, at $k=2$ the solution of the third layer is determined, etc..

Thus, the conditions Eqn. 2.2, Eqn. 2.3 turned out to be sufficient for determination of all the desired values of all the layers. From here it follows, that the solution of the boundary layer (I_b) will be determined independently and will remove in coordination when satisfying the boundary conditions on the lateral surface. As it is denoted above, in practical applications as a rule, the boundary layer is neglected.

4. ON MATHEMATICALLY EXACT SOLUTIONS

If the functions u_j^+ entering the boundary conditions Eqn. 2.2 are polynomials from $x, y, (\xi,\eta)$, the iterated process of determination $\sigma_{ij}^{(k,s)}$, $u^{(k,s)}$, $\mathrm{v}^{(k,s)}, w^{(k,s)}$ cuts off on certain approximation, depending on the degree of the polynomial. As a result we obtain a mathematically exact solution in the inner problem. For the illustration of what has been said above the solution of the boundary problem Eqn. 2.1 – Eqn. 2.3 at $\theta^{(k)}=0,\ F_j^{(k)}=0$ will be brought, and

$$u^+=\ell(a_1+a_2\xi+a_3\eta),\ \mathrm{v}^+=\ell(b_1+b_2\xi+b_3\eta),\ w^+=\ell(c_1+c_2\xi+c_3\eta) \tag{4.1}$$

Using the formulae Eqn. 3.6 – Eqn. 3.8, it is easy to be convinced, that the approximations $s=0,1$ will be different from zero. Calculating these approximations for three-layered packet, according to the formulae Eqn. 3.1, Eqn. 3.2, the following exact solution will be obtained:

the values of the first layer ($0\le z\le h_1$)

$$\begin{aligned}
&\sigma_{xz}^{(1)\mathrm{int}}=0,\quad \sigma_{yz}^{(1)\mathrm{int}}=0,\quad \sigma_{zz}^{(1)\mathrm{int}}=0\\
&\sigma_{xx}^{(1)\mathrm{int}}=\varepsilon^{-1}\sigma_{xx}^{(1,0)}+\varepsilon^{0}\sigma_{xx}^{(1,1)}=\frac{1}{A_{11}^{(1)}}(a_2a_{22}^{(1)}-b_3a_{12}^{(1)})\\
&\sigma_{yy}^{(1)\mathrm{int}}=\sigma_{yy}^{(1,1)}=\frac{1}{A_{11}^{(1)}}(b_3a_{11}^{(1)}-a_2a_{12}^{(1)}),\qquad \sigma_{xy}^{(1)\mathrm{int}}=\sigma_{xy}^{(1,1)}=\frac{1}{a_{66}^{(1)}}(a_3+b_2)\\
&u_x^{(1)\mathrm{int}}=\ell(\varepsilon^0u^{(1,0)}+\varepsilon u^{(1,1)})=\ell(a_1+a_2\xi+a_3\eta)-c_2z\\
&u_y^{(1)\mathrm{int}}=\ell(\varepsilon^0\mathrm{v}^{(1,0)}+\varepsilon \mathrm{v}^{(1,1)})=\ell(b_1+b_2\xi+b_3\eta)-c_3z\\
&u_z^{(1)\mathrm{int}}=\ell(\varepsilon^0w^{(1,0)}+\varepsilon w^{(1,1)})=\ell(c_1+c_2\xi+c_3\eta)+\\
&+\frac{1}{A_{11}^{(1)}}(a_{13}^{(1)}(a_2a_{22}^{(1)}-b_3a_{12}^{(1)})+a_{23}^{(1)}(b_3a_{11}^{(1)}-a_2a_{12}^{(1)}))z
\end{aligned} \tag{4.2}$$

the values of the k-rd layer ($\sum_{j=1}^{k-1}h_j\le z\le\sum_{m=1}^{k}h_m$)

$$\sigma_{xz}^{(k)\mathrm{int}}=0,\quad \sigma_{yz}^{(k)\mathrm{int}}=0,\quad \sigma_{zz}^{(k)\mathrm{int}}=0,\ k=2,3,\dots,N$$

$$
\begin{aligned}
&\sigma_{xx}^{(k)\mathrm{int}} = \frac{1}{A_{11}^{(k)}}(a_2 a_{22}^{(k)} - b_3 a_{12}^{(k)}), \quad \sigma_{yy}^{(k)\mathrm{int}} = \frac{1}{A_{11}^{(k)}}(b_3 a_{11}^{(k)} - a_2 a_{12}^{(k)}), \quad \sigma_{xy}^{(k)\mathrm{int}} = \frac{1}{a_{66}^{(k)}}(a_3 + b_2) \\
&u_x^{(k)\mathrm{int}} = \ell(a_1 + a_2\xi + a_3\eta) - c_2 z \\
&u_y^{(k)\mathrm{int}} = \ell(b_1 + b_2\xi + b_3\eta) - c_3 z \\
&u_z^{(k)\mathrm{int}} = \ell(c_1 + c_2\xi + c_3\eta) + \\
&\quad + \left[\sum_{j=1}^{k} \frac{1}{A_{11}^{(j)}}(a_{13}^{(j)}(a_2 a_{22}^{(j)} - b_3 a_{12}^{(j)}) + a_{23}^{(j)}(b_3 a_{11}^{(j)} - a_2 a_{12}^{(j)}))h_j\right](z - \sum_{j=1}^{k-1} h_j)
\end{aligned} \tag{4.3}
$$

On the base of the obvious regularities in formulae Eqn. 4.2, Eqn. 4.3, it is not difficult to write out the exact solution of the inner problem for *N*-layered packet. For the packet of the finite tangential dimensions, close to the lateral surface to this solution the solution of the boundary layer should be added. The above brought property of the solution may have an important applied significance. Really, let for the moment of time $t = t_n$ the data of seismic stations and GPS systems $u_x^+(\xi_i, \eta_i, t_n),\ u_y^+(\xi_i, \eta_i, t_n),\ u_z^+(\xi_i, \eta_i, t_n)$ known in *m* points of the facial surface of *N*-layered packet foundation-base. Then the displacements of the facial surface may be represented in the form of Lagrange polynomial

$$
u_x^+(\xi, \eta, t_n) = \sum_{i=1}^{m} u_x^+(\xi_i, \eta_i, t_n) \frac{\prod\limits_{j=1, j\neq i}^{m} (\xi - \xi_j)(\eta - \eta_j)}{\prod\limits_{j=1, j\neq i}^{m} (\xi_i - \xi_j)(\eta_i - \eta_j)}, \quad (x, y, z) \tag{4.4}
$$

where Π means product.

Substituting Eqn.4.4 into the formulae Eqn. 3.1, Eqn. 3.2, Eqn. 3.4 – Eqn. 3.8, after the final number of iteration we determine mathematically exact solution of the problem, corresponding to the dimensions of seismic stations and GPS systems, i.e. the stress-strain state of the whole packet, corresponding to time $t = t_n$. Modern computational tools may determine this solution in a few minutes.

Conducting the monitoring of the solution by time and observing the change of the stress-strain state of responsible structures construction area in seismic dangerous zones, the full representation about the construction area state may be composed and the possibility of critical situations rise may be revealed.

5. CONCLUSION

Modern science mainly relates the emergence of strong earthquakes to Lithospheric plates tectonics of the Earth (≈95% of earthquakes). In this paper the problem of determining stress-strain states of Lithospheric plates based on equations and relations of the three-dimensional problem of elasticity theory and data of seismic stations and GPS systems is considered. The corresponding non-classical three-dimensional problem is solved by the asymptotic method. The data from GPS systems and seismic stations on the values of displacements of points of the face surface of a plate is approximated by Lagrange polynomial, and the corresponding mathematically exact solution of the internal problem is derived.

Tracing the behavior of stress-strain state of Lithospheric plates over time (monitoring) provides an opportunity for establishing the place and time of critical stress-strain states leading to a global destruction. Together with the analysis of anomalous phenomena accompanying earthquake it opens a way for prediction strong earthquakes.

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