Force Reduction Factors for Building Structures Equipped with Added Viscous Dampers

S. Silvestri, L. Landi, G. Gasparini, M. Palermo & T. Trombetti
Department DICAM, University of Bologna, Italy

SUMMARY:
This paper focuses on the study of the hysteretic behaviour of inelastic SDOF system equipped with viscous dampers aimed at obtaining a practical tool useful for the seismic design of building structures with added dampers, within the framework of the seismic design based on ductility. The objective is to evaluate the appropriate force reduction factor for higher damped (i.e. damping ratio greater than 5%) SDOF system able to guarantee a prescribed value of structural safety.

Keywords: viscous dampers; ductility demand; force reduction factor; non linear dynamic analysis.

1. INTRODUCTION

The fundamental objective of the traditional structural design for seismic actions characterised by high intensity (SEAOC Vision 2000 1995, Bertero and Bertero 2002, Piesteley 2000) is the human life protection. This performance objective (Bertero and Bertero 2002) requires that the structure, when subjected to a strong seismic input, even if heavily damaged, does not collapse. This approach leads to the base concept of structural ductility. However, after a strong earthquake, the structure can lose his entire functionality and its retrofitting may be very difficult or even not possible to apply. The present design approach results to be adequate (Eurocode 8, NTC 2008) in the aim to limit the cost.

Despite these concepts are basic in the seismic design, the research of better performance objectives (Bertero and Bertero 2002) and innovative design approaches has been encouraged within socio-economic reality of developed countries. Important structures like hospitals, police stations, fire department barracks, communication centres, airports, nuclear power plants and all buildings strategic for public safety must be designed to reach higher protection levels under strong earthquakes: they should undergo limited or even no structural damage. For this objective the traditional design approach (based on the adoption of a force reduction factor, Newmark and Hall 1982 and Miranda 1977) may be often economically prohibitive. This problem, which is relevant for new buildings, becomes particularly evident for the retrofit of existing buildings. In this case high cost and large impact on the architectural aspects can be produced even by little improvements in the structural behaviour.

The solution to the problem of obtaining higher performance objectives (strategic buildings) and improved safety levels (historical buildings) may be found in the use of innovative technologies (Christopoulos and Filiatrault 2006, Soong and Dargush 1997, Constantinuou et al. 1998), such as viscous dampers (Silvestri et al. 2010). From the design point of view (NTC 2008, Eurocode 8, FEMA 450), a structure coupled with a damping system is usually designed to remain in the elastic field. Although this approach leads to higher level of structural safety when compared to those typically required by the traditional design approach, on the other hand it had strongly limited the use of this design approach to not-common building typologies (e.g. strategic buildings).

It clearly appears that a design approach able to couple the advantages of the traditional approach and the innovative approach may be very effective, especially from the point of view of the costs reduction and may lead to a wide diffusion of the use of dissipative devices in building structures.

In order to allow the applicability of this coupled-design approach within the actually widespread seismic design approach (i.e. response spectrum analysis with force reduction factor) some questions need to be investigated. From one hand, it is really acquired that the ductility capacity of a structure (Paulay and Priestley 1992) is not affected by the presence of added dampers. On the other hand it is clear that the insertion of dampers into a structure reduces the overall ductility demand. Thus, if the designer decides to account for both the ductility capacity of the structural members and the dissipative properties of the added viscous dampers, he cannot use the actual simple tools suggested by code; in other words, he cannot reduces the elastic design spectrum simply adding the two effect of the ductility (trough the behaviour factor q) and the higher damping ratio (trough the reduction coefficient η). As a consequence, only one analysis method is actually available: a fully non-linear time-history analysis. This method, even if technically feasible, involves various difficulties in the practical application: (i) the commercial software are not always able to simply develop non-linear time-history analysis; (ii) the definition of the non-linear cyclic response of structural members is often far from the knowledge of practical engineers.

With the purpose of extending the use of dissipative devices to a wider range of building structures, the present paper proposes a simple formulation for the force reduction factor R in the case of buildings equipped with added dampers. The use of such reduction factor in conjunction with viscous dampers allows to satisfy a criterion of equal safety between the bare structures and the structure equipped with added viscous dampers.

2. ANALYTICAL PROBLEM FORMULATION

2.1. Seismic demand of elastic and inelastic damped SDOF systems

Two equal inelastic SDOF system (i.e. same mass m and same initial stiffness k) equipped with two viscous dampers leading to a damping ratio equal to ξ=5% and to an higher damping ratio (generally indicated as ξ) are considered. Clearly, if the systems are subjected to the same base input (i.e. ground motion) they will exhibit a different dynamic response (i.e. different seismic demand). Regarding to Fig. 2.1 the following notation will be adopted in this paper:

- $F_{e\text{-}5}$: strength demand of the elastic SDOF system ($SDOF_{E\text{-}5}$);
- $\delta_{e\text{-}5}$: displacement demand of the elastic SDOF system ($SDOF_{E\text{-}5}$).
- $F_{e\text{-}\xi}$: strength demand of the elastic SDOF system ($SDOF_{E\text{-}\xi}$) with the same mass m and stiffness of system $SDOF_{E\text{-}5}$, but different damping coefficient c;
- $\delta_{e\text{-}\xi}$: displacement demand of the elastic SDOF system ($SDOF_{E\text{-}\xi}$);
- $F_{y\text{-}5}$: yield strength of the inelastic SDOF system ($SDOF_{EP\text{-}5}$) equivalent to the $SDOF_{E\text{-}5}$ system (same mass m, same stiffness k, same damping coefficient c); for a force reduction factor R_5, $F_{y\text{-}5}$ is equal to $F_{y\text{-}5} = F_{e\text{-}5} / R_5$;
- $\delta_{y\text{-}5}$: yielding displacement of the $SDOF_{EP\text{-}5}$ system;
- $\delta_{u\text{-}5}$: displacement seismic demand for the $SDOF_{EP\text{-}5}$ system;
- $F_{y\text{-}\xi}$: yield strength of the inelastic SDOF system ($SDOF_{EP\text{-}\xi}$) equivalent to the $SDOF_{E\text{-}\xi}$ system (same mass m, same stiffness k, same damping coefficient c); for a force reduction factor R_ξ, $F_{y\text{-}\xi}$ is equal to $F_{y\text{-}\xi} = F_{e\text{-}\xi} / R_\xi$;
- $\delta_{y\text{-}\xi}$: yielding displacement of the $SDOF_{EP\text{-}\xi}$ system;
- $\delta_{u\text{-}\xi}$: seismic displacement demand of the $SDOF_{EP\text{-}\xi}$ system;

The ductility demand for the two inelastic systems ($SDOF_{EP\text{-}5}$ and $SDOF_{EP\text{-}\xi}$) can be expressed by the following relations:

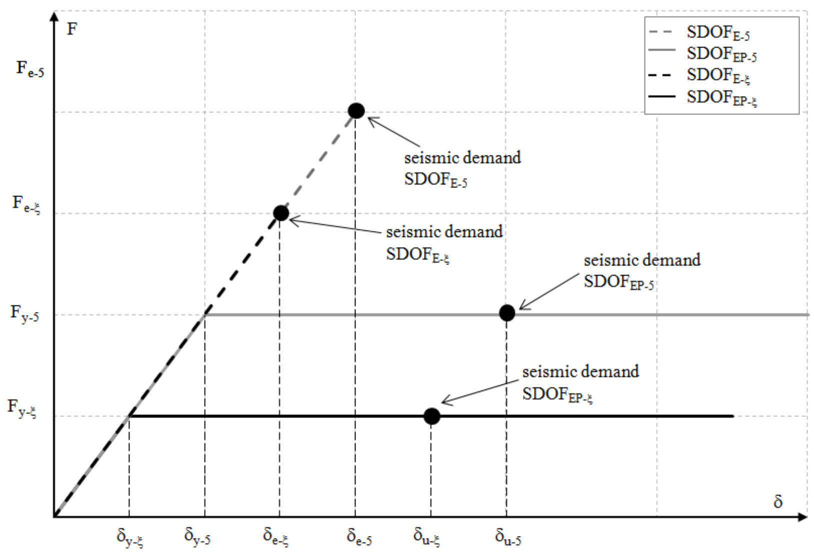

Figure 2.1. Seismic demand for elastic and inelastic SDOF systems (with and without added damping).

$$\mu_{\delta-5} = \frac{\delta_{u-5}}{\delta_{y-5}} \qquad \mu_{\delta-\xi} = \frac{\delta_{u-\xi}}{\delta_{y-\xi}} \tag{2.1 a, b}$$

2.2. Objective

Actually, the response spectrum analysis with force reduction factor R is the widespread seismic design procedure used by the practical engineers. Numerical values of the force reduction factor R are typically given by codes only for the case of structures without added dissipative devices (thus considering only the inherent damping, conventionally equal to 5%). Therefore, in the case of a structure equipped with added viscous dampers is not possible to perform a response spectrum analysis reducing the elastic spectrum according to: (i) the effect of the higher viscous dampers (commonly known in the scientific literature as reduction coefficient η) and (ii) the effect of the ductility of the structural elements (force reduction factor R or behaviour factor q provided by codes).

The objective of the present paper is to study the influence of higher damping ratio on the force reduction factor R. In more details the main purpose is to obtain a relationship between the force reduction factor given by code (referred as R_5) and the force reduction factor for structures equipped with added viscous dampers (referred as R_ξ) which satisfy the following criterion of equal structural safety: the ductility demand of the system with higher damping ratio must be less (or at least equal) than the ductility demand of the system with only inherent damping. In order to accomplish the proposed objective an extensive parametric study has been developed (i.e. $\mu_{\delta-\xi} \leq \mu_{\delta-5}$).

2.3. Methodology

A number of 126000 non-linear Time-History analyses have been performed on inelastic damped SDOF systems. The parametric study has been carried out varying: natural elastic period of the system T, target ductility $\bar{\mu}$, damping ratio ξ and seismic inputs. In particular 100 ground motions have been chosen as base input (Hatzigeorgiou and Beskos 2009). Table 2.1 gives the range of the parameters considered in the analysis. The equations of motion have been integrated using the “Alpha” Method, (Hilber et al. 1977; the adopted “alpha” coefficient was 0.05).

Table 2.1. Range of the parameters adopted for the numerical analysis.

Parameter	Min	Max	Step
T	0.1	3	0.1
$\bar{\mu}$	1.0	6.0	1.0
ξ	0.05	0.35	0.05

Each numerical analysis consisted in an iterative procedure, that is composed by the following steps (referring to a certain SDOF system, i.e. a certain T, and ξ):

STEP 1: linear time-history analysis of two elastic SDOF systems (one with ξ= 0.05 and one with the generic ξ) in order to obtain:

$F_{e\text{-}5}$: strength demand of the $SDOF_{E\text{-}5}$ system;

$F_{e\text{-}\xi}$: strength demand of the $SDOF_{E\text{-}\xi}$ system;

STEP 2: calculation, for a given value of the force reduction factor R_5, of the yielding point ($F_{y\text{-}5}$, $\delta_{y\text{-}5}$) of the inelastic SDOF system ($SDOF_{EP\text{-}5}$) equivalent to the linear system $SDOF_{E\text{-}5}$.

STEP 3: development of non-linear time-history analysis of the $SDOF_{EP\text{-}5}$ system aimed at obtaining the ductility demand $\mu_5 = \delta_{u\text{-}5} / \delta_{y\text{-}5}$;

STEP 4: development of an iterative procedure for the evaluation of the force reduction factor R_ξ which provides the system $SDOF_{EP\text{-}\xi}$ the same ductility demand of the system $SDOF_{EP\text{-}5}$. The iterative procedure is composed of the following sub-steps (for each required iteration):

1. calculation, adopting a first attempt value of R_ξ (referred as R_ξ^1, where the apex 1 indicates the 1st iteration) equal to R_5, of the yielding point ($F_{y\text{-}\xi}^1$, $\delta_{y\text{-}\xi}^1$) of the inelastic SDOF system ($SDOF_{EP\text{-}\xi}$) equivalent to the elastic $SDOF_{E\text{-}\xi}$;
2. development of non linear time-history analysis of the $SDOF_{EP\text{-}30}$ system aimed at obtaining the ductility demand $\mu_\xi = \delta_{u\text{-}\xi} / \delta_{y\text{-}5}$ (μ_ξ^1 at the 1st iteration);
3. evaluation of the difference $\Delta\mu = \mu_\xi - \mu_5$ ($\Delta\mu^1$ at the first iteration);

The sub-steps 1-2-3 are repeated (varying the value of R_ξ^i, where i indicates the i-th iteration) until the absolute value of the difference $\Delta\mu^i$ satisfies the inequality $\Delta\mu^i \leq \bar{\Delta}$ where $\bar{\Delta}$ indicates the maximum allowable error.

Sub-steps 1 to 4 are repeated for each ground motion and varying all the parameters as indicated in Table 2.1.

3. RESULTS

This section presents the main results obtained from the parametric analysis. In detail:

- subsection 3.1 is focused on the relationship between R_ξ and R_5 for fixed values of $\bar{\mu}$, thus accounting for the influence of the period T;
- subsection 3.2 is focused on the relationship between R_ξ and R_5 for fixed values of T, thus accounting for the influence of the ductility $\bar{\mu}$;
- subsection 3.3 is focused on the full relationship between R_ξ and R_5 (also practical observations are provided).

3.1. Force reduction factors R_ξ and R_5 for fixed values of T

The present subsection discusses the influence of T (for fixed values of $\bar{\mu}$) on R_5 and R_ξ as obtained from the numerical analysis.

As an example, Figs. 3.1 shows the mean value of R_5 and R_{30} versus T for fixed values of $\bar{\mu}$ =, 2, 3, 4, 5 and the corresponding standard deviation as obtained from the parametric analysis (same results appeared for the other ξ, see section 3.2). Inspection of the graphs allow the following observations:

- for T < 0.25 sec mean values of R_5 and R_ξ are less than $\bar{\mu}$, for all values of $\bar{\mu}$;
- for T < 0.25 sec mean values of R_5 and R_ξ are greater than $\bar{\mu}$, for all values of $\bar{\mu}$;
- for T < 2.0 sec mean values of R_5 are greater than mean values of R_ξ, for all values of $\bar{\mu}$;
- for 2.0 < T > 3.0 sec mean values of R_5 are less than mean values of R_ξ, for all values of $\bar{\mu}$;
- standard deviation of R_5 is high for T < 2 sec, while decrease for T > 2.0;
- standard deviation of R_5 is significantly less than standard deviation of R_{30} and approximately constant for all period T.

In the light of the proposed objective it is useful to introduce the ratio between R_ξ and R_5:

$$\alpha_\xi = \frac{R_\xi}{R_5} \tag{3.1}$$

which allows to express R_ξ as a function of R_5:

$$R_\xi = \alpha_\xi \cdot R_5 \tag{3.2}$$

As an example, Figs. 3.2 display the mean values of α_{20} and α_{30} versus T for fixed values of $\bar{\mu}$ =, 2, 3, 4, 5 and the corresponding standard deviation as obtained from the parametric analysis. Inspection of the graph allows the following observations:

- mean values of α_{20} and α_{30} are very close to each other;
- mean values of α_{20} and α_{30} are included between 0.85 and 1.15;
- for T < 1.5 sec mean values of α_{20} and α_{30} are less than 1;
- for T < 1.5 sec mean values of α_{20} and α_{30} are greater than 1;
- standard deviations of α_{20} and α_{30} are similar and almost constant with T for all values of $\bar{\mu}$

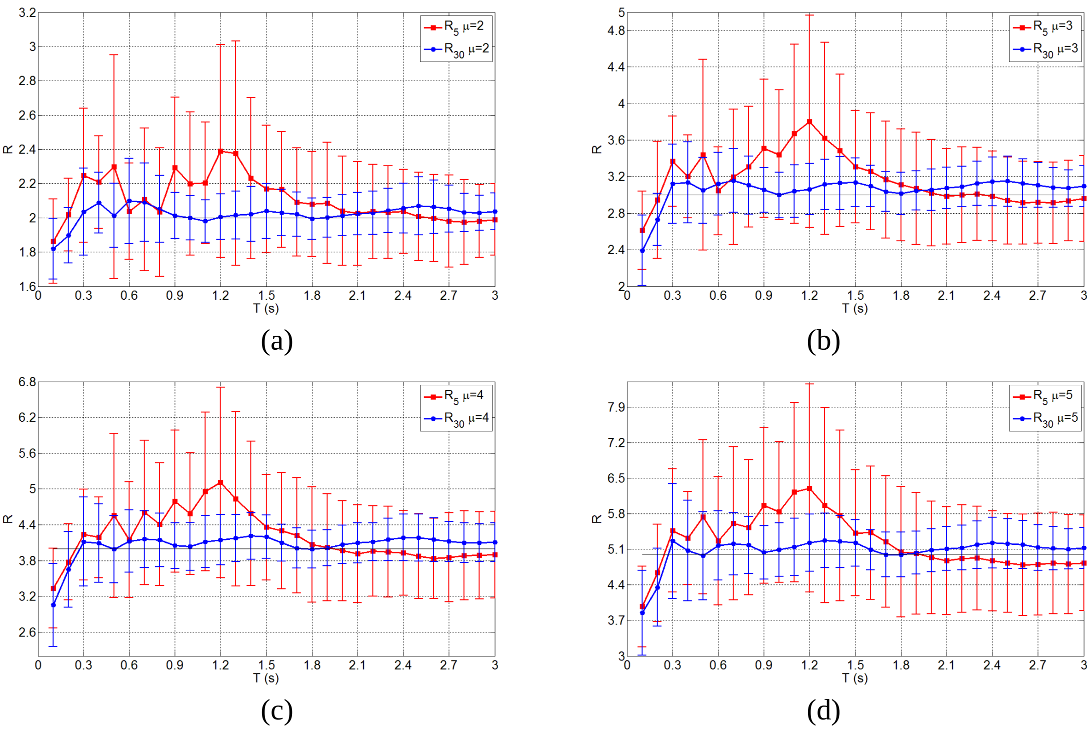

Figure 3.1. R_5 and R_{30} versus T for fixed values of target ductility: (a) $\bar{\mu}$ =2; (b) $\bar{\mu}$ =3; (c) $\bar{\mu}$ =4; (d) $\bar{\mu}$ =5;

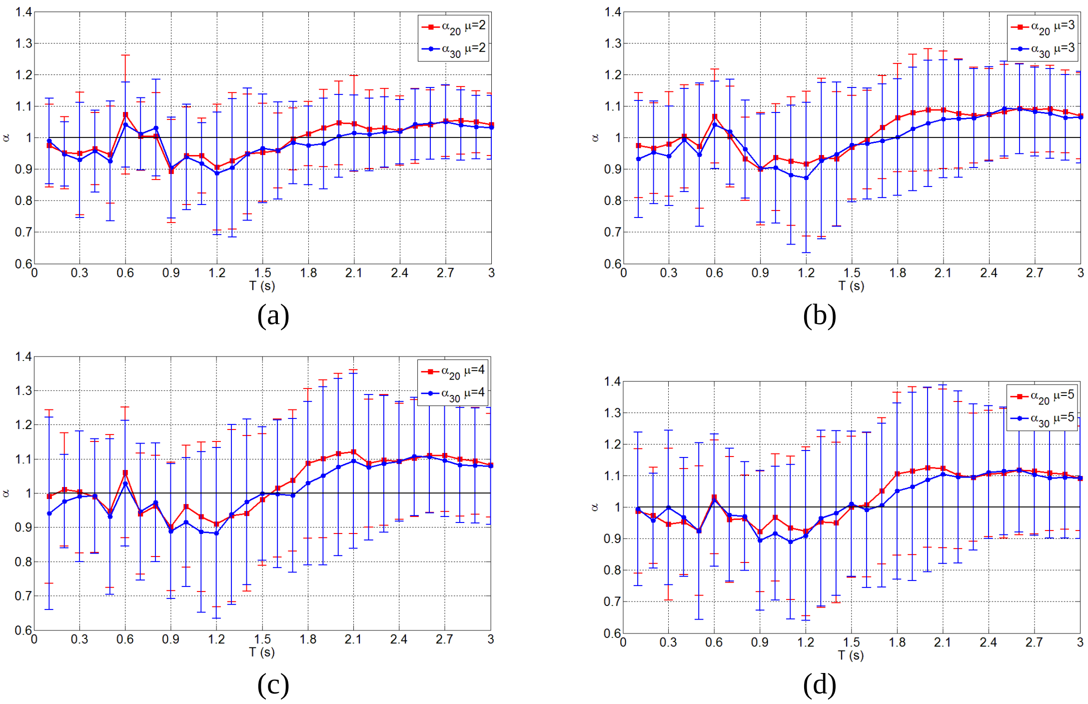

Figure 3.2. α_{20} and α_{30} versus T for fixed values of target ductility: (a) $\bar{\mu}$ =2; (b) $\bar{\mu}$ =3; (c) $\bar{\mu}$ =4; (d) $\bar{\mu}$ =5;

3.2. Force reduction factors R_ξ and R_5 as a function of $\bar{\mu}$

The present subsection presents the results obtained from the analysis in terms of R_5 and R_ξ (for all values of ξ) as a function of $\bar{\mu}$ for selected values of T.

Figs. 3.3 display the mean value of R_5 and R_ξ versus $\bar{\mu}$ for T = 0.2 sec; 1.0 sec; 1.5 sec and 2.5 sec and the corresponding standard deviation as obtained from the parametric analysis. Inspection of the graphs allows the following observations:

- for all values of period T (0.2 < T < 3.0 sec) R_ξ is close to $\bar{\mu}$. This is an expected result confirming the so-called "equal displacement" rule (Newmark and Hall 1973).
- for very short periods (T < 0.2 sec) mean values of R_5 and R_{30} are very close to each other and slightly less than $\bar{\mu}$;
- for 0.5 < T < 1.4 sec mean values of R_5 and R_{30} are slightly higher than $\bar{\mu}$. Moreover mean values of R_5 are higher than mean values of R_{30};
- for 1.5 < T < 3.0 sec mean values of R_5 and R_{30} are slightly higher than $\bar{\mu}$. Moreover mean values of R_{30} are higher than mean values of R_5;

Figs. 3.4 display the mean values of α_ξ (for all ξ) versus $\bar{\mu}$ for T = 0.2 sec; 1.0 sec; 1.5 sec and 2.5 sec and the corresponding standard deviation as obtained from the parametric analysis. Inspection of the graphs allows the following observations:

- for $T \cong 0.5$ mean values of α_ξ are between 0.95 and 1.0, decrease as ξ increases while are almost constant with $\bar{\mu}$;
- for 0.5 < T < 1.4 sec mean values of α_ξ are between 0.85 and 1.0, decrease as ξ increases while are almost constant with $\bar{\mu}$;
- for 1.5 < T < 3.0 sec mean values of α_ξ are between 0.95 and 1.15, increase as ξ increases and also slightly increase as $\bar{\mu}$ increase;

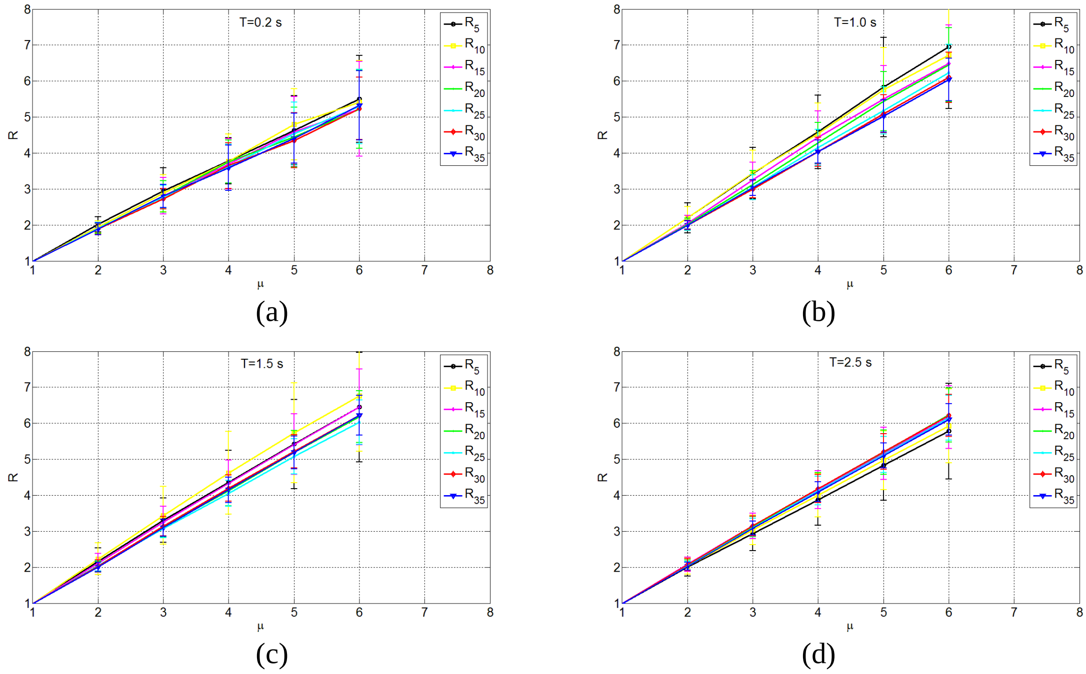

Figure 3.3. R_5 and R_ξ versus $\bar{\mu}$ for fixed values of T: (a) T =0.2 sec; (b) T=1.0 sec; (c) T=1.5 sec; (d) T=2.5 sec;

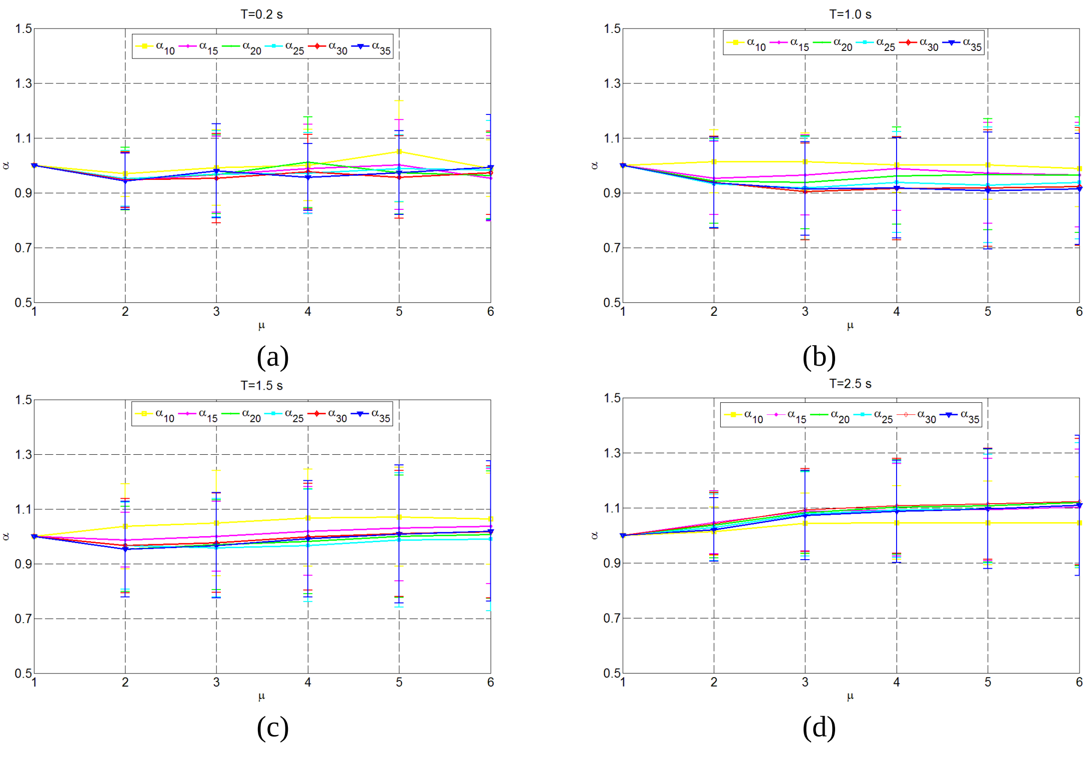

Figure 3.4. α_{20} and α_ξ versus $\bar{\mu}$ for fixed values of T: (a) T =0.2 sec; (b) T=1.0 sec; (c) T=1.5 sec; (d) T=2.5 sec;

3.3. Force reduction factors R_ξ and R_5 as a function of T and $\bar{\mu}$

The present subsection presents a summary of the results in terms of R_5 and R_ξ as functions of T and $\bar{\mu}$.

Based on all the results commented in the previous subsections clearly appears that α_ξ is slightly influenced by $\bar{\mu}$ and thus, from a practical point of view, the assumption of a constant α_ξ for all $\bar{\mu}$ appears reasonable. Moreover, also the influence of the period can be simplified introducing two constant range of α_ξ for periods T higher or less than 1.5 sec. Tables 3.1 and 3.2 provide the mean values of α_ξ, the coefficient of variation of R_ξ, and the ratio between COV R_ξ and COV R_5 over all values of $\bar{\mu}$ and for the two period ranges (i.e. $T < 1.5$ sec and $T > 1.5$ sec) above identified.

As expected for period $T < 1.5$ sec mean values of α_ξ are slightly less than 1.0 (in mean 0.96) while for $T > 1.5$ sec mean value of α_ξ are slightly larger than 1.0 (in mean 1.06). COV R_ξ decreases as ξ increases. The last columns of Tables 3.1 and 3.2 (i.e. COV R_ξ / COV R_5) shows that for high damping ratio (i.e. ξ=0.30 - 0.35) COV R_ξ reduces to approximately 0.5 COV R_5.

Table 3.1. Mean values of α_ξ and COV R_ξ and COV R_ξ / COV R_5 over all values $\bar{\mu}$ and periods $T < 1.5$ sec.

ξ	α_ξ	COV R_ξ	COV R_ξ /COV R_5
10	0.9897	0.189	0.82
15	0.9702	0.161	0.70
20	0.9607	0.146	0.63
25	0.9556	0.135	0.58
30	0.9528	0.122	0.52
35	0.9499	0.112	0.48

Table 3.2. Mean values of α_ξ and COV R_ξ and COV R_ξ / COV R_5 over all values $\bar{\mu}$ and periods $T > 1.5$ sec.

ξ	α_ξ	COV R_ξ	COV R_ξ /COV R_5
10	1.0524	0.164	0.85
15	1.0758	0.134	0.69
20	1.0790	0.111	0.58
25	1.0675	0.092	0.48
30	1.0605	0.079	0.41
35	1.0582	0.069	0.38

4. THE GLOBAL REDUCTION FACTOR PROPOSED FOR BUILDING STRUCTURES EQUIPPED WITH ADDED VISCOUS DAMPERS.

The actual Italian building code (NTC 2008) allows to obtain the design spectrum dividing the elastic design spectrum by the reduction coefficient η equal to:

- for structures equipped with additional dampers (i.e. providing a damping ratio greater than 0.05, Boomer et al. 2000):

$$\eta = \frac{1}{\sqrt{1+\xi[\%]}} = \eta_{visc} \tag{4.1}$$

- for ductile structures without added damping (i.e. ξ=5%):

$$\eta = \frac{1}{q} = \eta_{ist} \tag{4.2}$$

All the results obtained from the parametric analysis and presented in the previous section, allows to introduce a global reduction factor, η_{tot}, for building structures equipped with added dampers able to couple both the effects do to the ductility of the structural elements and the dissipation of the viscous dampers. In detail η_{tot} can be expressed as a function of the behaviour factor q (typically provided by codes), the reduction coefficient η_{visc} (typically provided by code) and the α coefficient (introduced in the present paper) with the following relationship:

$$\eta_{tot}(\xi) = \eta_{visc}(\xi) \cdot \eta_{ist}(q) = \frac{\eta(\xi)}{q \cdot \alpha(\xi)} \tag{4.3}$$

Using the above defined global reduction coefficient η_{tot} for conventional response spectrum analysis the ordinate of the design spectrum S_d can be evaluated with the following relationship:

$$S_{d,\xi} = S_{e,5} \cdot \eta_{tot}(\xi) = S_{e,5} \cdot \frac{\eta(\xi)}{\alpha(\xi)q} \tag{4.4}$$

Fig. 4.1 provides a qualitative comparison between the elastic and inelastic design spectrum as per Italian building code (NTC 2008) and according to Eqn. 3.6 (for a behaviour factor q equal to 4 and a damping ratio equal to 0.30).

For this specific case (q = 4 and a damping ratio ξ = 0.30) the ordinate of the inelastic design spectrum with 0.30 damping ratio (Eqn. 4.4) are reduced approximately to 0.5 of the ordinate of the inelastic design spectrum with 5% damping ratio.

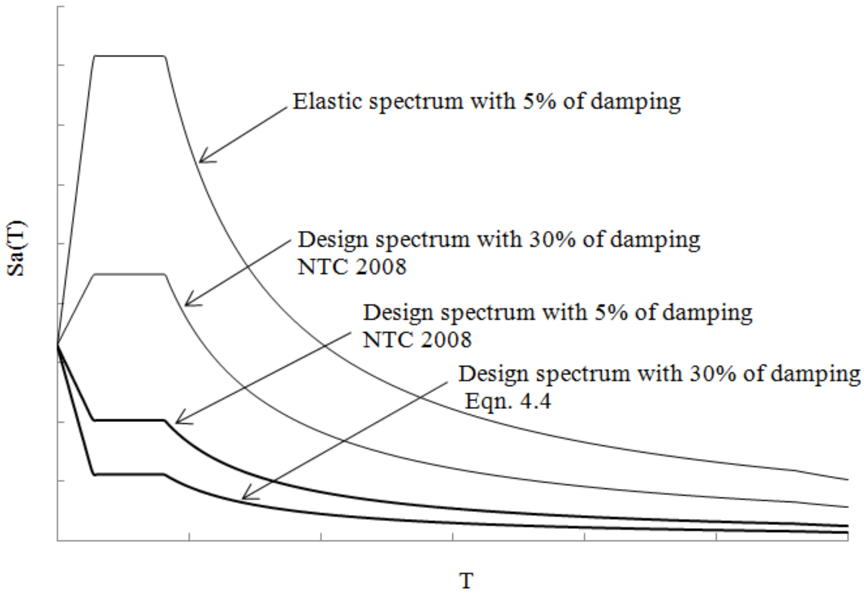

Figure 4.1. Comparison between elastic and inelastic spectrum (as per NTC 2008 and Eqn. 4.4)

CONCLUSIONS

The present paper aims at investigating how the force reduction factor is affected by high values of damping ratio with the purpose of providing a simple design tool, within the framework of the response spectrum seismic analysis, for the seismic design of building structures equipped with added viscous dampers. A simple formulation for the force reduction factors to be adopted for high damped system, able to provide the structure equipped with dissipative devices the same level of structural safety of the structural system without added dissipative devices, is proposed as a function of the force reduction factor actually given by codes (i.e. the force reduction factor calibrated for structures provided only with the inherent damping, conventionally equal to 5%) and the reduction coefficient η.

The adoption of the proposed force reduction factor *R* in conjunction with the reduction coefficient, related the presence of added dampers, leads to a significant reduction (with respect to those resulted following the actual code prescriptions, NTC 2008) of the design forces on the structural elements.

REFERENCES

Bertero, R.D. and Bertero V.V. (2002). Performance-based seismic engineering: the need for a reliable conceptual comprehensive approach. *Earthquake Engineering and Structural Dynamics* **31:3**, 627-652.

Bommer, J.J., Elnashai A.S., Weir, A. G. (2000). *Compatible acceleration and displacement spectra for seismic design codes*, Proc. of the 12th World Conference on Earthquake Engineering, Auckland, New Zealand.

Constantinuou, M.C., Soong T.T. and Dargush G.F.(1998). *Passive Energy Dissipation Systems for Structural Design and Retrofit*, Monograph No. 1, Multidisciplinary Center for Earthquake Engineering Research, Buffalo, New York.

Christopoulos, C. and Filiatrault, A. (2006). *Principles of Passive Supplemental Damping and Seismic Isolation*, IUSS Press, Pavia, Italy.

Eurocode 8. (2003). *Design of structures for earthquake resistance*, prEN 1998-1, draft January.

Elnashai, A.S. and Mwafy, A.M. (2002). *Overstrength and force reduction factors of multistorey reinforced-concrete buildings*, The Structural Design of Tall Buildings, **11:5,** 329 - 351.

Hatzigeorgiou, G.D., Beskos, D.E. (2009). *Inelastic displacement ratios for SDOF structures subjected to repeated earthquakes*, Engineering Structures **31:** 2744-2755.

Hilber, H.M., Hughes, T.J.R. and Taylor, R.L. (1977). *Improved numerical dissipation for time integration algorithms in structural dynamics*. Earthquake Engineering and Structural Dynamics, **5**, 283-292.

Miranda, E. (1977). *Strength Reduction Factors in Performance-Based Design*, National Center for Disaster Prevention, EERC-CUREe Symposium in Honor of Vitelmo V. Bertero, January 31 - February 1, 1997, Berkeley, California.

NEHRP (2003). *Recommended provisions for seismic regulations for new buildings and other structures* (FEMA 450), 2003 edition. Building Seismic Safety Council of the National Institute of Building Sciences, Washington, D.C.

Newmark, N. M. and Hall, W. J. (1973).*Seismic design criteria for nuclear reactor facilities* Report No. 46, Building Practices for Disaster Mitigation, National Bureau of Standards, U.S. Department of Commerce, 209-236.

Newmark, N.M., and Hall, W.J., (1982). *Earthquake Spectra and Design*, Earthquake Engineering Research Institute, Berkeley, California, 35-36.

NTC 2008 (2008). *Norme Tecniche per le Costruzion*, Italian building code, adopted with D.M. 14/01/2008, published on S.O. n. 30 G.U. n. 29 04/02/2008.

Paulay, T. and Priestley, M.J.N. (1992). *Seismic Design of Reinforced Concrete and Masonry Buildings*, John Wiley & Sons, New York.

Priestley, M.J.N. (2000). *Performance Based Seismic Design*, Proceeding of 12th World Conference on Earthquake Engineering, Auckland, New Zealand, Upper Hutt, New Zealand, 2000, Paper No. 2831.

SEAOC Vision 2000 Committee (1995). *Performance-based seismic engineering for buildings*. Report prepared by Structural Engineers Association of California, Sacramento, CA.

Silvestri, S., Gasparini, G. and Trombetti, T. (2010). *A Five-Step Procedure for the Dimensioning of Viscous Dampers to Be Inserted in Building Structures*, Journal of Earthquake Engineering **14: 3**, 417 - 447.

Soong ,T.T. and Dargush, G.F. (1997). *Passive Energy Dissipation Systems in Structural Engineering*, John Wiley & Sons, Baffins Lane, Chichester, West Sussex PO19 1UD, England.