Period Shifting Effect on the Corner Displacement Magnification of One-Storey Asymmetric Systems

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SUMMARY:
This paper gives a new insight into the dynamic behaviour of one-storey eccentric systems, with particular attention devoted to provide a comprehensive physically-based formulation of the maximum corner displacement amplification, which involves three contributions (translational response, torsional response and their combination). It is shown that the largest amplifications, which mainly occur for the class of torsionally-flexible systems, are due to the translational contribution through to the shift in the fundamental period of the eccentric system with respect to the one of the equivalent not-eccentric system. A simplified method for the estimation of the maximum corner displacement based on the physical properties of the system is finally obtained.

Keywords: in-plane irregularity, torsionally-stiff , torsionally-flexible, period shifting, seismic response.

1. INTRODUCTION

Since the late 1970s, it is known that structures characterized by non coincident center of mass and center of stiffness, commonly defined as eccentric (or asymmetric) systems, when subjected to dynamic excitation develop a coupled lateral-torsional response that may considerably increase their local peak response, such as the corner displacements (Kan and Chopra 1977 (a and b) Rutenberg 1992, Hejal and Chopra 1987).

In order to effectively apply the performance-based design approach to seismic design, there is a growing need for code-oriented methodologies aimed at predicting deformation parameters. Thus, the estimation of the displacement demand at different locations, especially for eccentric structures, appears a fundamental issue. Furthermore, the ability to predict the torsional response of eccentric systems can be also useful to improve the capability of one of the most actually used seismic design approaches (i.e. push-over analysis, Perus and Fajfar 2005).

Since the early 1990s Nagarajaiah et al. 1993, investigating the torsional coupling behavior of base-isolated structures, observed that, for the specific class of torsionally-stiff asymmetric structures, the maximum center mass displacement can be well approximated by the maximum displacement of the equivalent not-eccentric system.

In previous research works (Trombetti 1994, Trombetti and Conte 2005, Trombetti et al. 2008), the authors identified a structural parameter, called “alpha”, capable of measuring the attitude of one-storey asymmetric systems to develop rotational responses and proposed a simplified procedure, called “Alpha-method”, for the estimation of the maximum torsional response. In its original formulation, the “Alpha-method” was based on the aforementioned assumption of equal maximum displacement response between the eccentric system and the equivalent not-eccentric system.

The object of the present paper is to provide a more comprehensive investigation on the dynamic properties of one-storey eccentric systems, with specific focus on the class of the so-called torsionally-flexible systems, which showed a greater attitude in developing consistent corner displacement amplifications (Trombetti et al. 2008).

2. PROBLEM FORMULATION

Let us consider the one-storey eccentric structure (i.e. a system characterized by non-coincident center of mass, CM, and center of stiffness, CK) displayed in Fig. 2.1 (the origin of the reference system is located at CM). It is assumed that the diaphragm is infinitely rigid in its own plane, and that the lateral-resisting elements (e.g. columns, shear walls, …) are massless and axially inextensible. The self torsional stiffness (k_θ) of each lateral-resisting element is also neglected. Under this assumption, the following three degrees of freedom are assumed: (i) longitudinal center mass displacement, $u_{y,CM}$, (ii) transversal center mass displacement, $u_{x,CM}$, (iii) center mass rotation, $u_{\theta,CM}$, which coincides with the floor rotation, u_θ. The system is subjected to a one-way dynamic excitation (e.g. free vibrations or seismic input) along the longitudinal direction (namely, the y-direction).

From simple trigonometric relationships, with reference to the plan view of the system given in Fig. 2.1, the longitudinal corner side displacement, i.e. the displacement of the flexible side of the system (e.g. point B, the farther from CK), $u_{y,B}$, at any generic instant t, is given by:

$$u_{y,B}(t) = u_{y,C_M}(t) - u_\theta(t) \cdot \frac{L}{2} \tag{2.1}$$

Estimating the corner displacement according to Eqn. 2.1 requires the development of time-history analyses. Nevertheless, the practical engineer is interested in the absolute maximum value, $u_{y,B,max}$, of the corner displacement response history. Thus, the main purpose of this research work is to provide a simple formulation for the evaluation of $u_{y,B,max}$, starting from:

$$u_{y,B,\max} = u_{y,C_M,\max} \oplus u_{\theta,\max} \cdot \frac{L}{2} \tag{2.2}$$

which highlights that the maximum corner displacement depends on the following three contributions:

- translational contribution, as given by the maximum absolute displacement response $u_{y,CM,max}$ of the center of mass governed by period shifting effect (see section 4);
- torsional contribution, as given by the product of the maximum absolute rotational response $u_{\theta,max}$ and the lever arm L/2 (see section 5);
- combination of the translational and torsional contributions of above, as indicated by symbol $\oplus$ (see section 6).

Manipulation of Eqn. 2.2 leads to:

$$u_{y,B,\max} = u_{y,C_M,\max,N-E} \cdot \frac{u_{y,C_M,\max}}{u_{y,C_M,\max,N-E}} \left(1 \oplus \frac{u_{\theta,\max}}{u_{y,C_M,\max}} \cdot \frac{L}{2} \right) \tag{2.3}$$

Introducing the following parameters:

- $\delta = \dfrac{u_{y,CM,\max}}{u_{y,CM,\max,N-E}}$, which indicates the center mass displacement amplification with respect to that of the equivalent not-eccentric system (N-E), $u_{y,CM,max,N-E}$;
  - $A \cdot \alpha_u = \rho_m \cdot \dfrac{u_{\theta,\max}}{u_{y,CM,\max}}$ which indicates a rotational parameter (ρ_m is the mass radius of

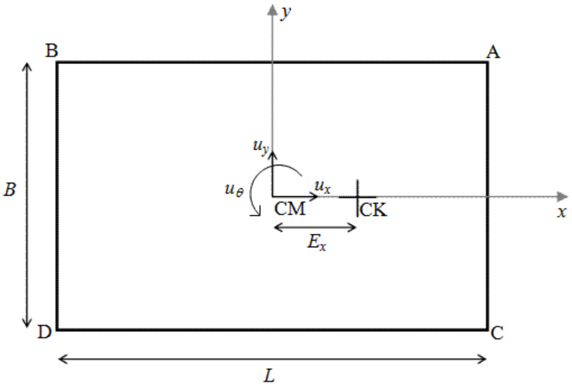


Figure 2.1. Plan view of the in-plane eccentric system with the indication of the degrees of freedom

gyration of the system);
- B, which is a parameter of simultaneity accounting for the time combination of the translational and torsional contributions;
- $\phi = \dfrac{L}{2\rho_m} = \sqrt{\dfrac{3(L/B)^2}{1+(L/B)^2}}$ which indicates a shape factor of the system.

Eqn. 2.3 reduces to:

$$u_{y,B,\max} = u_{y,C_M,\max,N-E} \cdot \delta \cdot (1 + A \cdot B \cdot \alpha_u \cdot \phi) \tag{2.4}$$

The objective of the work is to quantifies the value of $\delta \cdot (1 + A \cdot B \cdot \alpha_u \cdot \phi)$ that represents the corner displacement magnification with respect to the equivalent N-E system.

3. THE DYNAMIC PROPERTIES OF ONE-STOREY ECCENTRIC SYSTEMS

3.1. The equation of motion

Under the following additional (with respect to those of section 2) assumptions:
- the total lateral stiffness k of the system is the same along the x- and the y-direction (i.e. k=k_x=k_y, where k_x and k_y are the translational stiffness along the x- and the y-direction, respectively);
- the rotational response u_θ developed under dynamic excitation is small enough to allow the approximation ;
- the longitudinal eccentricity is equal to zero (i.e. E_y = 0). This case maximizes the rotational response of the system in free vibrations (Trombetti and Conte 2005);

the dynamic coupled lateral-torsional response of the system under consideration (Fig. 2.1) is governed by the following set of coupled differential equations of motion (Trombetti and Conte 2005), written in a reference system with origin located at CM:

$$m\begin{bmatrix} \ddot{u}_x(t) \\ \ddot{u}_y(t) \\ \rho_m \ddot{u}_\theta(t) \end{bmatrix} + [C]\begin{bmatrix} \dot{u}_x(t) \\ \dot{u}_y(t) \\ \rho_m \dot{u}_\theta(t) \end{bmatrix} + m\omega_L^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & e_x\sqrt{12} \\ 0 & e_x\sqrt{12} & \Omega_\theta^2 + 12 e_x^2 \end{bmatrix}\begin{bmatrix} u_x(t) \\ u_y(t) \\ \rho_m u_\theta(t) \end{bmatrix} = \begin{bmatrix} p_x(t) \\ p_y(t) \\ p_\theta(t)/\rho_m \end{bmatrix} \tag{3.1}$$

where:

m is the mass of the system; $e_x=E_x/D_e$ is the relative eccentricity (hereafter it will be simply indicated as e); D_e is the equivalent diagonal equal to $12\rho_m$; $\Omega_\theta = \omega_\theta / \omega_L$ is a dimensionless parameter that measures the torsional flexibility of the system (ω_L and ω_θ are the uncoupled translational natural frequency of vibration and the uncoupled torsional natural frequency of vibration, defined in a reference system with origin located at CK, respectively); $[C]$ is the damping matrix (classical damping is assumed).

The parameter Ω_θ represents a physical property of the eccentric system, leading to the two following classes: (i) torsionally-stiff systems: $\Omega_\theta \geq 1.0$; (ii) torsionally-flexible systems: $\Omega_\theta < 1.0$

3.2. The eigenproblem

The solution of the eigenvalues problem governing the undamped free vibrations of the system gives the following closed-form expressions of natural frequencies ω_1, ω_2, ω_3, normalized with respect to the uncoupled longitudinal frequency ω_L and squared (Trombetti and Conte 2005):

$$\begin{gathered}\Omega_1 = \left(\omega_1 / \omega_L\right)^2 = 1/2\left(1+\Omega_\theta^{\,2} + 12e^2 - \sqrt{\left(\Omega_\theta^{\,2} + 12e^2 - 1\right)^2 + 48e^2}\right) \\ \Omega_2 = \left(\omega_2 / \omega_L\right)^2 = 1 \\ \Omega_3 = \left(\omega_3 / \omega_L\right)^2 = 1/2\left(1+\Omega_\theta^{\,2} + 12e^2 + \sqrt{\left(\Omega_\theta^{\,2} + 12e^2 - 1\right)^2 + 48e^2}\right)\end{gathered} \tag{3.2}$$

Fig. 3.1 a plots the normalized natural frequencies versus e and Ω_θ showing that: (i) $\omega_2=\omega_L$; (ii) ω_1 is generally close to ω_L; (iii) ω_3 can be quite larger than ω_L.

The solution of the eigenproblem also provides the following vibration mode shapes (eigenvectors):

$$\{\phi_1\} = \begin{bmatrix} 0 \\ 1 \\ \dfrac{\Omega_1 - 1}{e\sqrt{12}} \end{bmatrix}; \ \{\phi_2\} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \ \{\phi_3\} = \begin{bmatrix} 0 \\ 1 \\ \dfrac{\Omega_3 - 1}{e\sqrt{12}} \end{bmatrix} \tag{3.3}$$

The first and third modes of vibration are coupled modes (i.e. translational component in y-direction coupled with a torsional component), while the second mode is purely translational in *x*-direction, due to the assumption of null eccentricity in *y*-direction.

From Eqn. 3.2 the following expressions of the natural periods of vibration, normalized with respect to the uncoupled lateral period T_L, can be obtained (Fig 3.1 b):

$$\begin{aligned}\frac{T_1}{T_L} &= \frac{1}{\sqrt{\frac{1}{2}\left(1+\Omega_\theta^{\,2} + 12e^2 - \sqrt{\left(\Omega_\theta^{\,2} + 12e^2 - 1\right)^2 + 48e^2}\right)}} \\ \frac{T_2}{T_L} &= 1 \\ \frac{T_3}{T_L} &= \frac{1}{\sqrt{\frac{1}{2}\left(1+\Omega_\theta^{\,2} + 12e^2 + \sqrt{\left(\Omega_\theta^{\,2} + 12e^2 - 1\right)^2 + 48e^2}\right)}}\end{aligned} \tag{3.4}$$

3.3. The modal contribution factors

In order to understand how each mode of vibration contributes to the dynamic response of the system, the closed-form expressions of the modal contribution factors MCF_i, i = 1,2,3, activated by a dynamic input characterized by influence vector {0,1,0} (i.e. input only along the y-direction), have been derived (Fig. 3.1 c):

$$
\begin{aligned}
MCF_1 &= \frac{1}{1+\left(\frac{\Omega_1 - 1}{\sqrt{12}}\right)^2} \\
MCF_2 &= 0 \\
MCF_3 &= \frac{1}{1+\left(\frac{\Omega_3 - 1}{\sqrt{12}}\right)^2}
\end{aligned}
\tag{3.5}
$$

Inspection of Fig. 3.1 c leads to the following observations:

- $MCF_2 = 0$ for all values of e and Ω_θ results from the assumptions of null eccentricity in the y-direction and influence vector along the y-direction;
- torsionally-stiff systems are principally governed by the first mode of vibration T_1 that, as showed in Fig. 3.1.a, is close to the second period of vibration, T_2, which in turn is equal to the uncoupled lateral period, T_L;
- torsionally-flexible systems with small eccentricity (e < 0.1) are mainly governed by the third mode of vibration that is approximately equal to T_L; torsionally-flexible systems with high eccentricity (e > 0.3) are substantially governed by the first mode of vibration that may be considerably higher than T_L; for torsionally-flexible systems characterized by eccentricity e between 0.1 and 0.3 both T_1 and T_3 contribute to the dynamic response of the system.

4. THE DISPLACEMENT AMPLIFICATION AT THE CENTER MASS: PERIOD SHIFTING

In the case of seismic excitation, the maximum center mass displacement can be predicted using the SRSS modal combination rule (Chopra 1995):

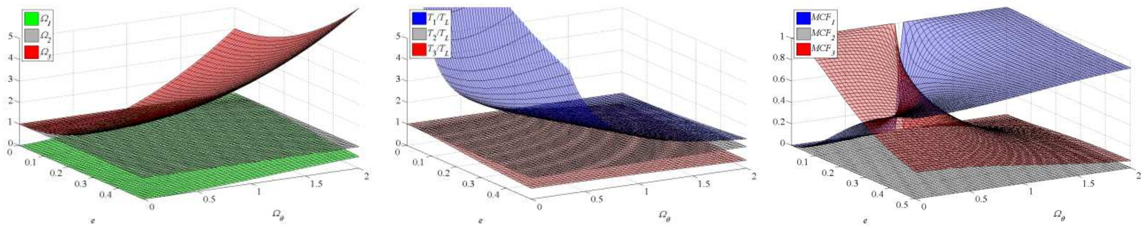
$$u_{y,CM,\max} \cong \sqrt{\sum_{i=1}^{3} (S_d(T_i) \cdot MCF_i)^2} \tag{4.1}$$

where $S_d(T_i)$ indicates the spectral displacement response at T_i (i=1,2,3). Under the assumption that $S_d(T)$ is a linear function of the period T ($S_d(T)=\varphi \cdot T$) Eqn. 4.1 yields to:

$$u_{y,CM,\max} \cong \varphi T_L \sqrt{\frac{MCF_1}{\Omega_1}^2 + MCF_2^{\,2} + \frac{MCF_3}{\Omega_3}^2} \tag{4.2}$$

Dividing Eqn. 4.2 by the center mass displacement of the equivalent N-E system ($u_{y,CM,max,N\text{-}E}$) the following closed-form expression of the displacement amplification, δ, as a function of e and Ω_θ (Fig. 5.1 a) can be derived:

$$\delta = \frac{u_{y,CM,\max}}{u_{y,CM,\max,N-E}} \cong 12e^2 \sqrt{\frac{1}{\Omega_1\left[12e^2 + (\Omega_1 - 1)^2\right]^2} + \frac{1}{\Omega_3\left[12e^2 + (\Omega_3 - 1)^2\right]^2}} \tag{4.3}$$



(a) (b) (c)

Figure 3.1. (a) normalized natural frequencies; (b) normalized natural periods; (c) modal contribution factors

It should be noted that, for sake of conciseness, Eqn. 4.3 is not directly expressed in terms of Ω_θ, but in terms of the normalized frequencies Ω_1 and Ω_3 (functions of e and Ω_θ). Inspection of Fig. 5.1.a reveals that:

- for a wide region of e and Ω_θ, δ is close to one;
- for high values of eccentricity e coupled with low values of Ω_θ, the displacement amplification δ can achieve values also larger than 5 (period shifting effect).

5. THE MAXIMUM ROTATIONAL RESPONSE

5.1. Undamped free vibration

In a previous research works (Trombetti and Conte 2005), the authors identified a rotational parameter called "alpha", governing the maximum rotational response of eccentric systems:

$$\alpha \overset{def}{=} \rho_m \frac{u_{\theta,\max}}{u_{y,CM,\max}} \tag{5.1}$$

In the case of undamped free vibrations from a given initial deformation, the alpha parameter assumes the following closed-form expression (Trombetti and Conte 2005):

$$\alpha_u = \frac{4e\sqrt{3}}{\sqrt{\left(\Omega_\theta{}^2 + 12e^2 - 1\right)^2 + 48e^2}} \tag{5.2}$$

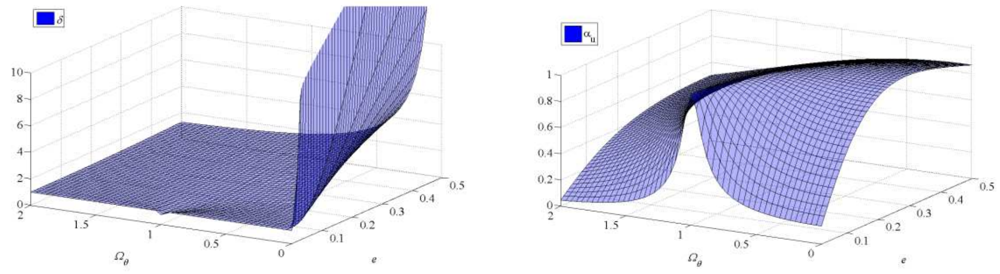
where the subscript u indicates "undamped conditions".

Fig. 5.1 b shows that α_u is bounded between zero and one. The above introduced rotational parameter allows to express the maximum rotational response as follows:

$$u_{\theta,\max} = \frac{\alpha_u}{\rho_m} \cdot u_{y,CM,\max} \tag{5.3}$$

upper bounded by:

$$u_{\theta,\max} \le u_{y,CM,\max} / \rho_m \tag{5.4}$$



(a) (b)

Figure 5.1. (a) δ versus e and Ω_θ; (b) α_u versus e and Ω_θ

5.2. Damped seismic response

In the case of damped systems subjected to seismic excitation, the alpha parameter is indicated as:

$$\alpha_{d,eqke} \overset{def}{=} \rho_m \left. \frac{u_{\theta,\max}}{u_{y,CM,\max}} \right|_{d,eqke} \tag{5.5}$$

where the subscript *d,eqke* indicates “damped conditions and earthquake input”.
By posing:

$$A \overset{def}{=} \frac{\alpha_{d,eqke}}{\alpha_u} \tag{5.6}$$

the maximum rotational response experienced by a damped eccentric system under seismic excitation can be expressed by the following simple relationship:

$$u_{\theta,\max} = \frac{\alpha_u}{\rho_m} \cdot A \cdot u_{y,CM,\max} \tag{5.7}$$

Parameter A should be obtained and calibrated by means of extensive numerical simulations, which are currently under development. Preliminary results indicates that for almost all values of e and Ω_θ parameter A is upper bounded by 1 (isolated cases of A <1 appeared for torsionally-flexible system).

6. THE COMBINATION OF THE MAXIMUM DISPLACEMENT RESPONSE WITH THE MAXIMUM ROTATIONAL RESPONSE

6.1. Undamped free vibration

The solution of the equations of motion of the studied eccentric system, in the case of undamped free vibrations from a given initial displacement a along the y-direction, is given by (Trombetti and Conte 2005):

$$\begin{aligned} u_y(t) &= a \cdot \left(R_1 \cos(\omega_1 t) + R_3 \cos(\omega_3 t) \right) \\ u_x(t) &= 0 \\ u_\theta(t) &= \frac{a}{\rho_m} \cdot \frac{\alpha_u}{2} \left(\cos(\omega_1 t) - \cos(\omega_3 t) \right) \end{aligned} \tag{6.1}$$

where R_1 and R_3 are defined as follows (Trombetti and Conte 2005):

$$
\begin{aligned}
R_1 &= \frac{1-\Omega_3}{\Omega_1-\Omega_3} \\
R_3 &= \frac{\Omega_1-1}{\Omega_1-\Omega_3}
\end{aligned}
\tag{6.2}
$$

Careful inspection of Eqns. 6.1 leads to the following observations:

- the maximum longitudinal displacement is developed for $\omega_1(t)=n\pi$ and $\omega_3(t)=m\pi$ (with n and m both odd or both even) and is equal to a. The corresponding rotation is zero;
- the maximum rotation is developed for $\omega_1(t)=n\pi$ and $\omega_3(t)=(m+1)\pi$ (with n and m both even or odd) and is equal to $(a/\rho_m)\cdot a_u$. The corresponding longitudinal displacement, $u_{y,CM@u_{\theta,max}}$, is equal to $a\cdot(R_1\text{-}R_3)$.

Based on the above mentioned observations, two limit assumptions (HP1 and HP2) are introduced:

1. the maximum corner displacement is calculated supposing a full correlation between the maximum rotational response and maximum center mass displacement response (HP1):

$$
u_{y,B,\max,HP1} = u_{y,CM,\max} + u_{\theta,\max}\frac{L}{2} \tag{6.3}
$$

which can be easily rewritten as:

$$
u_{y,B,\max,HP1} = \delta\cdot u_{y,CM,\max,N\text{-}E}\cdot(1+\alpha_u\cdot\phi) \tag{6.4}
$$

2. the maximum corner displacement is calculated combining the maximum rotational response with the center mass displacement achieved at the instant of maximum rotation, $u_{y,CM@u_{\theta,max}}$ (HP2):

$$
u_{y,B,\max,HP2} = u_{y,CM@u_{\theta,\max}} + u_{\theta,\max}\frac{L}{2} \tag{6.5}
$$

which can be easily rewritten as:

$$
u_{y,B,\max,HP2} = \delta\cdot u_{y,CM,\max,N\text{-}E}\cdot\left((R_1\text{-}R_3)+\alpha_u\cdot\phi\right) \tag{6.6}
$$

It is clear that Eqns. 6.5 and 6.6 represent an upper bound and a lower bound for the maximum corner displacement, respectively; thus:

$$
u_{y,B,\max,HP2} \le u_{y,B,\max} \le u_{y,B,\max,HP1} \tag{6.7}
$$

The following closed-form expressions of the corner displacement amplifications result from the two limiting assumptions HP1 and HP2 (Figs. 6.1):

Corner displacement magnification with respect to the center mass displacement (Figs. 6.1 a and b):

$$
\begin{aligned}
\Delta_1 &= \frac{u_{y,B,\max,HP1}}{u_{y,CM,\max}} = 1+\alpha_u\phi \\
\Delta_2 &= \frac{u_{y,B,\max,HP2}}{u_{y,CM,\max}} = (R_1\text{-}R_3)+\alpha_u\phi
\end{aligned}
\tag{6.8}
$$

Corner displacement magnification with respect to the center mass displacement of the equivalent *N-E* system (Figs. 6.1 c and d):

$$\Delta_{N\text{-}E,1} = \frac{u_{y,B,\max,HP1}}{u_{y,CM,\max,N\text{-}E}} = \delta \cdot \Delta_1 = \delta\left(1+\alpha_u \phi\right)$$
$$\Delta_{N-E,2} = \frac{u_{y,B,\max,HP2}}{u_{y,CM,\max,N\text{-}E}} = \delta \cdot \Delta_2 = \delta\left[\left(R_1 - R_3\right)+\alpha_u \phi\right] \tag{6.9}$$

Careful examination of the graphs plotted in Figs. 6.1 lead to the following fundamental observations:

- both Δ_1 and Δ_2 are larger than one for all values of e and Ω_θ . This result justifies the introduction of the assumption HP2 (lower bound). The maximum corner displacement amplifications Δ_1 and Δ_2 are limited to values around 2.3 and 1.6, with reference to HP1 and HP2, respectively;
- both $\Delta_{N\text{-}E,1}$ and $\Delta_{N\text{-}E,2}$ are basically governed by δ (i.e. period shifting). In detail: (i) high torsionally-stiff systems (i.e. Ω_θ >1.5) exhibit maximum corner displacement amplifications approximately equal to 2.5 and 2.0 with reference to HP1 and HP2, respectively; (ii) low torsionally-stiff systems ($\Omega_\theta \cong 1$) exhibit maximum corner displacement amplifications approximately equal to 3.5 and 3.0 with reference to HP1 and HP2, respectively; (iii) torsionally-flexible systems exhibit maximum corner displacement amplifications larger than 5;

6.2. Damped seismic response

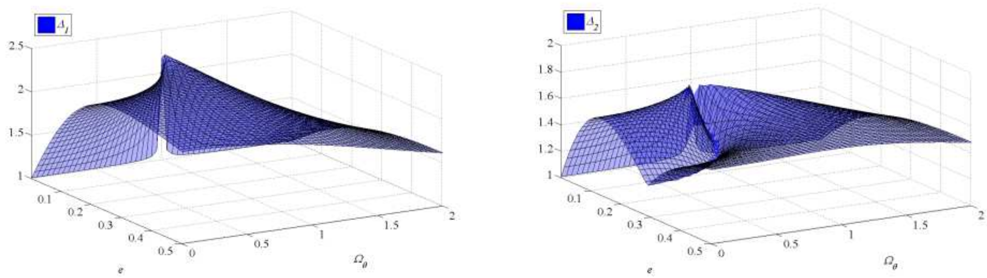
In the case of seismic excitation, a parameter of simultaneity *B* is introduced to account for the time correlation between the rotational and displacement seismic responses.

Parameter *B* should be obtained and calibrated by means of extensive numerical simulations, which are currently under development. However, it should be noted that parameter *B* is certainly less than 1 and therefore, from a conservative design point of view, it can be taken equal to 1.

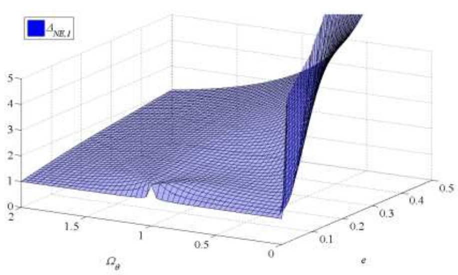
7. "ALPHA METHOD" FOR THE PREDICTION OF THE MAXIMUM CORNER DISPLACEMENT OF ECCENTRIC SYSTEMS

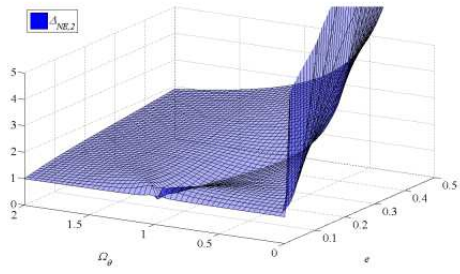
In a previous research work (Trombetti and Conte), the authors proposed a simplified method, called "Alpha-method", for the prediction of the maximum rotational response of eccentric systems. The original formulation of the method was developed limiting to the study of torsionally-stiff system and thus assuming that the maximum center mass displacement of the eccentric system can be reasonably approximated by the corresponding displacement of the equivalent not-eccentric system.

The results presented in this paper lead to comprehensive understanding of the dynamic behaviour of eccentric system. The analytical tools detailed in previous sections allow to extend the original formulation of the "Alpha-method" to a generic eccentric system, removing the assumption of equal center mass displacement between the eccentric system and its equivalent N-E system.



(a) (b)





(c) (d)

Figure 6.1. Corner displacement magnification with respect to: (a) center mass displacement based on HP1; (b) center mass displacement based on HP2; (c) center mass displacement of the equivalent N-E system based on HP1 (d) center mass displacement of the equivalent N-E system based on HP2

On the light of all the results reported in previous sections, the following formula for the evaluation of the maximum corner displacement of an eccentric system under seismic excitation is proposed:

$$u_{y,B,\max} = u_{y,C_M,\max,N-E} \cdot \delta \cdot \left(1 \ + \ A \cdot B \cdot \alpha_u \cdot \phi\right) \tag{7.1}$$

CONCLUSIONS

This paper provides a comprehensive insight into the dynamic behaviour of one-storey eccentric systems, aimed at increasing the knowledge about this class of structures, as well as providing simple tools for their seismic design. For the specific case of undamped eccentric systems in free vibrations, closed-form expressions for an upper bound and a lower bound of the maximum longitudinal corner displacement have been derived. Based on these results, a simplified approach for the seismic design of eccentric systems, originally proposed by the author for the evaluation of the torsional response of torsionally-stiff eccentric systems, has been revised accounting for all classes of eccentric systems.

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