Seismic Design of Moment Resisting Frame Structures Equipped With Viscous Dampers

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SUMMARY:
The effectiveness of viscous dampers in mitigating the seismic excitation impacts upon building structures has been widely proved. Recently, a direct practical procedure for dimensioning viscous dampers that are inserted in building structures has been proposed. The procedure, originally developed with reference to a shear-type structures schematization, provides an easy identification of the mechanical characteristics of the manufactured viscous dampers. In detail this paper presents a rational approach aimed at obtaining simple direct design formula for the dimensioning of each added viscous dampers that are to be inserted in generic moment-resisting frame structures.

Keywords: Moment-resisting frames; viscous dampers; design procedure; seismic response; damping ratio.

1. INTRODUCTION

Manufactured viscous dampers are hydraulic devices which can be installed in structures in order to mitigate the seismic effects through dissipation of the kinetic energy transmitted by the earthquake to the structure (Soong and Dargush 1997, Constantinou et al. 1998, Christopoulos and Filiatrault 2006,). These devices have been the objective of several research works since the 1980's (Constantinou and Tadjbakhsh 1983, Constantinou and Symans 1993, Singh and Moreschi 2002, Levy and Lavan 2006). Nevertheless, even if all the above cited are remarkable from a scientific point of view (development of sophisticated algorithms and complex procedures), they hardly represent a direct and immediate help for the practical engineers. Indeed, investigations about developing a practical method for sizing viscous dampers, which are capable of achieving a target level of seismic performance, are still open.

Regarding to this aspect of assessing the viscous dampers, only few contributions can be found in the scientific literature. Among these, the most remarkable are: (i) Christopoulos and Filiatrault 2006 suggested a practical design approach for estimating the damping constants of individual dampers consisting in a trial and error procedure; (ii) Silvestri et al. 2010 proposed a direct design approach, defined as five-step procedure.

The latter (five-step procedure) aims at guiding the professional engineer from the choice of the target objective performance to the identification of the mechanical characteristics (i.e. damping coefficient) of commercially available viscous dampers. The analytical developments presented in the original version of the procedure (Silvestri et al. 2010) have been carried out with reference to a Shear-Type (referred hereafter with the acronym ST) structure schematization. On the other hand, in the same work (Silvestri et al. 2010) the authors added two applicative example of the procedure developed on two moment resisting frame, thus removing the assumption of shear-type schematization.

Therefore, from a theoretical point of view, the purpose of the present work is to extend the validity of the proposed approach for a generic Flexible-Type (FT) structure schematization, i.e. a structural model which considers the actual stiffness of the beams. This purpose will require a further insight into the damping properties of systems typically characterized by not proportional damping (Cheng 2001, Occhiuzzi 2009).

2. OVERVIEW OF THE FIVE-STEP PROCEDURE

Recently (Silvestri et al. 2010) the authors proposed a direct five-step procedure for the dimensioning of the damping coefficient *c* of viscous dampers which was simply based on the knowledge of the floor masses and the fundamental period of vibration of the structure.

The procedure, called Five-step procedure, is composed of the following steps:

STEP 1. Identification of the target damping ratio $\bar{\xi}$ of the structure on the basis of a chosen target level $\bar{\eta}$ of structural performances; $\bar{\eta}$ is the reduction factor used to reduce the spectral ordinates as a function of the damping ratio.

STEP 2. Identification of the tentative characteristics of the linear viscous dampers for preliminary design (i.e. linear damping ratio, $c_L = \overline{c_L}$; damping exponent, $\alpha = 1.0$; oil stiffness, $k_{oil} = \infty$), i.e. first dimensioning of the linear damping coefficients.

STEP 3. Development of a series of preliminary time-history analyses of the building structure equipped with the viscous dampers identified in Step 2. This step allows to: (i) sizing the linear damping coefficients of the dampers to be added to the structure in order to achieve the desired level of actions (axial forces, shear forces, bending moments, etc.) on the structural members of the building; and (ii) identify the range of “working” velocities for the linear added viscous dampers.

STEP 4. Identification of the characteristics of the “equivalent” non-linear viscous dampers ($c_{NL} = \overline{c_{NL}}$, $\alpha = \bar{\alpha}$, $k_{oil} = \bar{k}_{oil}$), i.e. identification of a system of manufactured viscous dampers capable of providing the structure with actions (on the structural members) comparable to those obtained in Step 3 using the linear viscous dampers identified in Step 2.

STEP 5. Development of a series of final time-history analyses of the building structure equipped with the viscous dampers identified in Step 4. This last step is necessary in order to verify the effectiveness of Step 4 and obtain the forces both trough the structural members and dampers which are to be used for the final design specifications.

The original contribution of this procedure lies, mainly, on the STEP 2, which provides simple relationships (Eqs. 27, 28 and 29 of Silvestri et al. 2010), for the calculation of the damping coefficient *c* of each viscous damper in order to achieve the proposed performance objective.

3. DAMPING PROPERTIES OF FRAME SYSTEMS WITH ADDED VISCOUS DAMPERS: OVERVIEW

In previous works (Silvestri et. al 2003, Trombetti and Silvestri 2004, 2006 and 2007, Silvestri and Trombetti 2007) the authors showed that the proportional damping and its two limiting cases (i.e. Mass Proportional Damping, MPD, and Stiffness Proportional Damping, SPD) do have a physical counterpart in ST structures equipped with added viscous dampers in the case of (i) equal lamped mass *m* at each floor, (ii) constant translational stiffness *k* at each floor. On the contrary, regarding to a generic flexible-type (FT) frame equipped with added viscous dampers, the damping system may not be always schematized using the classic Rayleigh theory (i.e. proportional damping, Chopra 1995).

In order to illustrate a tangible instance relevant to the above mentioned concept, a 3-storey FT structure equipped with Inter-Story dampers (represented in Fig. 3.1) has been posed as a typical

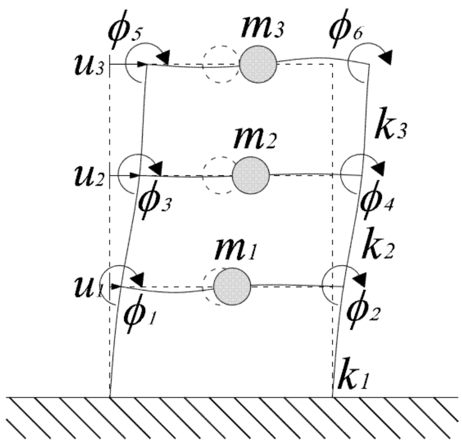

Figure 3.1. 3-storey flexible-type structure equipped with interstorey dampers.

exemplification for all same cases. In Fig. 3.1 m_i (i=1,…,3) indicates the floor mass of the i-th story; K_i (i=1,…,3) indicates the translational lateral stiffness of the i-th storey , θ_i (i=1,…,6) indicates the i-th rotational degree of freedom; u_i (i=1,…,3) indicates the translational degree of freedom (the beams axial flexibility is neglected). Under the assumption of: (i) neglecting the axial flexibility of the system; (ii) static condensation of the rotational degrees of freedom θ_i (Clough and Penzien 1993); the system showed in Fig. 3.1 has three degrees of freedom. The mass, stiffness and damping matrixes of the existing system presented in the example have the following matrix form:

$$[M]=\begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix} \tag{3.1}$$

$$[K]=\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \tag{3.2}$$

$$[C]=\begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \tag{3.3}$$

From Eqns. 3.1 to 3.3 it can clearly deduced that [K] is a fully populated matrix; [M] is a diagonal matrix; [C] is a band matrix. Therefore, in this case it is not possible to obtain the damping matrix [C] in the proportional form (i.e. [C] = $\alpha \cdot$ [M] + $\beta \cdot$ [K]).

To cut a long story short, Table 3.1 provides a scheme of the type of damping related to the coupling of a certain structure schematization (i.e. ST or FT) with a certain dampers placement (i.e. Fixed-Point, FP, or Inter-Storey, IS).

In the next sections the relationship between the damping ratio c and the damping coefficient ξ will be investigated starting from frame structures equipped with Fixed-Point dampers and then moving to frame structures equipped with Inter-Storey dampers.

Table 3.1. Type of damping for different typologies of “system”.

Dampers placement	Type of damping	
	ST schematization	FB schematization
FP placement	Proportional (MPD) System C (see Sec. 4)	Proportional (MPD)
IS placement	Proportional (SPD) System B (see Sec. 4)	Not-proportional System A (see Sec. 4)

4. PROBLEM FORMULATION

The “system” defined here is composed of a specific frame structure (characterized by floor mass m_i, with i=1,..,N , column moment of inertia J_i, with i=1,...,N , N indicates the number of stories) equipped with a specific damping system (i.e. damping coefficients c_j, j=1,...N x n, where n indicates the number of dampers per each floor).

System A, graphically represented in Fig. 3.2, is defined as a flexible-type frame structure (floor mass $m_{A,i}$, column moment of inertia $J_{A,i}$) equipped with inter-storey viscous dampers (damping coefficients $c_{A,j}$, total damping coefficient $c_{A,tot}$, sum of the $c_{A,j}$).

The objective of the research work is the identification of the values of the total damping coefficient $c_{A,tot}$ or the single damping coefficients $c_{A,j}$ of the dampers of system A in order to obtain a target value of damping ratio, $\bar{\xi}$ (Step 1, section 4):

$$c_{A,tot} = f\left(\bar{\xi}\right) \tag{4.1}$$

Or:

$$c_{A,j} = f\left(\bar{\xi}\right) \qquad \forall j \tag{4.2}$$

In order to achieve this objective, it is necessary to introduce the following systems:

- System B, that is graphically represented in Fig. 3.2, is defined as the shear-type frame structure with the same properties (i.e. $m_{B,i} = m_{A,i}$, $J_{B,i} = J_{A,i}$) as defined in system A, but different translational stiffness ($k_{B,i} \neq k_{A,i}$, due to the restrained rotation of the nodes) equipped with inter-storey viscous dampers, characterised by the same total damping coefficient of the damping system of system A ($c_{B,tot} = c_{A,tot}$). It should be mentioned that the fundamental frequencies of the two systems , ω_B and ω_A, are different.
- System C, that is graphically represented in Fig. 3.2 is defined as the shear-type frame structure with the same properties ($m_{C,i} = m_{B,i}$, $J_{C,i} = J_{B,i}$, and same restrained rotations of the nodes) as defined in system B, equipped with fixed-point viscous dampers, characterised by the same total damping coefficient of the damping system of system B ($c_{C,tot} = c_{B,tot}$). Clearly systems B and C are characterized by the same fundamental frequency (ω_B is equal to ω_C).

According to structural dynamics (Chopra 1995), it is well known that the total damping coefficient of system C can be expressed as a function of the corresponding damping ratio by the following equations (MPD schematization):

$$c_{C,tot} = 2 \cdot \xi_c \cdot \omega_C \cdot m_{tot} \tag{4.3}$$

Since no analytical relationships are currently available in order to express the total damping coefficient of system A as a function of its fundamental damping ratio, then the procedure schematically illustrated in the flowchart reported in Fig. 3.3 is introduced. The first stage of the flow chart consists in the derivation of a relationship between the fundamental damping ratio of system A and system B and represents the core of the present research work, since the second stage (consisting

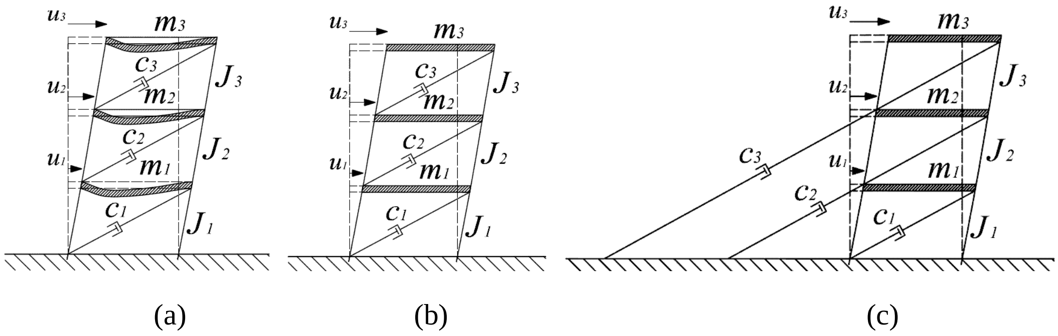

Figure 3.2. Schematic representation of: (a) System A; (b) System B; (c) System C.

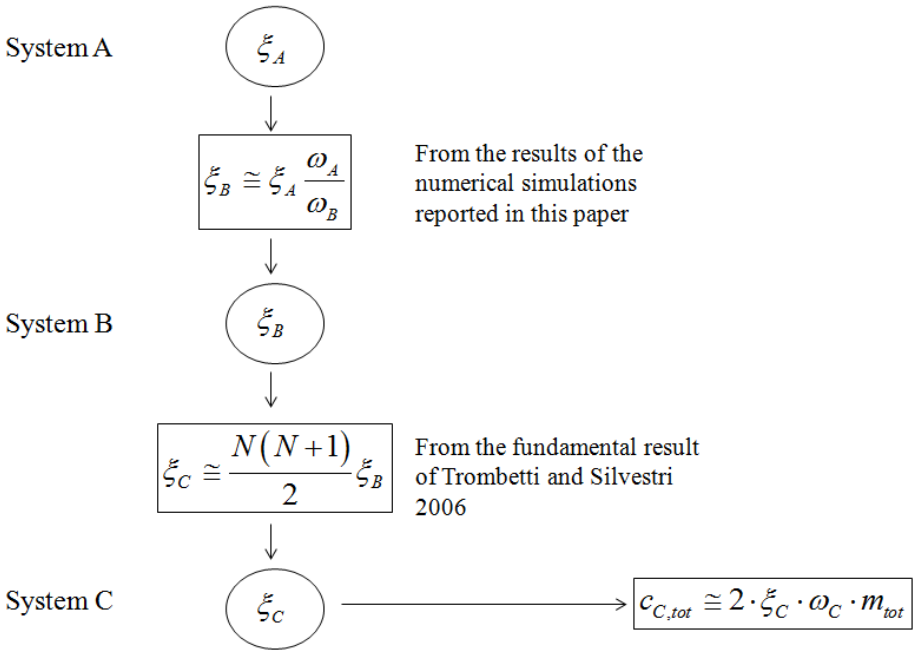

Figure 3.3. Flowchart of the scheme adopted to reach the objective of the research work.

in determining the correlation between the fundamental damping ratio of systems B and C) was the fundamental result of a previous research work developed by the authors (Trombetti and Silvestri 2006).

Details of the two stages of the flow chart will be given in the following sections.

5. THE RELATIONSHIP BETWEEN THE FUNDAMENTAL DAMPING RATIOS OF SYSTEMS A, B AND C

In a previous research work (Trombetti and Silvestri 2006), under the assumption of equal lateral stiffness and lateral mass at every storey ($m_i = m \; \forall$ i and $k_i = k \; \forall$ i), the authors demonstrated that the fundamental damping ratio between system B and C (under the constrain that the two systems have equal total damping coefficients) can be expressed by the following exact relationship:

$$\xi_C = \frac{k}{m \cdot \omega_c^{\,2}} \cdot \xi_B \tag{5.1}$$

that can be well approximated by (Trombetti and Silvestri 2004):

$$\xi_C \cong \frac{N(N+1)}{2} \cdot \xi_B \tag{5.2}$$

It has been showed in section 3 that, while for system B and system C it is possible to define the damping matrix on the basis of the SPD or MPD limiting cases (i.e. physical counterpart), respectively, as far as system A is concerned, it is necessary to recur to the theory of complex damping (Cheng 2001, Occhiuzzi 2009). In detail, instead of searching for an exact analytical relationship between the fundamental damping ratio of system A and system B, a numerical procedure has been preferred.

For damped SDOF systems, an analytical relationship between the fundamental damping ratios of systems A and B can be drawn starting from the basic concepts of structural dynamics (the "basic idea", as detailed in next section 5.1).

For damped MDOF systems, a numerical analysis performed in the field of complex damping has been conducted in order to verify if the same relationship still holds (next section 5.2).

5.1. The basic idea

Two equivalent (same mass *m*, same column moment of inertia *J* and same damping coefficient *c*) SDOF systems are considered (Fig. 5.1): the first one represents a one-storey one-bay shear-type (ST) structure equipped with an interstory viscous damper, and the second one represents a one-story one-bay flexible type (FT) structure equipped with an interstory viscous damper. Due to the different stiffness of the beams, the fundamental periods of the two defined systems (defined as T_{ST} and T_{FT}) are not equal. Under the abovementioned assumption, it is easy to show that the ratio of the modal damping ratios pertained to the two systems is exactly the same as the corresponding ratio of the fundamental periods:

$$\rho_\xi = \frac{\xi^{FT}}{\xi^{ST}} = \frac{c/2 \cdot m \cdot \omega^{FT}}{c/2 \cdot m \cdot \omega^{ST}} = \frac{\omega^{ST}}{\omega^{FT}} = \frac{T^{FT}}{T^{ST}} = \rho_T \tag{5.3}$$

5.2. Numerical analysis: main results

The numerical analysis has been carried out on the flexible-type structures schematized in Fig. 5.2, which are characterized by the following main properties:

- Number of stories, *N*, variable from 2 to 6;
- Number of bays equal to 1;
- Bay width equal to 6 m;
- Interstory height equal to 3 m;
- Square Columns with constant cross section 40x40 cm at each storey (fixed for all the models);
- Beams with constant cross section for all stories (different for each model);
- Floor mass *m* equal to 80000 kg;
- Beam–Column Stiffness ratio, $\rho_R = k_{beam} / k_{column}$, variable from 0,5 to ∞ (shear type system);
- Elastic material with Young's modulus, *E*, equal to 20000 MPa.

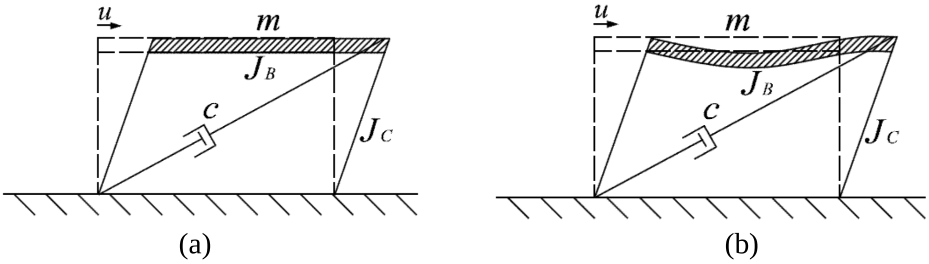

Figure 5.1. (a) Shear-type SDOF system equipped with viscous damper; (b) Flexible-type SDOF system equipped with viscous damper.

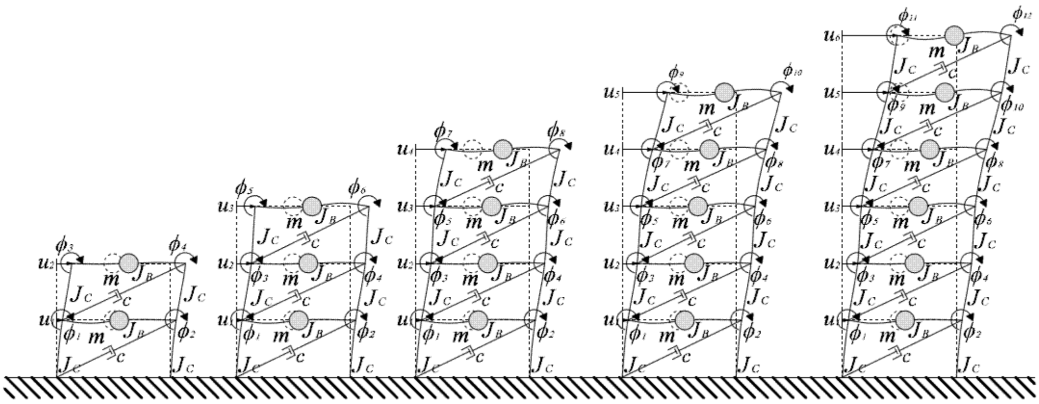

Figure 5.2. Schematic representation of the MDOF frame systems used to perform the numerical analysis.

Considering the *k*-th FT system each single analysis included the following phases: (i) evaluation of the fundamental modal damping ratio, ξ_k^{FT} , by means of the complex damping theory (Cheng 2001) (all the details regarding to the calculations can be found in Muscio 2009); (ii) evaluation of the first modal damping ratio of the equivalent ST system by means of the Rayleigh theory (Chopra 1995), ξ_k^{ST} ; (iii) evaluation of the ratio of the fundamental modal damping ratios between the FT structure and the equivalent ST structure, $\rho_{\xi-k}$; (iv) evaluation of the ratio of the fundamental periods between the FT structure and the equivalent ST structure, ρ_{T-k} .

The main results are briefly illustrated through Figs. 5.3 which display the relationship between ρ_ξ and ρ_R (Fig 5.3 a) , ρ_T and ρ_R (Fig 5.3 b) and ρ_ξ and ρ_T (Fig 5.4 c). Inspection of the graphs leads to the following deductions:

- for all values of flexibility ratio, ρ_R , ρ_ξ is higher than the corresponding ρ_T ;
- as might be reasonably expected, for a fixed value of ρ_R , both ρ_ξ and ρ_T increase with the increase of the total number of storey *N*;
- as might be reasonably expected, both ρ_ξ and ρ_T increase as ρ_R decreases;
- the values of ρ_ξ and ρ_T exhibit an high linear correlation (correlation coefficient equal to 0.98);
- the values of the ratio ρ_ξ are higher than the corresponding (i.e. the value calculated to the same structure) values of the ratio ρ_T ; thus, ρ_T can be assumed as lower bound for ρ_ξ ;
- for practical application ρ_ξ can be assumed equal to ρ_T ; the assumption leads to slightly conservative results.

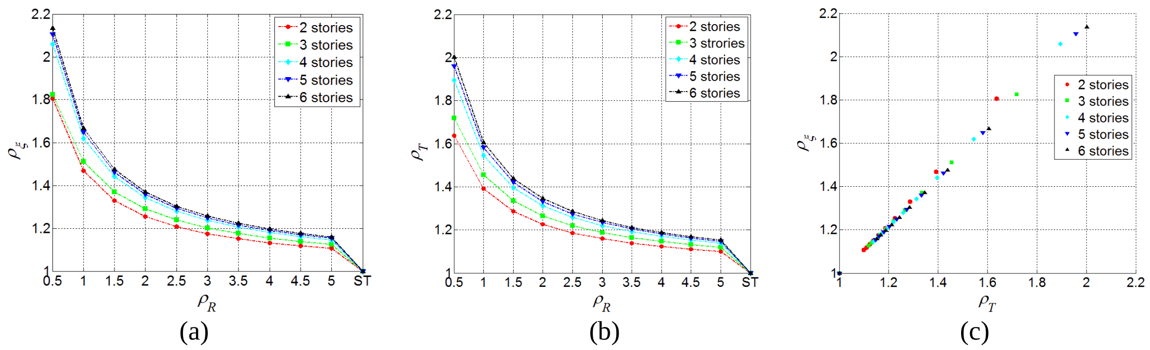

Figure 5.3. (a) ρ_ξ versus ρ_R; (b) ρ_T versus ρ_R; (c) ρ_ξ versus ρ_T.

5.3. The “approximate” relationship

Based on the fundamental result of the numerical analysis commented above (and summarized in Figs. 5.3), the following approximate relationship between the fundamental modal damping ratio of system A and B, can be assumed for design purpose:

$$\xi_B \cong \xi_A \cdot \frac{T_B}{T_A} \tag{5.4}$$

Or in terms of fundamental frequencies:

$$\xi_B \cong \xi_A \cdot \frac{\omega_A}{\omega_B} \tag{5.5}$$

Eqns. 5.4 or 5.5 represent the objective relationship of the stage 1 of the flow chart given in Fig. 3.3.

5.4. The total damping coefficient for system A

The fundamental result provided by the numerical analysis (Eqn. 5.4 or 5.5) allows us to obtain a simple formula for the dimensioning of the total damping coefficient c_{tot} of system A in order to achieve the target damping ratio $\overline{\xi}$. In more details, according to the flowchart represented in Fig. 3.3, the following simple (like design formula) expression of the total damping coefficient $c_{A,tot}$ for system A, leading to the target damping ratio $\overline{\xi}$, is engendered by merely substitution of Eqn. 5.5 into 5.2 and then Eqn. 4.3 (note that $\omega_B = \omega_C$):

$$c_{A,tot} \cong \overline{\xi} \cdot \omega_A \cdot m_{tot} \cdot N \cdot (N+1) \tag{5.6}$$

Eqn. 5.6 represents the fundamental result of the present research work. It should be mentioned that Eqn. 5.6 is identical to Eqn. 27 of (Silvestri et al. 2010). However, while the latter was derived based on the assumption of shear-type structure schematization, Eqn. 5.6 keeps its validity for a generic flexible-type frame structure.

6. THE DIMENSIONING OF EACH VISCOUS DAMPER

In the previous section a simple analytical relationship (Eqn. 5.6) for the evaluation of the total damping coefficient c_{tot} of a system composed by interstorey dampers to be added to moment-resisting frame structures has been provided. However, from a design point of view, the practical engineer is interested in sizing each damper rather than evaluating the total damping coefficient. Therefore, in order to obtain simple design relationships for the dimensioning of each damper the following assumptions are considered:

- for Fixed-Point damper placement, based on the correspondence and compatibility (i.e. physical counterpart) which exist between the MPD theory and the FP dampers placement, then the i-th storey damping coefficient c_i is proportional to the i-th floor mass, m_i:

$$c_i = \frac{m_i}{m_{tot}} \cdot c_{tot} \tag{6.1}$$

- for Inter-Story damper placement, the i-th storey damping coefficient can be only approximately assumed proportional to the i-th lateral storey stiffness (however for practical purpose the assumption is reasonable):

$$c_i \cong \frac{k_i}{k_{tot}} \cdot c_{tot} \tag{6.2}$$

where k_{tot} is the sum of the storey lateral stiffness over all stories.

Obviously it can be simply noticed that, if n equal dampers are placed at each storey, the damping coefficient c of each damper results equal to:

$$c = \frac{c_i}{n} \tag{6.3}$$

Thus:
(i) in the case of IS dampers placement the substitution of Eqns. 6.2 into 6.3 (assuming equal lateral stiffness k at each floor) and then into Eqns. 5.6 leads to:

$$c = \frac{\bar{\xi} \cdot \omega \cdot m_{tot} \cdot (N+1)}{n} \tag{6.4}$$

(ii) in the case of FP dampers placement the substitution of Eqns. 6.1 in 6.3 (assuming equal lamped mass m at each floor) and then into Eqn. 4.3 (imposing $\xi_c = \bar{\xi}$) leads to:
In the case of Fixed-Point dampers placement

$$c = \frac{2 \cdot \bar{\xi} \cdot \omega \cdot m_{tot}}{n \cdot N} \tag{6.5}$$

Eqns. 6.4 and 6.5 are direct design formulas for the dimensioning of viscous dampers that are to be inserted in moment resisting frame structures in order to make them able to satisfy a prescribed performance objective (i.e. the achievement of a target damping ratio reduction $\bar{\xi}$, i.e. a target reduction coefficient $\bar{\eta}$):

CONCLUSIONS

This paper demonstrates the effectiveness of applying a practical procedure for dimensioning added viscous dampers that are inserted in moment resisting frame systems, known as five-step procedure.

The procedure, which was originally developed by assuming a Shear-Type structure schematization, the proposed a direct simple design formula that allows to size the damping coefficient *c* which is pertained to each damper, according to the knowledge of the building mass and fundamental period, which can be evaluated easily by the practical engineers.

The demonstration of the validity of the proposed approach for a generic moment resisting frame, thus removing the assumption of Shear-Type structure schematization, has been provided through an insight into the damping properties of the studied systems in accordance with the theory of complex damping.

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