Evaluation of Soil Heterogeneity Effects on the Coherency Function of Seismic Motions.

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SUMMARY:
This paper deals with the analytical evaluation of soil lateral heterogeneity effects, especially the random fluctuations of predominant layer natural frequency, on the spatial ground motion coherency and the seismic response of multi-support structures. A coherency function probabilistic model is proposed. In this model, the spatial variation of the motions is attributed to the wave passage effects with constant apparent propagation velocity, the effects of loss of coherence in the bedrock motion, and particularly the site-response effects that are based on the assumption of vertical propagation of shear waves through a horizontal layer with random characteristics overlying an elastic half-space to the ground surface. Results indicate that the variability of local site condition tends to cause diminution of the values of the total coherency function; this diminution is notable near the mean resonant frequency of the layer and begins to decrease as the frequency increases. The decrease of the total coherence is significant even for relatively low values of CV (5 to 15%) and the site effects change significantly the trend of the total coherency function. Also, as the heterogeneity of the medium increases, as the site contribution is important and the seismic motions become less similar.

Keywords: coherency function; spatial variability; soil lateral heterogeneity; site effect; seismic motion.

1. INTRODUCTION

In the current seismic design of engineering structures, the seismic ground motion excitations are supposed to be identical at their supports. However, this assumption can yield overestimated structural translational responses and neglects the rotational motions for the extended structures. Consequently, it can have a significant impact on their dynamic response characteristics. Thus, the design and the analysis of structures have to include the spatial variability of the seismic motion. which

With the availability of recorded data at dense accelerograph arrays, several empirical models of the spatial variability, generally in coherency function term, have been developed (Abrahamson and al, 1991; Hao and al, 1989; Harichandran and Vanmarcke, 1986; Hindy and Novak, 1980; Loh and Yeh, 1988; Nakamura and Yamazaki, 1995; Novak and Hindy, 1979; Zerva and Zervas, 2002). Nevertheless, empirical coherency models depend on sites for which they have been developed and cannot be reliably extrapolated to other different sites. On the other hand, some researchers have developed analytical or semi empirical models (Der Kiureghian, 1996; Laouami and Labbe, 2001; Luco and Wong, 1986). These models are more flexible and can be extrapolated to other sites.

Nevertheless, much of these theoretical models suppose horizontal soil stratification with lateral homogeneity of its characteristics. However it’s clear that the lateral heterogeneity influences the incident seismic motion and is responsible on a part of the spatial variation of the surface motions. Zerva and Harada (1997) approximated the site topography by a horizontally extended layer with random characteristics overlaying a half-space to study the local site effect on the coherency function. Their works show that the site effect is concentrated in the vicinity of the stochastic layer natural frequency and yields a drop in the value of the coherence. Liao and Li (2002) introduced the orthogonal expansion method into a numerical approach to evaluate analytically the effect of the uncertainty in soil properties on the coherency function. They pointed out that the stochasticity in the soil layer causes diminution of the coherency function values near the sites' resonant frequencies.

The aim of this paper is the analytical evaluation of lateral heterogeneity effect in soil layer on the spatial ground motion coherency. A coherency function probabilistic model is proposed. In this model, the spatial variation of the motions is attributed to the wave passage effects with constant apparent propagation velocity, the effects of loss of coherence in the bedrock motion by means of Luco and Wong's model (Luco and Wong, 1986), and particularly the site-response effects that are based on the assumption of vertical propagation of shear waves through a horizontal layer with random characteristics to the ground surface. The applicability of the approach is limited to sites which can be represented by horizontal soil layers and possessing a quasi-regular topography.

2. STOCHASTIC MODEL OF SPATIAL VARIABILITY OF GROUND MOTION

2.1 Random Field For Soil Properties

Based on the assumption of vertical shear wave propagation through horizontal layer with deterministic values of soil properties and extending laterally to infinity, overlying a bedrock, the free field seismic ground motion accelerations are modeled by Kanai-Tajimi power spectral density (PSD) of a white process filtered by the soil layer whose transfer function is (Kanai, 1957):

$$H(i\omega)=\frac{\omega_g^2+2i\xi_g\omega_g\omega}{(\omega_g^2-\omega^2)+2i\xi_g\omega_g\omega} \tag{1}$$

And the PSD (Kanai, 1957):

$$S_0(\omega)=\frac{1+4\xi_g^2\dfrac{\omega^2}{\omega_g^2}}{(1-\dfrac{\omega^2}{\omega_g^2})^2+4\xi_g^2\dfrac{\omega^2}{\omega_g^2}}A \tag{2}$$

In which A is the amplitude of the white process bedrock excitation, and ω_g and ξ_g are the predominant natural frequency and damping coefficient of the soil layer. $i=\sqrt{-1}$.

Generally, the dispersion observed in soil data comes from both the spatial variability and from errors in testing. Thus, it is hardly surprising that mechanical properties of soils vary from place to place within resulting deposits. In principle, spatial variation of soil properties can be characterized in detail, but only if a large number of samples is available. In reality, the number of tests required far exceeds that which would be practical (Nour and al, 2002). Since deterministic descriptions of this spatial variability are in general not feasible, the overall characteristics of the spatial variability and the uncertainties involved are mathematically modeled using stochastic fields (Assimaki and al, 2003). Thus, for engineering purposes, one assumes that spatial variability of soil properties is decomposed into a deterministic trend, and a random component describing the variability about that trend.

In order to investigate the heterogeneous character of soil, a 2D soil model with stochastic properties is studied herein. The modeling consists in discretizing the soil medium into spaced parallel springs (Winkler model) having different natural frequencies fluctuating randomly around a mean ω_0 along the horizontal x-axis (fig. 2.1); thus, the predominant natural frequency can be expressed as follows

$$\omega^*(x)=\omega_0\left(1+\omega\ (x)\right) \tag{3}$$

Where $\omega\ (x)$ is a homogeneous random field with zero mean and a standard deviation $\sigma_{\omega\omega}$.

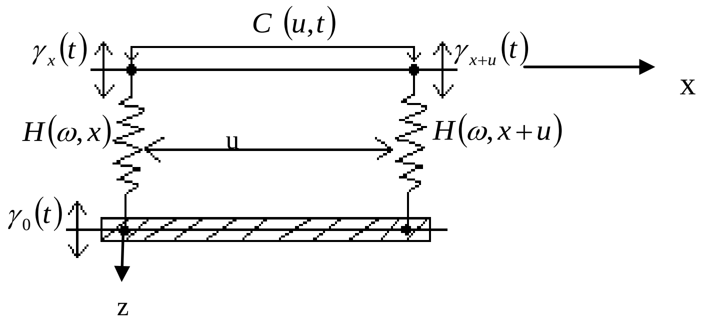

Figure. 2.1. Winkler model; case of a layer with stochastic properties. $C(u,t)$: inter correlation function, $\gamma_0(t)$: white process,. $\gamma_x(t)$ and $\gamma_{x+u}(t)$: filtered white process, $H(\omega,x)$ and $H(\omega,x+u)$: transfer functions.

The soil predominant frequency is modeled herein as spatially random field. For the random definition of the medium, the chosen random variable is defined by its mean, and variance. One supposes that the soil is modeled by the Winkler model consisting of infinity of soil columns with different frequencies varying randomly and excited by a white process solicitation $\gamma_0(t)$. In order to develop an analytical model, one needs to simplify the problem by considering two soil columns separated by the distance u, and characterized respectively by the fundamental frequencies $\omega^*(x)$ and $\omega^*(x+u)$ and by the transfer functions $H(\omega,x)$, $H(\omega,x+u)$. Based on the random vibration theory, the surface responses of the two soil columns $\gamma_x(t)$ and $\gamma_{x+u}(t)$ are correlated by the inter correlation function $C\ (u,t)$. The analytical estimate of $C\ (u,t)$ and the associated cross-PSD allows to connect analytically the soil properties spatial variability with that of the free field responses.

2.2 Correlation Functions

By introducing the stochasticity into the medium (eqn 3) assuming that the damping coefficient of the medium ξ_g is equal to a mean value ξ , the transfer function $H(i\omega)$ (eqn 1) becomes

$$H(x,\omega)=\frac{\left(\omega^*(x)\right)^2+2i\xi\omega\left(\omega^*(x)\right)}{\left(\left(\omega^*(x)\right)^2-\omega^2\right)+2i\xi\omega\left(\omega^*(x)\right)} \tag{4}$$

It should be noted that this approach is applicable for sites which can be represented by horizontal soil layers and having a quasi-regular topography for which the homogeneity assumption for the random field characterizing the heterogeneous soil is valid. In the time domain, the impulse response function is determined via an inverse Fourier transformation of the transfer function as

$$h(x,t)=IFT\left[H(x,\omega)\right]=\begin{cases}\dfrac{\omega^*(x)}{\sqrt{1-\xi^2}}.e^{-\xi\omega^*(x)t}\cos\left(\omega^*(x)\sqrt{1-\xi^2}t+\phi\right) & for\ t\geq 0\\ 0 & for\ t<0\end{cases} \tag{5}$$

with :

$$\phi=-arctg\left[\frac{1-2\xi^2}{2\xi\sqrt{1-\xi^2}}\right] \tag{6}$$

IFT indicates inverse Fourier transform, the impulse response function $h(x,t)$ is developed into a Taylor series around ω_0 and results $\left(for\ t \geq 0\right)$ in

$$h(x,t)\Big|_{\omega^*(x)=\omega_0} = \frac{\omega_0 e^{-\xi\omega_0 t}}{\sqrt{1-\xi^2}} \cdot \left\{ \begin{array}{l} \cos\left(\omega_0\sqrt{1-\xi^2}t+\phi\right)+ \\ \omega(x)\left[\begin{array}{l} \cos\left(\omega_0\sqrt{1-\xi^2}t+\phi\right)-\omega_0\xi\ t\cos\left(\omega_0\sqrt{1-\xi^2}t+\phi\right)- \\ \omega_0\sqrt{1-\xi^2}t\sin\left(\omega_0\sqrt{1-\xi^2}t+\phi\right) \end{array}\right] \end{array} \right\} \quad (7)$$

By definition, the autocorrelation function $C_{xx}(x,\tau)$ and the cross-correlation function $C_{xy}(x,\tau,u)$ between the two columns separated by a distance u, are respectively given as

$$C_{xx}(x,\tau) = \int_{\tau}^{\infty} h(x,t)h(x,t-\tau)dt \quad (8)$$

$$C_{xy}(x,\tau,u) = \int_{0}^{\infty} h(x,t)h(x+u,t+\tau)dt \quad (9)$$

Using the equation (7) and after lengthy algebraic manipulations, one can obtain the analytical expressions of the autocorrelation and the cross-correlation mean functions of the ground acceleration:

$$\hat{C}_{xy}(\tau,u) = \frac{\omega_0 e^{-\xi\omega_0|\tau|}}{4\xi\left(1-\xi^2\right)} \times \left\{ \begin{array}{l} \cos\left(\omega_0\sqrt{1-\xi^2}|\tau|\right)+ \\ \xi\sin\left(\omega_0\sqrt{1-\xi^2}|\tau|+2\phi+\phi_1+\pi\right) \end{array} \right\} + C_{\omega\omega}(u) \times \left\{ \begin{array}{l} a(\tau)-b(\tau)+ \\ c(\tau)+d(\tau)+e(\tau) \end{array} \right\} \quad (10)$$

And

$$\hat{C}_{xx}(\tau) = \frac{\omega_0 e^{-\xi\omega_0|\tau|}}{4\xi\left(1-\xi^2\right)} \times \left\{ \begin{array}{l} \cos\left(\omega_0\sqrt{1-\xi^2}|\tau|\right)+ \\ \xi\sin\left(\omega_0\sqrt{1-\xi^2}|\tau|+2\phi+\phi_1+\pi\right) \end{array} \right\} + \sigma^2_{\omega\omega} \times \left\{ \begin{array}{l} a(\tau)-b(\tau)+ \\ c(\tau)+d(\tau)+e(\tau) \end{array} \right\} \quad (11)$$

With:

$$a(\tau) = \left\{ \cos\left(\omega_0\sqrt{1-\xi^2}|\tau|\right) \times \left[\frac{\omega_0 e^{-\xi\omega_0|\tau|}\left(1-2\xi^3\omega_0|\tau|+\xi\omega_0|\tau|\right)}{8\xi^3\left(1-\xi^2\right)} \right] \right\},\ b(\tau) = \frac{\omega_0^2|\tau|e^{-\xi\omega_0|\tau|}}{4\xi\sqrt{1-\xi^2}}\sin\left(\omega_0\sqrt{1-\xi^2}|\tau|\right)$$

$$c(\tau) = \frac{\omega_0 e^{-\xi\omega_0|\tau|}}{4\left(1-\xi^2\right)}\sin\left(\omega_0\sqrt{1-\xi^2}|\tau|+2\phi+\phi_1+\pi\right), d(\tau) = \cos\left(\omega_0\sqrt{1-\xi^2}|\tau|+2\phi\right)\left[\frac{-\omega_0 e^{-\xi\omega_0|\tau|}\left(\xi+\omega_0|\tau|\right)}{8\left(1-\xi^2\right)}\right]$$

$$e(\tau) = \sin\left(\omega_0\sqrt{1-\xi^2}|\tau|+2\phi\right) \times \left[\frac{\omega_0 e^{-\xi\omega_0|\tau|}}{8\sqrt{1-\xi^2}}\right],\ \phi_1 = arctg\left(\frac{-\xi}{\sqrt{1-\xi^2}}\right)$$

$C_{\omega\omega}(u) = E\left[\omega(x)\omega(x+u)\right]$ is the autocorrelation function of the homogeneous random field $\omega\ (x)$ and represents the fluctuation of the predominant frequency of the layer around its mean value.

The proposed correlation functions describe the spatial variability in the time domain. However, in the earthquake engineering practices, it is of practical interest to describe the spatial variability in the frequency domain (Laouami and Labbe, 2001). thus, the PSD functions will be derived.

2.3 PSD Functions

By definition, the cross-PSD function is obtained via the Fourier transform of the cross-correlation function, it can be written as

$$S_{xy}(u,\omega) = FT\left[\hat{C}_{xy}(u,\tau)\right] = \int_{-\infty}^{\infty} \hat{C}_{xy}(u,\tau)\exp(-i\omega\tau)d\tau \qquad (12)$$

From eqns (10) and (12) and after lengthy algebraic manipulations, the analytical expression of the cross-PSD function of the ground acceleration is determined as

$$S_{xy}(u,\omega) = \frac{f(\omega)}{g(\omega)} + \frac{C_{\omega\omega}(u)}{p(\omega)} \times \{j(\omega) + k(\omega) + l(\omega) + m(\omega) + n(\omega)\} \qquad (13)$$

With:

$$f(\omega) = \left[\omega_0^2 + \omega^2\right]\cdot\left[\omega_0^2 + \xi\omega_0^2\sin(2\phi + \phi_1 + \pi)\right] + \omega_0^2\sqrt{1-\xi^2}\left[\omega_0^2 - \omega^2\right]\cos(2\phi + \phi_1 + \pi)$$

$$g(\omega) = 2(1-\xi^2)\left[\omega_0^4 + \omega^4 - 2\omega_0^2\omega^2(1-2\xi^2)\right],\ l(\omega) = 2\xi\omega_0^4\sqrt{1-\xi^2}\left(\omega_0^4 - \omega^4\right)\sin(2\phi)$$

$$p(\omega) = 2(1-\xi^2)\left[\omega_0^2 + \omega^2 - 2\omega_0\omega\sqrt{1-\xi^2}\right]^2 \times \left[\omega_0^2 + \omega^2 + 2\omega_0\omega\sqrt{1-\xi^2}\right]^2$$

$$j(\omega) = \omega_0^2\omega^2\left[4\omega_0^2\omega^2(1-\xi^2) + \left(\omega_0^2 + \omega^2\right)^2\right],\ k(\omega) = \omega_0^4\left[-2\xi^2\left(\omega_0^4 + \omega^4\right) + \left(\omega_0^2 - \omega^2\right)^2\right]\cos(2\phi)$$

$$m(\omega) = \xi\omega_0^2\sin(2\phi + \phi_1 + \pi) \times \left[\omega_0^6 + \omega^6 + \omega_0^2\omega^2\left(4\xi^2 - 1\right)\left(\omega_0^2 + \omega^2\right)\right]$$

$$n(\omega) = \omega_0^2\sqrt{1-\xi^2}\cos(2\phi + \phi_1 + \pi) \times \left[\omega_0^6 - \omega^6 + \omega_0^2\omega^2\left(4\xi^2 - 3\right)\left(\omega_0^2 - \omega^2\right)\right]$$

Considering that the auto-PSD of the mouvement is spatially invariant, it is obtained by setting $u = 0$ in the equation (13) or via the Fourier transform of the autocorrelation function, as:

$$S_{xx}(\omega) = \frac{f(\omega)}{g(\omega)} + \frac{\sigma_{\omega\omega}^2}{p(\omega)} \times \{j(\omega) + k(\omega) + l(\omega) + m(\omega) + n(\omega)\} \qquad (14)$$

It is interesting to note that for soil layer with deterministic properties $(\sigma_{\omega\omega} = 0)$, the auto-PSD expression (eqn 14) would be equal to the Kanai-Tajimi spectrum.

2.4. Coherency Function

The spatial variation of seismic ground motions is caused by their apparent propagation on the ground surface and the change in their shape at various locations. It has been recognized that the spatial variation can be described by a function exponentially decaying with separation distance and frequency named coherency function. In general, the total spatial variation of seismic ground motions is composed of terms corresponding to wave passage effects, effects of loss of coherence in the bedrock motion, and site response contribution.

The normalized cross-PSD, namely coherency function of site response contribution to spatial variability is defined mathematically as follows

$$\gamma_{site}(\omega,u)=\frac{S_{xy}(\omega,u)}{S_{xx}(\omega)} \tag{15}$$

The substitution of eqns (13) and (14) into eqn (15) yields the site response contribution:

$$\gamma_{site}(\omega,u)=\frac{H_1(\omega)+C_{\omega\omega}(u)H_2(\omega)}{H_1(\omega)+\sigma^2_{\omega\omega}H_2(\omega)} \tag{16}$$

In which,

$H_1(\omega)=p(\omega)\times f(\omega)$ and $H_2(\omega)=g(\omega)\times\{j(\omega)+k(\omega)+l(\omega)+m(\omega)+n(\omega)\}$

The soil heterogeneity affects the coherence of the seismic motions according to the eqn (16).
Indeed, the surface motion is the result of the superposition of the incident motion at the bedrock-layer interface and the relative motion between the bedrock and the ground surface. Furthermore, it is assumed that the incident motion is independent of reflected and refracted waves from the soil layer. Thus, the total spatial variation is composed of terms corresponding to wave passage effects, bedrock motion coherence effects, and site response contribution (Der Kiureghian, 1996) as follows

$$\gamma_{tot}(u,\omega)=\gamma_{site}(u,\omega)\cdot\gamma_{i\,coh}(u,\omega)\cdot\gamma_{i\,prop}(u,\omega) \tag{17}$$

with: $\gamma_{i\,coh}(u,\omega)$ is the loss of coherence of the incident motions resulting from the scattering of the waves as they travel from the source to the bedrock-layer interface, which can be approximated by the well known model of Luco and Wong (1986) as:

$$\gamma_{i\,coh}(u,\omega)=\exp\left(-(\alpha\omega u/\beta)^2\right)=\exp\left(-\bar{\alpha}^2\omega^2u^2\right) \tag{18}$$

In which, $\alpha=\mu(R/r_0)^{0.5}$, β is an estimate for the elastic shear wave velocity in the random media; r_0 is the scale length of random inhomogeneities along the wave propagation path; μ^2 is a measure of the relative variation of the elastic properties in the medium; and $\bar{\alpha}$ is the coherency drop parameter which controls the exponential decay ratio of the function. The model has been used extensively for the description of the spatial coherence of the surface motions (Luco and Wong, 1986; Zerva, 1992) and for the representation of the coherence of the incident motion at the bedrock-layer interface (Der Kiureghian, 1996; Der Kiureghian and Neuenhofer, 1992).

And $\gamma_{i\,prop}(u,\omega)$ is the apparent propagation of the motion described by (Luco and Wong,1986; Zerva and Harada, 1997; Der Kiureghian and Neuenhofer, 1992):

$$\gamma_{i\,prop}(u,\omega)=\exp\left(-\frac{i\omega u}{c}\right) \tag{19}$$

Where: c is the apparent propagation velocity of the incident motion at the bedrock-layer interface. For a single type of wave dominating the window analyzed, as is most commonly the case for the strong motion S-wave window used in spatial variability evaluations, the consideration that the waves propagate with constant velocity on the ground surface is a valid (Zerva and Zervas, 2002).
From eqns (16), (18), (19) and (17), the total coherency function of the surface motions is rewritten as:

$$\gamma_{tot}(u,\omega)=\frac{H_1(\omega)+C_{\omega\omega}(u)H_2(\omega)}{H_1(\omega)+\sigma^2_{\omega\omega}H_2(\omega)}\cdot\exp\left(-\bar{\alpha}^2\omega^2u^2\right)\cdot\exp\left(-\frac{i\omega u}{c}\right) \tag{20}$$

The amplitude of the function (20) which describes the loss of coherence in the surface motions is:

$$\gamma_{tot}(u,\omega)=\frac{H_1(\omega)+C_{\omega\omega}(u)H_2(\omega)}{H_1(\omega)+\sigma_{\omega\omega}^2 H_2(\omega)}\cdot \exp\left(-\overline{\alpha}^2\omega^2u^2\right) \qquad (21)$$

2.5. Soil Properties Spatial Variability

The parameters of the proposed model $\xi, \omega_0, \sigma_{\omega\omega}$ *and* $C_{\omega\omega}(u)$ depend on the soil properties and need to be estimated. Mean value and standard deviation can be determined using standard techniques. Several methods for estimating the correlation structure from field data are reported in the literature, with the most popular in geotechnical engineering applications being the method of moments, inverse estimation, and maximum likelihood. Apart from these three methods, there are a series of practical procedures which are appropriate for analyzing data provided by in situ soil tests (Assimaki and al, 2003). The mean frequency and the damping ratio are assumed to be equal to the ones reported by Der Kiureghian and Neuenhofer (1992) : $\xi = 0.2$, $\omega_0 = 05\ (rd/s)$ for soft soil conditions.

Zerva and Harada (1997) proposed a soil profile of an example site and determined its properties from a probabilistic analysis of the spatially variable soil characteristics. They considered that the soil layer heterogeneity results from the random variability of thickness of the sub-layers constituent the soil profile. The numerical spatial autocorrelation of soil predominant frequencies $C_{\omega\omega}(u)$ obtained by the authors in their analysis is used herein to produce the behavior of the spatial variability:

$$C_{\omega\omega}(u_k)=\frac{1}{N-k}\sum_{n=1}^{N-k}\left[\frac{\omega^*(x_n+u_k)-\omega_0}{\omega_0}\right]\cdot\left[\frac{\omega^*(x_n)-\omega_0}{\omega_0}\right] \qquad (22)$$

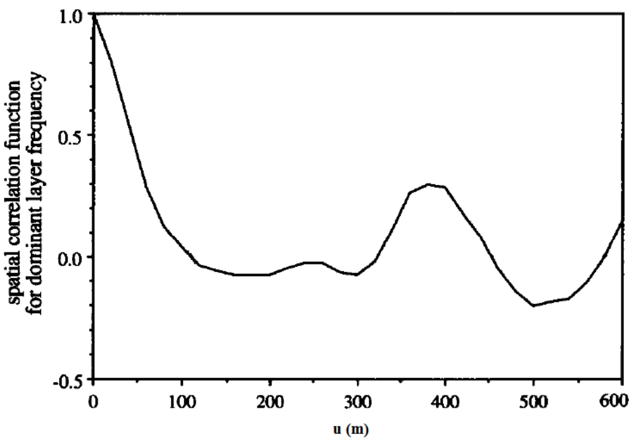

Figure. 2.2. Spatial autocorrelation function for the soil natural frequency estimated by Zerva & Harada (1997).

Concerning the standard deviation $\sigma_{\omega\omega}$, one can take a value of 0.5 for the evaluation of coherency function, i.e., the coefficient of variation CV is equal to 10%. Lumb and Holt (1968) examined the results of sear strenght of soft marine clay from Hong Kong based on 270 test results, they proposed a CV around 20%. The resulting spatial autocorrelation function (eqn 22) normalized by the layer frequency variance is presented in fig. 2.2. Several functions have been used to model the correlation and are, in general, exponentially decaying. Using these soil properties, the PSD and the coherency functions are discussed in the following section.

2.6. Discussion

The effect of soil heterogeneity on the auto-PSD seismic motion (eqn 14) for the soft soil profile at various values of CV is shown in fig. 2.3a. It is observed that the auto-PSD maximum values, concentrated in the vicinity of the mean value of the layer resonant frequency, increase as the coefficient of variation increases too. This indicates that the soil heterogeneity amplifies the incident motion. This result has also been noted by Wang and Hao (2002), who pointed out that random variation of site properties may cause higher amplitude and wider frequency content of the surface ground motion, and by Hadid and Afra (2000), who concluded that the inhomogeneity in the soil profile can cause important amplification. Semblat and his co-workers (2005) denoted that taking into consideration lateral heterogeneities in the numerical modeling of site effects in Volvi basin (EuroSeisTest) may be the reason that the amplification was strong for the medium range frequency.

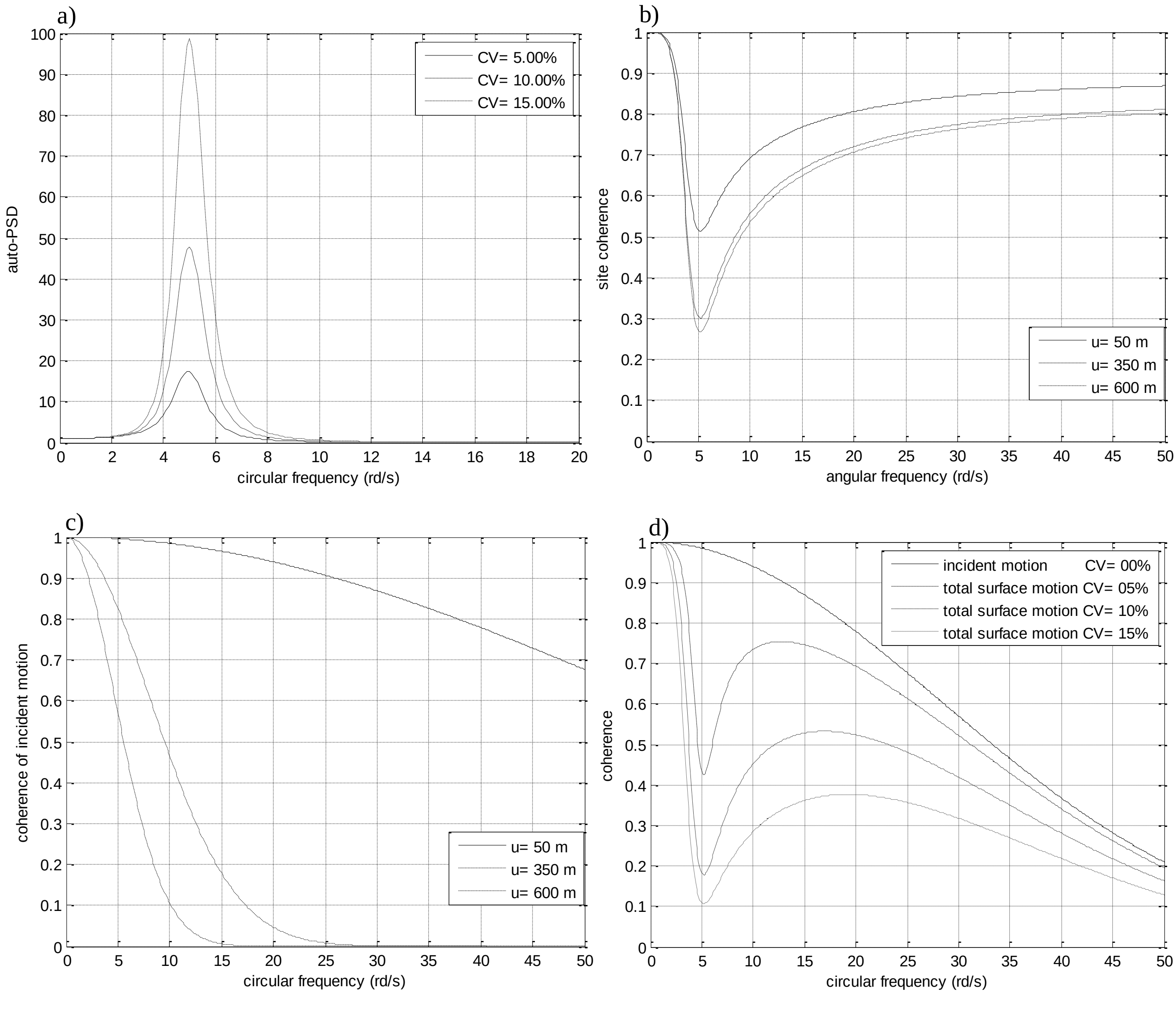

Figure. 2.3. a) Auto-PSD functions of site effect at various values of CV. b) Coherence of site effects at various separation distances. c) Loss of coherence of incident motion at various separation distances. d) Loss of total coherence at various values of CV. u= 100m.

In order to fully gain insight into the effects of incident motion variability and soil layer heterogeneity on the total coherence of the surface motions, they are studied separately. The contribution of soil layer's heterogeneity to the spatial variation (eqn 16) for the soft soil profile at various separation distances is presented in fig. 2.3b. It's observed that the coherency functions decrease abruptly in the vicinity of the first resonant frequency of the layer and maintain a slight upward trend as the frequency increases. The coherency values tend to decrease significantly with the separation distance.

This shape of the coherence is explained by the fact that the heterogeneous soil layer responds to incident motion as a series of single degree of freedom systems with frequencies that fluctuate slightly around ω_0 and are correlated. For exciting frequencies close to the mean resonant frequency of the oscillators, the response of the system is affected by the variability in this eigenvalue which induces a loss of coherence (Zerva and Harada, 1997). Whenever frequency increases past the mean resonant frequency, the effect of this variability on the response of the system is less pronounced.

In the absence of the layer with stochastic properties or if this layer has deterministic properties, the variability of surface motion becomes identical to that of incident motion (eqn 18). The loss of coherence of motion at various separation distances; for a median value $\overline{\alpha} = 2.5\times10^{-4} s.m^{-1}$ from the ones suggested by Luco and Wong (1986); is given in fig. 2.3c. It is worth noting that the loss of coherence is more pronounced at high frequencies for incident motion (fig. 2.3c), in contrary to the site effects where the loss is more pronounced at low frequencies (fig. 2.3b).

The effect of the wave passage on the incident motion shows oneself by deterministic phase difference in the arrival of the waves at various stations (eqn 19) whose value is:

$$\theta(u,\omega) = -\omega.u / c \qquad (23)$$

The loss of the total surface motions coherence (eqn 21) for the soft soil profile is shown in fig. 2.3d. It is shown that the overall trend of the loss of coherence with and without the effect of layer heterogeneity decreases with the frequency. The variability of local site condition tends to cause diminution of the values of the total coherency function; this diminution is notable near the mean resonant frequency of the layer and begins to decrease as the frequency increases. The contribution of soil layer increases with the increase of the coefficient of variation, so, as the heterogeneity of the medium increases, as seismic motions become less similar. Liao (2006) indicated from the spatial variation estimation in the Parkway valley that the station pairs located at different site conditions generally yield the lowest coherency estimates. It is important to underline that the decrease of the coherence is significant even for relatively low values of CV (5 to 15%). The coherence-hole phenomenon in the vicinity of the resonant frequency has been noted by Zerva and Harada (1997), who pointed out that the site stochasticity essentially controls the strains at higher-body wave-apparent propagation velocities for soft soil conditions, and by Liao and Li (2002), who suggested that the possible raison of the non-repetitive observation of coherency drop phenomenon in seismic data recorded at dense instrument arrays is the complex geologic condition underneath the arrays and the complexity of the incoming waves. Such drops in coherency have been observed from analyses of recorded data between two stations by Cranswick (1988) in his geophysics research.

3. CONCLUSION

The consideration of spatial variation of seismic input motion is imperative in seismic design of extended structures. In this paper, the spatial variation of seismic ground motions is quantified taking into account the effects of soil lateral heterogeneity. For this purpose, an analytical coherency function probabilistic model is presented. In this model, the spatial variation of the motions is attributed to the wave passage effects with constant apparent propagation velocity, the effects of loss of coherence in the bedrock motion by means of Luco and Wong's model (Luco and Wong, 1986), and particularly the site-response effects that are based on the assumption of vertical propagation of shear waves through a horizontal layer with random characteristics to the ground surface.

Results indicate that the soil layer heterogeneity tends to amplify the incident motion and causes diminution of the values of the total coherency function; this diminution is notable near the mean resonant frequency of the layer and begins to decrease as the frequency increases. It is important to underline that the decrease of the total coherence is significant even for relatively low values of CV (5 to 15%) and the site effects change significantly the trend of the total coherency function. Also, it's observed that as the heterogeneity of the medium increases, as the site contribution is important and the seismic motions become less similar. The applicability of the presented approach is limited to sites which can be represented by horizontal soil layers and possessing a quasi-regular topography.

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