Uniform Annual Failure Rate Spectra for Friction Pendulum Isolated Structures

J. Luz Rivera-Salas
Universidad Autónoma de Sinaloa

Sonia E. Ruiz
Instituto de Ingeniería, Universidad Nacional Autónoma de México

Alfredo Reyes-Salazar
Universidad Autónoma de Sinaloa

SUMMARY:
An algorithm for the calculation of Uniform Annual Failure Rate spectra corresponding to a simple structure with a Friction Pendulum Isolation System is proposed. The structural failure is defined as the annual exceedance of a target structural base displacement. The maximum seismic response is calculated numerically from the solution of a system of simultaneous equations of motion corresponding to a two-degree-of-freedom system. The main structure is supposed to present linear elastic behavior, while the friction pendulum isolation system undergoes hysteretic behavior under cyclic loads. The isolation system behavior is represented by means of the Bouc-Wen model. The numerical algorithm proposed can be easily systematized in a computer program.

Keywords: Uniform annual failure rate spectra, friction pendulum isolation system, Bouc-Wen model

1. INTRODUCTION

Seismic events occurred in the past have shown that earthquakes with moderate intensity can produce damage and important economical losses due to repair or replacement of buildings contents. Base isolation constitutes a solution to prevent damage in structures or in their contents, due to the decoupling between structure and ground motion (Kelly 1990; Zayas 1987).

One of the most commonly used isolation devices is the Friction Pendulum System (FPS) which consists of a spherical surface of steel coated with teflon and of a component which slides on the surface (Zayas, 1987). One advantage of the FPS, compared for example with the elastomeric system, is that it does not need an extra device to introduce damping, and also that their displacements are limited by the retainers and can not exceed the maximum value; however, a disadvantage is the dependency between the bearing pressure and the kinetic friction coefficient, which is uncertain.

In this study a methodology to calculate Uniform Annual Failure Rate (UAFR) spectra corresponding to a two-degree-of-freedom (2DOF) FPS structure is proposed. Here, the structural failure is defined as the exceedance of a target structural base displacement. Maximum response values are calculated numerically from the solution of a system of simultaneous equations (which correspond to the main structure and to the FPS). It is assumed that the main structure presents linear elastic behavior, while the isolation system presents nonlinear behavior. The structural cyclic behavior of the FPS is represented by means of the Bouc-Wen model (Bouc 1967, Wen 1976, Morgan and Mahin 2011). An illustrative example is presented.

2. NONLINEAR DYNAMIC ANALYSIS OF A TWO-DEGREE-OF-FREEDOM SYSTEM WITH VISCOUS DAMPING

The model of 2DOF system (one degree corresponding to the isolator device and the other to the structure) with viscous damping can be represented as shown in Figure 1 (Kelly, 1990):

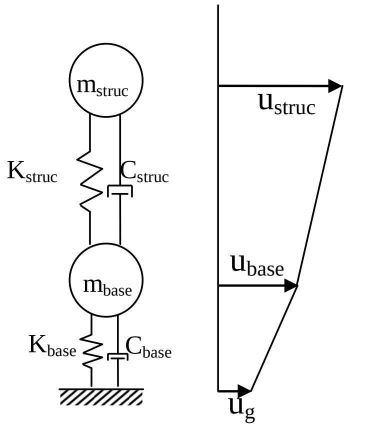

Figure 1. Two-degree-of-freedom system used to represent the isolator and the structure

where m, C, and K represent the mass, the viscous damping and the stiffness, respectively, and u_g, u_{base}, and u_{struc} are the relative displacements of the ground, the isolator and the structure, respectively.

The equation of motion of the structure-isolator system shown in Figure 1 is:

$$m_{struc}\ddot{u}_{struc} + m_{base}\ddot{u}_{base} + c_{base}(\dot{u}_{base} - \dot{u}_g) + k_{base}(u_{base} - u_g) = 0 \tag{2.1}$$

$$m_{struc}\ddot{u}_{struc} + c_{struc}(\dot{u}_{struc} - \dot{u}_{base}) + k_{struc}(u_{struc} - u_{base}) = 0 \tag{2.2}$$

Next, we define the following relative displacements:

$$v_{struc} = u_{struc} - u_{base} \tag{2.3}$$

$$v_{base} = u_{base} - u_g \tag{2.4}$$

and then substituting into equation (2.1), it is obtained:

$$m_{struc}\ddot{v}_{struc} + (m_{base} + m_{struc})\ddot{v}_{base} + c_{base}\dot{v}_{base} + k_{base}v_{base} = -(m_{base} + m_{struc})\ddot{u}_g \tag{2.5}$$

Similarly, substituting equation (2.3) in (2.2):

$$m_{struc}\ddot{v}_{struc} + m_{struc}\ddot{v}_{base} + c_{struc}\dot{v}_{struc} + k_{struc}v_{struc} = -m_{struc}\ddot{u}_g \tag{2.6}$$

Dividing equation (2.5) between ($m_{base} + m_{struc}$) and equating to zero, results:

$$\frac{(m_{base} + m_{struc})\ddot{u}_g}{(m_{base} + m_{struc})} + \frac{m_{struc}}{(m_{base} + m_{struc})}\ddot{v}_{struc} + \frac{(m_{base} + m_{struc})}{(m_{base} + m_{struc})}\ddot{v}_{base} +$$

$$\frac{c_{base}}{(m_{base} + m_{struc})}\dot{v}_{base} + \frac{k_{base}}{(m_{base} + m_{struc})}v_{base} = 0$$

From the above equation, it is obtained:

$$\ddot{v}_{base} = -\left[\ddot{u}_g + \frac{m_{struc}}{m_{base} + m_{struc}}\ddot{v}_{struc} + \frac{c_{base}}{m_{base} + m_{struc}}\dot{v}_{base} + \frac{k_{base}}{m_{base} + m_{struc}}v_{base}\right] \tag{2.7}$$

Following the same procedure with equation (2.6) it is obtained:

$$\frac{m_{struc}}{m_{struc}}\ddot{u}_g + \frac{m_{struc}}{m_{struc}}\ddot{v}_{struc} + \frac{m_{struc}}{m_{struc}}\ddot{v}_{base} + \frac{c_{struc}}{m_{struc}}\dot{v}_{struc} + \frac{k_{struc}}{m_{struc}}v_{struc} = 0$$

$$\ddot{v}_{struc} = -\left[\ddot{u}_g + \ddot{v}_{base} + \frac{c_{struc}}{m_{struc}}\dot{v}_{struc} + \frac{k_{struc}}{m_{struc}}v_{struc}\right] \quad (2.8)$$

3. COULOMB DAMPING DEVELOPED IN THE SIMPLE FRICTION PENDULUM ISOLATION SYSTEM

3.1. Coulomb damping

The simple FPS is composed of a slider that rests on a spherical surface of polished stainless steel. The surface is usually coated with teflon. Figure 2 shows the simple FPS device, and Figure 3 shows its cross section and the forces (*F*, *W*, *N* and F_f) acting on it (Symans, 2008).

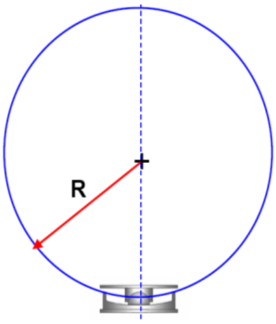

Figure 2. Radius of curvature for a typical FPS

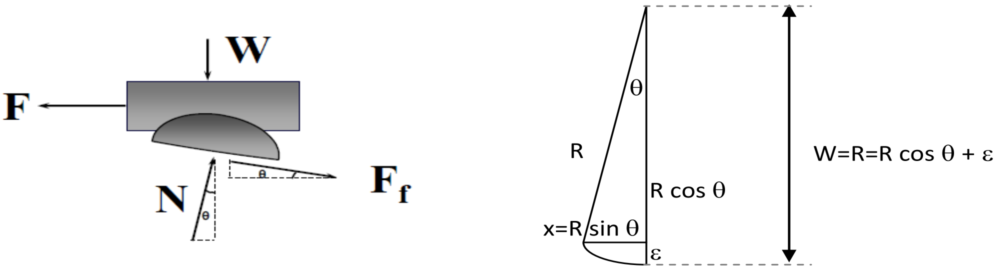

Figure 3. Balance of forces and displacement configuration

From Figure 3, it is obtained:

$$\begin{aligned} F_f &= -\mu W \quad if \quad \dot{x} > 0 \\ F_f &= -F \quad\;\; if \quad \dot{x} = 0 \end{aligned} \quad (3.1)$$

where $W = m.g$, W is the weight, m is the mass, g is the acceleration of gravity, and μ is the coefficient of kinetic friction.

Considering the forces acting on the horizontal axis (x), we have the following:

$$-F + W\tan\theta + F_f\cos\theta = 0 \quad (3.2)$$

We know that $\sin\theta = \frac{x}{R}$ and $\cos\theta = \sqrt{1-\left(\frac{x}{R}\right)^2}$

Simplifying, we obtain: $\cos\theta = \left[\frac{1}{1} - \frac{x^2}{R^2}\right]^{1/2} = \left[\frac{R^2 - x^2}{R^2}\right]^{1/2} = \sqrt{R^2 - x^2}\left(\frac{1}{R}\right)$. Repeating the same process results $\tan\theta = \frac{x}{\sqrt{R^2 + x^2}}$.

Substituting the foregoing values and the F_f value (equation 3.1) in equation (3.2), we get:

$$F = \frac{Wx}{\sqrt{R^2 + x^2}} + \mu W \operatorname{sgn}(\dot{x})\left(\frac{1}{R}\sqrt{R^2 - x^2}\right) \tag{3.3}$$

If x<<R then $\sqrt{R^2 - x^2} \approx R$ and $\sqrt{R^2 + x^2} \approx R$. From these approximations we obtain:

$$F = \frac{Wx}{R} + \mu W \operatorname{sgn}(\dot{x})\left(\frac{R}{R}\right) = \frac{Wx}{R} + \mu W \operatorname{sgn}(\dot{x}) \tag{3.4}$$

where R is the radius, μ is the coefficient of friction, x and $\operatorname{sgn}(\dot{x})$ is the displacement and signum function of the velocity, respectively. As shown in equation (3.4), the restoring force of the pendulum is composed of two parts: one associated with weight *W*, and the other associated with the frictional forces developed by the isolator system.

The stiffness of the friction pendulum bearing is given by the following equation (Zayas and Low, 2000):

$$K = \frac{W}{R} = \frac{mg}{R} \tag{3.5}$$

Substituting equation (3.4) in (2.7), we have the following expression:

$$\ddot{v}_{base} = -\left[\ddot{u}_g + \frac{m_{struc}}{m_{base} + m_{struc}}\ddot{v}_{struc} + \frac{\mu W}{m_{base} + m_{struc}}\operatorname{sgn}(\dot{v}_{base}) + \frac{K}{m_{base} + m_{struc}} v_{base}\right]$$

Substituting equation (3.5) in the above equation, it is obtained:

$$\ddot{v}_{base} = -\left[\ddot{u}_g + \frac{m_{struc}}{m_{base} + m_{struc}}\ddot{v}_{struc} + \frac{\mu W}{m_{base} + m_{struc}}\operatorname{sgn}(\dot{v}_{base}) + \frac{g}{R} v_{base}\right] \tag{3.6}$$

Equation (3.6) is the linear equation considering that the isolator displacements are smaller than 0.2R, which guarantees an error smaller than 2% (Symans 2008, Morgan and Mahin 2011).

3.2. Coefficient of friction

One of the main disadvantages of the friction isolation bearings lies in the uncertainty implicit in the coefficient of friction μ . Constantinou et al. (1990) found that the friction coefficient is a function of the maximum as well as of the minimum velocity. FEMA 450 (2003) considers a constant value of μ

over time in designing the sliding isolation system, and proposes an equivalent damping constant value. In this study it is considered that the coefficient μ depends on the velocity, and is given by (Constantinou et al. 1990):

$$\mu = \mu_{\max} - (\mu_{\max} - \mu_{\min}) \exp(-b|\dot{v}|) \tag{3.7}$$

where $\dot{v}$ is the velocity and *b* is a constant that is considered equal to 1.524 in this study.

4. SYSTEM OF SIMULTANEOUS EQUATIONS OF MOTION

4.1. Bouc-Wen model

In the following it is considered that the total restoring force is a nonlinear hysteretic function represented by the inertial forces and the restoring force, which depends on the ground acceleration $\ddot{u}_g(t)$:

$$m\ddot{u}(t) + q(\dot{u}, u, t) = -m\ddot{u}_g(t) \tag{4.1}$$

where $q(\dot{u}, u, t)$ is the restoring force, *u* is the relative displacement of the system and the dot indicates derivative with respect to time *t*.

The restoring force can be decomposed into two parts (Wen, 1976): a non-hysteretic and a nonlinear hysteretic component $q(\dot{u},u,t) = g(\dot{u},u) + h(\dot{u})$. A general representation of the hysteretic restoring force (Constantinou et al. 1990) is μWz in which *z* is the hysteretic component that in this work is considered to be nonlinear (see equation 3.7).

The hysteretic component can be represented by means of different mathematical models. One of them is the Bouc-Wen model (proposed by Bouc 1967 and modified by Wen 1976) represented by the following mathematical expression:

$$\dot{z} = \frac{\Gamma_3 \dot{v} - \upsilon_s\left(\Gamma_4 z|\dot{v}|\,|z|^{\Gamma_6 - 1} + \Gamma_5 \dot{v}|z|^{\Gamma_6}\right)}{\eta} \tag{4.2}$$

where Γ_3, υ and η are parameters that consider the damage; Γ_4 and Γ_5 are parameters that depend on the physical properties, and Γ_6 takes into account the transition between elastic and inelastic behavior.

4.2. System of simultaneous equations of motion

The system of equations that governs the two-degree-of-freedom FPS is presented in this section. It is assumed that the structure presents linear behavior and the isolator has nonlinear behavior. The following change of variables is adopted in the formulation: $y_1 = v_{base}, y_2 = \dot{v}_{base}, y_3 = z_{base}, y_4 = v_{struc}, y_5 = \dot{v}_{struc}$.

The resulting system of simultaneous equations of motion is:

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = -\left[\ddot{u}_g + \frac{m_{struc}}{m_{base}+m_{struc}}\dot{y}_5 + \frac{\mu W}{m_{base}+m_{struc}} y_3 + \left(\frac{g}{R}\right) y_1\right]$$

$$\dot{y}_3 = \Gamma_{3,base} y_2 - \upsilon_{base}\left(\Gamma_{4,base} y_3 |y_2| |y_3|^{\Gamma_{6,Ibase}-1} + \Gamma_{5,Ibase} y_2 |y_3|^{\Gamma_{6,base}}\right)/\eta \qquad (4.3)$$

$$\dot{y}_4 = y_5$$

$$\dot{y}_5 = -\left[\ddot{u}_g + \dot{y}_2 + 2\xi_{struc}\omega_{struc} y_5 + \omega^2_{struc} y_4\right]$$

5. ILLUSTRATIVE EXAMPLE

In order to verify the applicability of the system given by equations (4.3), a simple example was developed. It corresponds 2DOF system with the following properties: T_{struc}= 1s, W_{struc}=1.526 ton, ξ_{struc}= 5%, $\Gamma_{2,base}$= 0.06, $\Gamma_{3,base}$ =1, $\Gamma_{4,base} = \Gamma_{5,base}$=0.5, $\Gamma_{6,base}$ = 15. The structure was subjected to the accelerogram recorded on 19 September 1985 in Mexico City in Viveros station. The hysteretic cycles of the structure and of the isolating system are shown in Figure 4. The isolator permits a maximum static load of 8000 kN assuming a curvature R=4m, and the maximum displacement permitted in any direction is 0.5 m (www.fip-group.it).

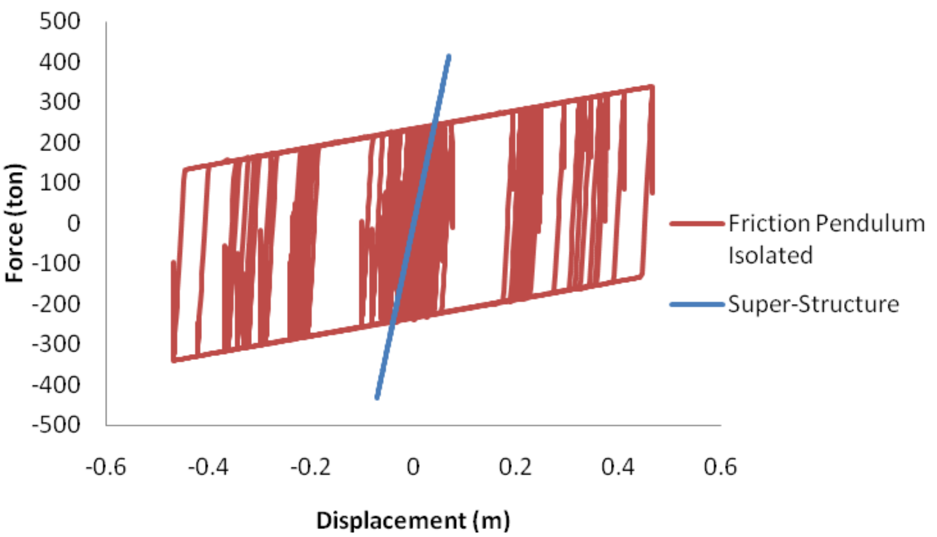

Figure 4. Hysteretic cycles

6. CALCULATION OF UNIFORM ANNUAL FAILURE RATE (UAFR) SPECTRA

Equations (4.3) can be used for different objectives. One of them is to calculate uniform annual rate (UAFR) spectra corresponding to structural systems with FPS, and another is to use the UAFR spectra for reliability-design purposes.

The algorithm proposed here to calculate the demand hazard curves and, from them, the uniform annual rate (UAFR) spectra corresponding to structural systems with FPS is inspired in that proposed by the authors (Rivera and Ruiz 2007) for structures with energy dissipating devices. The steps to follow are:

1. Values of the following parameters corresponding to the structure-isolator system are proposed: $R, m_{struc}, m_{base}, K_{struc}, K_{base}, \mu_{max}$ and μ_{min} and $\Gamma_{4,base} = \Gamma_{5,base} = 0.5$.
2. The values of the stiffness K_{base} is calculated
3. Each combined 2DOF system is subjected to a different accelerogram (recorded or simulated), scaled so that the spectral acceleration associated with the fundamental period of the system under study corresponds to a specified return interval (T_R) (Shome and Cornell 1995). The ratio between the spectral acceleration value and the inverse of the return interval is given by the site seismic hazard curve, which is assumed to be known.

4. The peak system base displacement (d_i) and the peak acceleration (a_i) at the top of the system, corresponding to the *i*-th accelerogram are calculated.

5. Structural failure of the 2DOF isolating system occurs when displacement in the isolating system is larger than d_a = target displacement. That is, when $d_i / d_a = Q_i \geq 1$. The annual structural failure rate is evaluated by means of (Esteva and Ruiz 1989):

$$\nu_F = \int \left| \frac{d\nu}{d\nu_y} \right| P\left(Q \geq 1 | y\right) dy \tag{6.1}$$

   where $\left| d\nu / d\nu_y \right|$ it is the absolute value of the derivative of the site seismic hazard curve (which is assumed to be known), and $P(Q \geq 1 | y)$ is the conditional probability that the structural failure occurs, given a seismic intensity y.
6. The integral is evaluated numerically for different values of the parameters. With the results, the demand hazard curves associated with 2DOF combined systems with different vibration periods are determined.

7. The uniform annual rate (UAFR) spectra are drawn on the basis of the demand hazard curves associated with different structural vibration periods.

7. RESULTS

In the following some results of the example mentioned above are shown. Figure 5 presents the probability $P\left(Q \geq 1 | y\right)$ of exceeding limiting displacements *da* = 0.1R, 0.15R and 0.2R, for different intensity values Sa/g, and in Figure 6 shows the base displacement hazard curve corresponding to the structure having structural period T_{struc} = 1s .

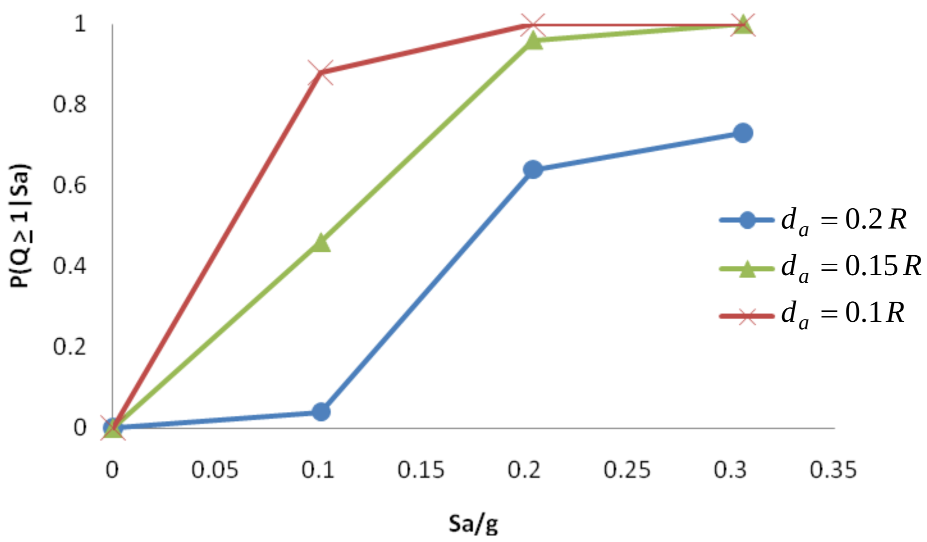

Figure 5. Vulnerability functions corresponding to different target displacements for T_{struc}=1 s.

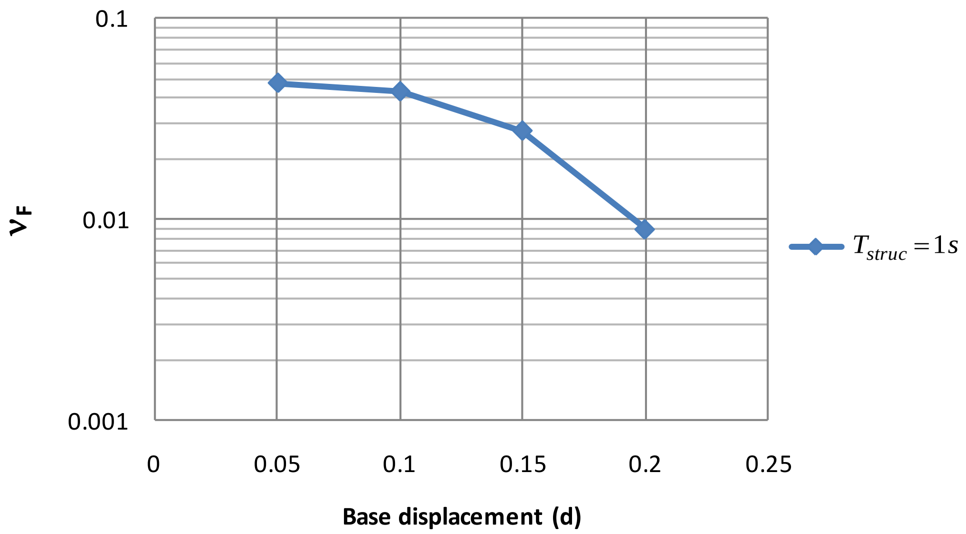

Figure 6. Demand hazard curve associated with the base displacement.

8. CONCLUSIONS

A system of simultaneous equations of motion that governs the dynamic response of a friction pendulum isolated structural system, represented by means a two-degree-of-freedom system and using the Bouc-Wen model, is presented.

The numerical algorithm proposed above for obtaining demand hazard curves and, from these, the uniform annual failure rate (UAFR) spectra for FPI systems can be easily systematized in a computer program.

The UAFR spectra corresponding to FPS can be useful for reliability-based design purposes.

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