Binomial logistic regression model for probabilistic assessment of failure of reinforced concrete beam-column joints subjected to seismic action

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SUMMARY:
A binomial logistic regression model has been developed to identify the failure initiation response mechanism of interior and exterior reinforced concrete beam column joints subjected to seismic action. The probabilistic model having good predictive efficiency is developed based on geometric, material and loading data of the experimental investigations. The simple to use model also quantitatively determines the probabilistic influence of each of the design parameters in response determination and can be utilised as an effective tool for both new construction as well as post-mortem analysis of existing damaged structures.

Keywords: probabilistic model, reinforced concrete, beam column joints, binomial logistic regression.

1. INTRODUCTION

ACI Code (2002, 2008) requirements are intended to ensure that connection response is determined by a ductile failure mechanism associated with flexural yielding of beam reinforcements and that connection strength is determined by beam flexural strength. These requirements include a minimum volume of transverse reinforcement, a minimum anchorage length for beam longitudinal reinforcement ratio, a minimum column-to-beam flexural strength ratio and a limit on the joint shear stress demand. However, some important questions are left unanswered by these ACI recommendations:

- *Probabilistic estimation* of failure initiation mechanism of reinforced concrete beam-column (RCBC) connections subjected to seismic loads.
- *Quantification of design parameters* that results in development of inelastic mechanisms leading to initiation of failure (strength degradation) in RCBC connections.
- *Recommendations* for structures designed with *high strength concrete*. Recently high strength concrete (compressive strength of concrete above 59 MPa (8.56 ksi), as specified by ACI-363 committee 1997 report) has been successfully utilized for cast-in-place concrete buildings and high-rise structures, which enables structural designers to design more slender reinforced concrete members (Sanada and Maruta 2004). Research studies by Noguchi *et al.* (1994) report that these smaller column sections with high strength concrete increases the potential of joint failure prior to beam yielding.
- *Recommendations* for structures designed with *high strength reinforcing steel*. Recently high strength reinforcing steel has also been used in recent constructions to prevent congested detailing of reinforcing bars in slender members. Research studies by Fujii and Morita (1991) conclude that high strength reinforcing steel in beams passing through joints prevents flexural yielding of the bars and thereby might result in development of joint failure prior to flexural yielding of longitudinal beam bars.

Apart from new constructions, joint failures (Pessiki *et al.* 1990, Lehman *et al.* 2004) have also been observed in old joints (joints designed prior to 1967) which typically have significantly low amount of joint transverse reinforcement, and thereby do not comply with the modern ACI Code (2005). Thus, in evaluating, retrofitting existing structures and designing new structures, it is appropriate to assess the potential for beam-column connections to exhibit different failure initiation mechanism response as well as the impact of design parameters on the connection response. The experimental investigations have revealed that inelastic mechanisms resulting in initiation of failure (strength degradation) in the connections of strong-column-weak-beam frames subjected to seismic loading are either yielding of longitudinal reinforcement in beams adjacent to the joint region or failure within the joint region prior to longitudinal steel yielding which is characterized by diagonal cracking of the joint region along with concrete spalling in the joint. It should be noted that the exact cause of joint failure prior to beam longitudinal bar yielding is still an open question; even though many researchers conclude this as shear failure based on visual observation from experimental investigations, it is yet to be proved through rigorous numerical simulations involving three-dimensional nonlinear finite element analysis considering the effects of concrete cracking, crushing and bond slip between the reinforcement bar and concrete. Specifically for a joint region, which is significantly different from shear panels analyzed by Vecchio and Collins (1986), the diagonal cracking can either be a result of shear failure or be a result of surfacing of radial cracks originating from slippage of beam bars through the concrete. It has been shown by several researchers (Tepfers 1979; Lundgreen and Gylltoft 2000) that apart from a slippage of the beam bars in concrete due to tangential forces, radial cracks are also observed due to bond slippage at an angle of 30 to 60 degrees to the direction of the bar around the entire circumference of the bar. It may be that diagonal cracking in joint and eventual concrete spalling is a result of surfacing of these radial cracks originating due to bond slippage between the reinforcement and concrete. Thereby, to properly characterize a connection response, it is necessary to classify them based on failure initiation rather than mechanism observed at or after failure.

A methodology is thereby developed in this manuscript which is simple, can be easily applied, is computationally efficient with which an engineer will be able to 1) identify probabilistically the inelastic mechanisms that might lead to catastrophic failure of reinforced concrete beam columns joints subjected to earthquake loading and 2) quantify the effect of different parameters which affects the failure initiation response. Based on this simple probabilistic model, the engineer may request the need for further investigation using more sophisticated and time consuming non-linear finite element analysis (such as Mitra and Lowes 2007). Readers are referred to papers by Mitra (Mitra 2012, Mitra and Samui 2011, Mitra et al. 2011, Kang and Mitra 2012) for an in-depth coverage of different types of probabilistic models developed in this regard.

2. EXPERIMENTAL DATASET AND MODEL PARAMETERS

An extensive dataset of experimental investigations of 110 interior and 49 exterior reinforced concrete beam column joints, which has been considered in the development of the probabilistic model, is presented in Mitra (2012). The dataset is limited to only two-dimensional connections in which a continuous column connects a beam and the specimen response is determined by flexural yielding of beams at the beam-joint interface and/or joint failure characterized by diagonal cracking in joint followed by concrete spalling. Too few tests were found in the literature in which connection failure was determined by column hinging to enable use of these data in the analysis. To improve the accuracy of the model, connections with slabs, eccentric beams (axis of the beam and column are not aligned), or out-of-plane beams were not included in the data set. Test specimens with plain round (smooth) reinforcing steel bars were also eliminated. Since too few tests were found in literature, the dataset does not include specimen in which beam-hinging effect has been shifted from the beam-column joint interface to a distance away from the interface and also experimental investigations in which column axial load was varied during the test. All specimens were subjected to quasi-static, cyclic loading to develop load distributions that are representative of those that develop in a frame under seismic loading.

2.1. Model parameters

The *nominal joint shear stress*, τ, is defined as the shear stress in the joint when beams reach nominal flexural strength on either side of the joint, normalized by the square root of the concrete compressive stress, as shown in Eqn 2.1.1:

$$\tau = \frac{1}{\sqrt{f_c} h_c b_j} \left(\frac{M_L + M_R}{h_b} - V_c \right) \tag{2.1.1}$$

where h_c is the height of the column, b_j is the maximum out of plane dimension of the beam or column, h_b is the height of the beam, M_L and M_R are the flexural strengths of the beam on the right and left of the joint computed in accordance with ACI 318-08, and V_c is the lateral load applied to the top of the column at the nominal strength of the beams. For exterior beam-column joints either M_L or M_R is taken as 0. The definition of joint shear stress is similar to that recommended by ACI Com. 352 with the exception that it 1) employs a slightly larger joint volume with the result that horizontal and vertical shear stresses are equal and 2) defines demand on the basis of frame member flexural strengths rather than longitudinal steel areas with the result that the determination of the frame member moments and column shear is consistent.

The *bond stress demand*, μ, is characterized using the bond index, proposed by Kitayama *et al.* (1987), as shown in Eqn 2.1.2

$$\mu = \frac{f_y \, d_b}{\alpha \sqrt{f_c} h_c} \tag{2.1.2}$$

where f_y is the actual yield strength of the beam reinforcement, d_b is the beam bar diameter, and h_c and f_c are as defined previously. The bond index is the maximum bond stress demand within the joint, normalized by the square root of the concrete compressive strength. A factor of α equal to 2 is applied for interior joints assuming that beam steel yields in tension and compression on opposite sides of the joint, whereas a factor of α equal to 1 is applied for exterior joints assuming that beam steel yields in tension on the side of the joint containing the beam.

For the current study, *transverse reinforcement ratio*, Φ, is included in the model as the total joint transverse steel force assuming yielding of the transverse steel normalized by the nominal joint shear force demand at beam reinforcement yielding, as shown in Eqn 2.1.3:

$$\Phi = \frac{A_{st\_T} \, f_{yt}}{\tau \sqrt{f_c} h_c b_j} \tag{2.1.3}$$

where A_{st_T} is the total area of joint transverse reinforcement passing through a plane normal to the beam axis, f_{yt} is the actual yield strength of joint transverse reinforcement, and τ , f_c, h_c and b_j are as defined previously.

The *column axial load ratio*, *p*, is included in the model, as shown in Eqn 2.1.4:

$$p = \frac{P}{A_g \, f_c} \tag{2.1.4}$$

where P is the column axial load, A_g is the gross cross-sectional area of the column and f_c is as defined previously.

The *beam top to bottom steel strength ratio* parameter, ϖ , was considered as a prospective factor in the development of the model, shown in Eqn 2.1.5 as:

$$\varpi = \frac{\left(n.f_y.A_s\right)_{bt}}{\left(n.f_y.A_s\right)_{bb}} \tag{2.1.5}$$

where the subscript $(.)_{bt}$ refers to the top longitudinal beam bars and subscript $(.)_{bb}$ refers to the bottom, n is the number of bars, f_y is the actual yield strength of the steel, and A_s the cross-sectional area of a single bar. It should be noted that the probable influence of the parameter on failure mechanism of the joint was first pointed out by Ichinose (1987) and Fujii and Morita (1991). If there is a significant difference in the strength of top and bottom steel, cyclic loading will result in significantly different steel and concrete stress-strain histories on the top and bottom of the beam and, thus, significantly different joint boundary and loading conditions. On the side of the beam with smaller steel strength, cyclic loading will result in yielding of steel in compression as well as tension, minimal accumulated plastic strain, and likely the closing of concrete cracks. This could be expected to result in premature deterioration of beam flexural strength. On the side of the beam with greater steel strength, cyclic loading will result in accumulated tensile strain in the steel, progressive widening of concrete cracks, and no closing of cracks under compressive loading. This could be expected to impact the formation of a concrete compression strut within the joint as well as increase yield penetration into the joint.

Another important parameter considered for the model development was the *aspect ratio of the joint*, ξ, which is defined as the ratio of the height of the beam section, h_b , to the height of the column section, h_c. The concept that aspect ratio may influence the behavior of a connection was first identified out by Pantazopoulou and Bonacci (1994) in which the authors suggested that the difference in behavior observed in experimental investigations from different countries might have a correlation with local design practice associated with increasing the dimension of connections to prevent congestion of transverse steel. Aspect ratio parameter was also considered by Kim and LaFave (2008) in developing simplified relations for joint shear strength based on Bayesian methodology. ACI doesn’t provide any limitations as to how much the increase can be, however logically this increase could have an impact in connection response mechanism (Pantazopoulou and Bonacci 1992).

2.2. Response parameters

For the current study, specimens are considered to exhibit either i) **J** : joint failure prior to beam yielding if connection strength is not sufficient to develop beam yield strength and the connection response is controlled by the response of the joint, ii) **B**: beam yielding prior to that of joint failure if the connection strength is sufficient to develop beam yield strength. Beam yield strength was defined by first yield of the beam reinforcement, in either positive or negative bending. Beam yield strength was computed by performing a moment curvature analysis of a fiber-type discretization of the beam section in which concrete fibers were modeled based on modified Kent-Park model and steel fibers were modeled by a bilinear steel hardening response. A detailed classification of all joints considered in the dataset can be obtained from Mitra (2012).

3. BINOMIAL LOGISTIC REGRESSION METHODOLOGY

The logistic models (Hosmer and Lemshow 2000; Agresti 2007) employ a regression relationship between independent quantitative variables and discrete qualitative events. In the present model, the two discrete qualitative events are “joint failure prior to beam yielding” (referred to as *Event* 1) and “beam yielding prior to joint failure” (referred to as *Event* 0). The likelihood of observing a discrete event of brittle joint failure prior to beam yielding is defined by the log of the odds ratio for that event. The odds ratio for *Event* 1 is the ratio of the probability of occurrence of *Event* 1, $P_{E=1}$, to the probability of occurrence of *Event* 0, $P_{E=0}$. Thus,

$$Y = \log\left(\frac{P_{E=1}}{1-P_{E=1}}\right) = \log\left(\frac{P_{E=1}}{P_{E=0}}\right) = \beta_0 + \sum_{k=1}^{K} \beta_k X_k \quad (3.1.1)$$

where β_i are logistic regression parameters, X_i are the covariates or quantitative joint design parameters and K is total number of design parameters considered. The method of maximum likelihood (Ben-Akiva and Lerman 1985), which provides a means of choosing an asymptotically efficient estimator for a set of parameters, is typically used to compute logistic regression parameters, β_i. As described in Ben-Akiva and Lerman (1985), Eqn 3.1.1 may be manipulated to define the probability of occurrence of *Events* 1 and 0 by Eqns 3.1.2 and 3.1.3 respectively as:

$$P_{E=1} = \frac{e^{\beta_0 + \sum_{k=1}^{K} \beta_k X_k}}{1 + e^{\beta_0 + \sum_{k=1}^{K} \beta_k X_k}} \quad (3.1.2)$$

$$P_{E=0} = \frac{1}{1 + e^{\beta_0 + \sum_{k=1}^{K} \beta_k X_k}} \quad (3.1.3)$$

4. RESULT DISCUSSION

Since behaviors of interior and exterior connections subjected to seismic load are significantly different (Paulay and Scarpas 1981), two different logistic regression models, one each for interior and exterior beam-column connections were developed. For each of the independent variable in the two models, Table 4.1(a) shows the computed regression parameters, β_i, as well as statistical parameters for use in model evaluation for the case of interior connections, whereas Table 4.1(b) shows the values for exterior connections. It should be noted that by obtaining the logistic regression parameters β_i, one can use Eqns 3.1.2 and 3.1.3 to evaluate the probability of occurrence of Event 1 and Event 0 respectively. Tables 4.1(a) and 4.1(b) also show the significance of the independent design variables by computing *t-statistic* and *p-values*. The *t-statistic* determines whether the coefficient of a particular parameter is significantly different from zero, i.e. whether the null hypothesis H_0: $\beta_i = 0$ can be rejected or not, and the *p-value* provides the probability at which the null hypothesis can not be rejected (Greene 2000). The *p-value* represents the smallest level of significance γ that leads to the rejection of the null hypothesis. If the *p-value* is more than or equal to γ, then the null hypothesis is rejected. Based on the wide variation of the dataset, in this investigation the value of γ is assumed to be 0.10; or in other words a significance level of 10% was chosen (i.e. at 90% confidence level) for this investigation. In other words, the *p-value* and *t-statistic* are interrelated, the smaller the *p-value*, the larger is the value of *t-statistic* and more significant is the parameter. In any statistical study, a question can be raised with regards to the model being biased on the data and more data are required to get a good prediction. However, for the present study, the research work of Peduzzi *et al.* (1996) and page 138 of Agresti (2007) should be highlighted in which it has been recommended that sample size required for estimating a logistic regression function should have atleast 10 cases per independent variable. Since the total number of available samples for interior beam-column joints is 110, a maximum of 11 independent variables can thereby be considered; and since total number of available samples for exterior beam-column joints is 49, a maximum of 4 independent variables can be considered.

It should be highlighted that based on *p-values* and *t-statistics* from Tables 4.1(a) and 4.1(b), all design variables selected for development of the probabilistic model for the interior and exterior joint are each statistically significant at least at 10% level (i.e. at 90% confidence level). Hence, the null hypothesis that each of these variables has no influence on determination of failure initiation mechanism can be rejected and the relationship between the dependent variable and each of the independent variables cannot be attributed by chance.

Table 4.1(a). Interior connection model results

covariate	Estimated β	t-stat	p-value	Influence factor
τ	5.24	3.24	0.0012	4.19
μ	2.82	3.39	0.0007	5.42
φ	-3.13	-2.08	0.0375	-1.97
p	-16.16	-2.52	0.0118	-2.1
ξ	11.87	3.56	0.0004	12.86
constant	-20.32			

Table 4.1(b). Exterior connection model results

covariate	Estimated β	t-stat	p-value	Influence factor
τ	42.35	2.35	0.0187	23.36
μ	-3.78	-1.92	0.0544	-16.97
ϖ	13.75	2.57	0.0101	14.58
p	-19.05	-2.2	0.0277	-2.45
constant	-21.72			

5. IMPACT OF DESIGN PARAMETERS ON CONNECTION RESPONSE

The computed regression parameters (β_i) in Tables 4.1(a) and 4.1(b) provide insight into the impact of individual design parameters on interior and exterior connection response respectively. Given the definition of Y in Eqn 3.1.1, the sign of a regression parameter (β_i) indicates whether an increase in the associated design parameter increases or decreases the likelihood of joint failure prior to beam yielding. A positive (negative) regression parameter indicates that increasing the associated design parameter increases (decreases) the likelihood of a joint failure prior to beam yielding. Similarly, a negative (positive) regression parameter indicates that increasing the associated design parameter increases (decreases) the likelihood of beam yielding prior to joint failure. The magnitude of a regression parameter multiplied with the mean of its corresponding design variables [“influence factor” column in Table 4.1(a) and 4.1(b)] indicates the relative importance of the design variables in determining connection failure initiation response. It should be noted that the sign of the influence factor is similar to the sign of the regression parameter and presents similar conclusions as that of the regression parameter. The following paragraphs discuss the influence of each parameter on the failure initiation mechanism within the beam-column joint region.

5.1. Nominal joint shear stress demand

Tables 4.1(a) and 4.1(b) demonstrates that higher shear stress demands results in an increased probability of brittle joint failure prior to beam yielding for both interior and exterior beam-column connections. However, the magnitude of the influence factor in Table 4.1(a) indicates that nominal joint shear stress is not the most critical design parameter in determining the likelihood of joint failure prior to beam yielding in an interior connection but is instead significantly less important than either bond stress demand or aspect ratio of the joint. On the other hand, the influence factor magnitudes in Table 4.1(b) suggest that nominal joint shear stress is the most critical parameter in determination of failure initiation mechanism of exterior beam-column connections. Based on *p-value* statistic, this variable was observed to be statistically significant at 0.12% (i.e. at 99.88%confidence level) for interior joints and 1.8% (i.e. at 98.2% confidence level) for exterior joints.

5.2. Beam bar bond stress demand

Analysis results [Table 4.1(a)] indicate that increased beam bar bond stress demand results in increased likelihood of joint failure prior to beam yielding determining interior connection response. On the other hand, based on Table 4.1(b) for exterior connection response, it was observed that increase in beam bar bond stress results in decreased likelihood of joint failure prior to beam yielding. It should be noted that in exterior connections, more effective anchorage can be attained by means of 90° hooks and thereby the loading conditions in exterior connections are more favorable and significantly different in comparison to interior connections (Durrani and Wight 1982). Based on the relative magnitude of the influence factors in Tables 4.1(a) and 4.1(b), bond stress demand is determined as the second most influential parameter in predicting both interior and exterior connection response mechanism. Based on *p-value* statistic, this variable was observed to be statistically significant at 0.07% (i.e. at 99.93% confidence level) for interior joints and 5.4% (i.e. at 94.6% confidence level) for exterior joints.

5.3. Transverse steel force normalized by nominal joint shear force

Statistical analysis results in Table 4.1(a) indicate that increasing the joint transverse steel strength normalized by nominal joint shear demand reduces the likelihood of joint failure prior to beam yielding in interior connections. Results from relative magnitude of influence factors in Table 4.1(a) identify that transverse steel ratio is one of the least important design parameters in determining the likelihood of joint failure prior to beam yielding in an interior connection. Based on observations from Table 4.1(b), it was observed that for exterior connections, transverse steel does not influence failure initiation mechanism. The *p-value* statistic reveals that this variable was observed to be statistically significant at 3.75% (i.e. at 96.25% confidence level) for interior joints. The effect of this variable on exterior joints was considered to be non-important since initial study based on *p-value* statistic showed that it was statistically significant beyond 10% (i.e. at a value lower than 90% confidence level).

5.4. Column axial load ratio

The data in Tables 4.1(a) and 4.1(b) show that increasing column axial load ratio reduces the likelihood joint failure prior to beam yielding both for interior as well as exterior connections respectively. Based on the influence factors it was observed that column axial load ratio is obviously not the most prominent factor determining connection response; however its contribution to connection response is significant for both interior and exterior connections and thereby cannot be entirely disregarded. Based on *p-value* statistic, this variable was observed to be statistically significant at 1.2% (i.e. at 98.8% confidence level) for interior joints and 2.8% (i.e. at 97.2% confidence level) for exterior joints.

5.5. Ratio of top to bottom beam longitudinal steel strength

Analysis results conclude that this design parameter significantly influences the connection response of exterior connections however not that of interior connections. The analysis results presented in Table 4.1(b) suggests that an increase in the ratio of the strength of beam top and bottom reinforcement will increase the likelihood of joint failure prior to beam yielding for exterior connections. The relative magnitude of the influence factor in Table 4.1(b) for exterior connections indicates that the ratio of beam steel strengths might have a significant contribution in failure response mechanism detection and thereby needs to be investigated more thoroughly through experimental and/or numerical investigations. The *p-value* statistic reveals that this variable was observed to be statistically significant at 1.0% (i.e. at 99% confidence level) for exterior joints. The effect of this variable on interior joints was considered to be non-important since initial study based on *p-value* statistic showed that it was statistically significant beyond 10% (i.e. at a value lower than 90% confidence level).

5.6. Aspect ratio of the connection

Based on Tables 4.1(a) and 4.1(b) results, it was observed that aspect ratio can be considered to be the most significant factor for interior connection response however, was not for exterior connections. The results from Table 4.1(a) suggest that an increase in the aspect ratio for an interior connection would result in an increase in the likelihood of joint failure prior to beam yielding. The *p-value* statistic reveals that this variable was observed to be statistically significant at 0.04% (i.e. at 99.96% confidence level) for interior joints. The effect of this variable on exterior joints was considered to be non-important since initial study based on *p-value* statistic showed that it was statistically significant beyond 10% (i.e. at a value lower than 90% confidence level). Since very few experimental results show the effect of this parameter and the significance of this parameter being proved by both this study as well as the study by Kim and LaFave (2008), it is thereby being recommended that effect of this parameter on connection response be further investigated more thoroughly through experimental and/or numerical investigations.

6. PREDICTIVE EFFICIENCY OF THE MODEL

To assess the predictive efficiency of the statistical model, the likelihood of joint failure prior to beam yielding (*Event* 1), computed using Eqn 3.1.2 with β_i from Table 4.1, was plotted versus the observed event in Figure 6.1. Specimens from the data set exhibiting beam yielding prior to joint failure (*Event* 0) are plotted as circles and specimens exhibiting joint failure prior to beam yielding (*Event* 1*)* are plotted as squares. If the model were perfect, all specimens exhibiting *Event* 0 would have a computed probability of occurrence of *Event* 1 of 0.0; while all specimens exhibiting *Event* 1 would have a computed probability of occurrence of 1.0. The data in Figure 6.1 indicate that the model is not perfect for interior and exterior beam column connections respectively. However for interior connections, using a probability of 50% as indicative of response, the model correctly predicts “joint failure prior to beam yielding” for 82% of the specimens and “beam yielding prior to joint failure” for 96% of the specimens. Thereby the model for interior beam-column connections is able to correctly predict the observed failure initiation mechanism for 92% of the specimens. For the exterior beam-column connections, using a probability of 50% as indicative of response, the model correctly predicts “joint failure prior to beam yielding” for 92% of the specimens and “beam yielding prior to joint failure” for 96% of the specimens. Thereby the model for exterior beam-column connections is able to correctly predict the observed failure initiation mechanism for 96% of the specimens.

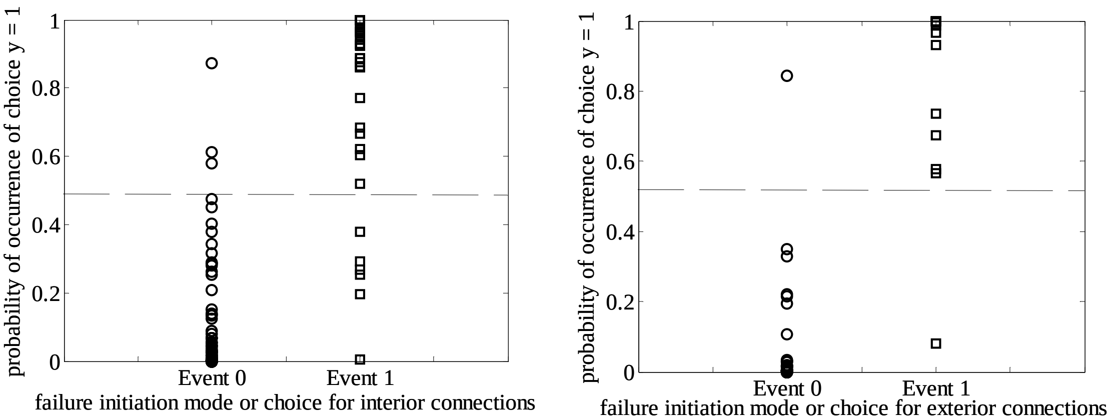

Figure 6.1. Probability of occurrence of Event 1 with different failure initiation modes

7. SUMMARY AND CONCLUSIONS

The research presented here employs an extensive experimental data set to calibrate a binomial logit model that enables prediction of inelastic mechanisms resulting in failure initiation within the connection region (either joint failure prior to or after beam yielding). The developed model also provides a relative quantitative estimate of the effect of each of the demand parameters that affects the connection response. The author suggests that the conclusions about the relative influence of the design parameters affecting response for both interior and exterior RCBC joints as presented in the paper are meant to serve as entry points for further inquiry rather than representing the final word on these parameters. Based on *p-value* statistic of the independent variables, it has been demonstrated that the variables selected for development of the models are statistically significant at least at 10%. The research also identified some design parameters, which are rarely considered in experimental investigations, to be of significance (such as aspect ratio of the joint and ratio of top to bottom beam reinforcement steel) for predicting the type of inelastic mechanism which initiates failure in interior and exterior RCBC connections. Further focused experimental researches on these parameters are suggested to validate and support the results obtained in this study. An overall predictive efficiency of 92% was observed for the interior joints and 97% for the exterior joints.

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