Residual seismic resistant performance of damaged braced steel frames after inelastic behavior

T. Inoue, T. Ito & Y. Shimoda
Tokyo University of Science, Japan

SUMMARY
There are a lot of discussions of sustainable use of buildings related to the resource and environmental problem. Robustness, redundancy and the repairability on the damaged buildings have been focused as a new design concept. To estimate the structural performance after inelastic seismic response behavior, in most previous studies, only residual deformation has been focused. In contrast, this study focuses on the residual strength and stiffness of the damaged buildings after inelastic seismic response for the purpose of sustainable use of buildings. In this paper, the residual seismic resistant performance of braced steel structure is investigated analytically. From the result, it is revealed that the residual strength and stiffness are strongly affected by the slenderness ratio and the share ratio of lateral resistant strength. Furthermore, it is indicated that the residual seismic resistant performance could be predicted at the design process.

Keywords: Braced steel structure, residual seismic resistant performance, aftershock, histogram

1. INTRODUCTION

Recently, the resource problem and the environmental problem have been focused in the world. There are a lot of discussions about the sustainable use of buildings. And also, the severe and terrible collapses of the buildings subjected to unexpected disturbance have been reported [e.g. Zdeněk P. Bažant, et al, 2008]. From these trend and the reasons, new design concepts, such as a robustness, redundancy and repairability on the damaged buildings, have been focused to sustain the structural robustness and safety. To estimate these structural performances, the seismic load effects at the ultimate states of the damaged buildings must be necessary.

Most past studies focused on the residual deformation of the damaged buildings after inelastic seismic response. If the small residual deformation of the damaged buildings is observed, it would be judged as safe for the sustainable use. However, if the residual strength is deteriorated, it may be not desirable to use aftertime, especially against to an aftershock. In this study, to ensure the enough seismic resistant performance against to an aftershock, the residual strength and rigidity of the damaged structures are focused.

In this study, the residual ratio of seismic resistant performance R is proposed to judge whether the damaged buildings have enough seismic resistant performance or not against to aftershock. Then, to obtain the index R, the residual strength and rigidity of damaged braced frame after inelastic response are analysed. And also, the performance evaluation curves of residual seismic resistant performance are obtained, which provide for the repairability limit state design.

2. RESIDUAL SEISMIC RESISTANT PERFORMANCE AND EVALUATION INDEX AFTER INELASTIC RESPONSE BEHAVIOR

2.1. Relation between Strength and Seismic Load before and after Inelastic Response Behavior

Considering the permanent usability and the seismic resistant performance against aftershock, it is necessary to make an accurate estimate of the residual seismic resistant performance after inelastic response. Figure 2.1(a), 2.1(b) shows the performance evaluation curves in the design process and after inelastic response.

The purpose of this study is to investigate performance evaluation curve before and after inelastic response behavior. In this paper, especially, the variation of the strength and stiffness of braced structure after inelastic response is focused. Additionally, the seismic load effect in the cause of variation of vibration characteristics is investigated.

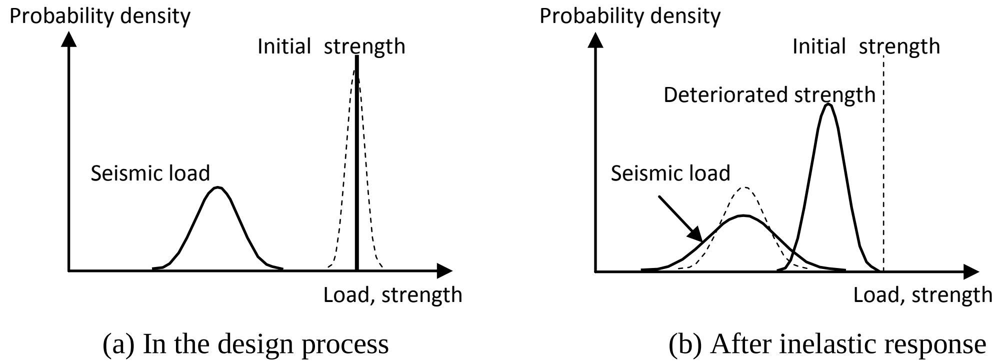


Figure 2.1. Performance evaluation curves in the design process and it after inelastic response behavior

2.2. Skeleton Curve and Residual Seismic Resistant Performance before and after Inelastic Response Behavior

Figure 2.2 shows the conceptual diagram of skeleton curve of braced structure before and after inelastic response behavior. The ultimate strength is the maximum point on the skeleton curve, and the yield strength is the point that the tangent stiffness becomes one-third initial stiffness (K_0). Here, the strength and stiffness of a brace member vary based on hysteretic response, and on the braced steel structure, the strength and stiffness differ according to the force direction. Therefore, in this study, the smaller strength and stiffness on plus and minus skeleton curve is adopted as the residual strength and stiffness after inelastic response.

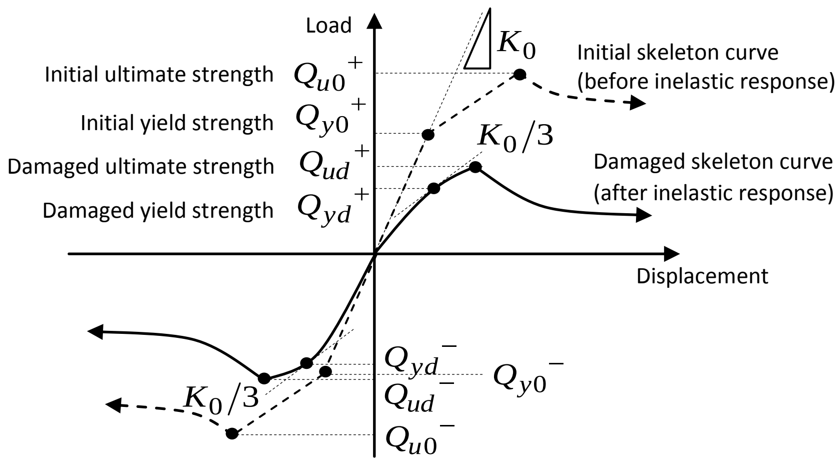


Figure 2.2. Skeleton curve of braced structure before and after inelastic response behavior

2.3. Evaluation Indexes of Residual Seismic Response Performance after Inelastic Response Behavior

In the previous studies [e.g. Feng, et al., 1986], the indexes of robustness and redundancy about variation of structural performance before and after a member disappeared have been proposed. And, the indexes about variation of strength and acceleration response spectrum are proposed.
Herein, this study considers the deterioration of structural performance of braced structure before and after inelastic response. Based on previous indexes, following indexes are defined:

Residual yield strength ratio: $$R_{ys} = \frac{Q_{yd}}{Q_{y0}} \tag{2.1}$$

Residual ultimate strength ratio: $$R_{us} = \frac{Q_{ud}}{Q_{u0}} \tag{2.2}$$

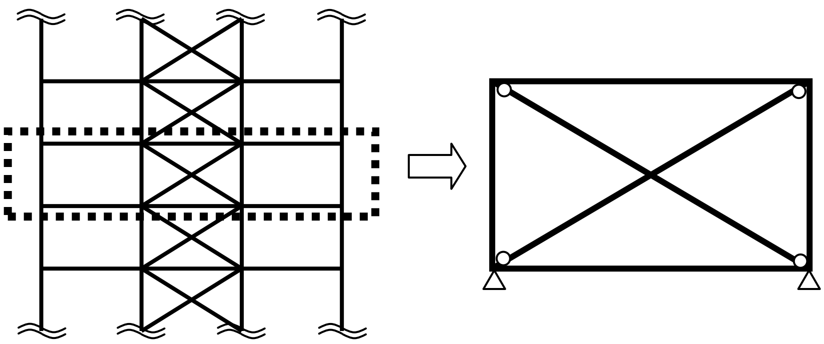
Residual stiffness ratio: $$R_k = \frac{K_d}{K_0} \tag{2.3}$$

where, K_d is initial tangent stiffness on skeleton curve after inelastic response

3. ANALYTICAL METHOD AND MODEL OF BRACED STEEL FLAMES

3.1. Analytical Model

Herein, multi-story multi-span steel frame equipped with diagonal bracing model is assumed, as shown Figure 3.1 (a). Then, on the model, the seismic load can be distributed to the beam-to-column (moment resisting) frames and the diagonal bracing system. So, in this study, the multi-story multi-span model can be substituted to an equivalent single-story single-span model, as shown Figure 3.1 (b). On the equivalent model, natural period is 0.2 seconds, and damping constant is 2%. As well, P-Δ Effect is considered.



(a) Multi-story multi-span steel frame equipped with diagonal bracing model

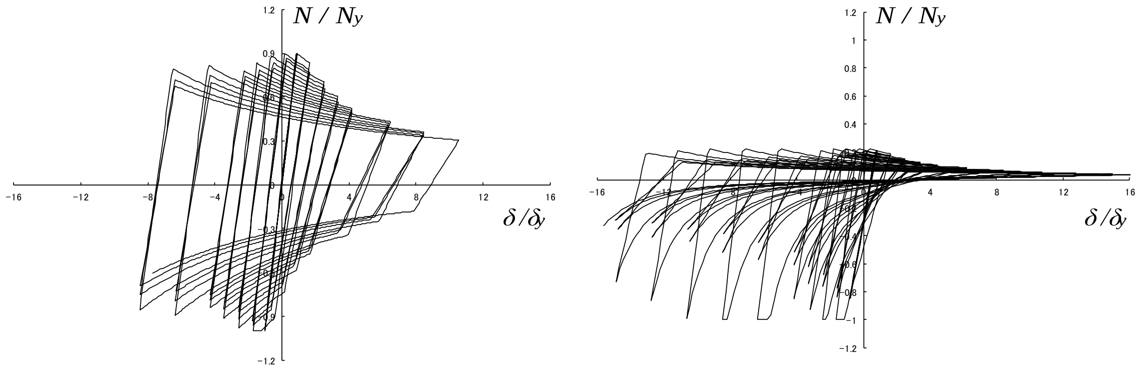
(b) Equivalent 1-story 1-span model

Figure 3.1. Analytical model of steel frame equipped diagonal bracing

3.2. Model of Restoring Force Characteristic

In our past papers [Shimoda Y., Takumi I., 2010], the strength, ductility and hysteretic loops of steel brace members subjected to inelastic cyclic loading was reviewed with a large number of past references, from which a database has been structured. Based on that data base, a model of restoring force characteristic of brace member has been formulated. The formulated model is based on

Wakabayashi model [Wakabayashi M., et al, 1982], which is proposed by Wakabayashi, and widely used for the hysteretic rule of steel brace member on response analysis. However, his model is restricted by the effective slenderness ratio. Therefore, we have modified his model so as to be able to simulate the hysteretic loops if the slenderness ratio of brace member is out of the application range of his model. In this paper, the modified model of restoring force characteristic is employed for brace members. As a reference, Figure 3.2 shows hysteretic behavior of two different types of slenderness ratio (20 and 80). Incidentally, vertical axis expresses the axial force ratio to the yield axial force *(N / N_y)*, and horizontal axis expresses the ductility factor (δ / δ_y).
Moreover, the hysteretic characteristic of the beam-column frame is the perfect elasto-plastic model, in order to investigate the effect of the brace member's hysteretic characteristic on the residual seismic resistant performance after inelastic response.



(a) Slenderness ratio: 20 (b) Slenderness ratio: 80

Figure 3.2. Model of restoring force characteristic of brace member

3.3. Analytical Variables

The analytical variables are as follows; the slenderness ratio of brace member (λ) and the lateral resistant strength share ratio of brace system to the frame (β). Then, β is defined from following equation.

$$\beta=Q_{u,B}/Q_u \tag{3.1}$$

where, $Q_{u,B}$ is demanded lateral strength of braces, and Q_u is that of the frame.
Figure 3.3 shows the relation of the skeleton curve among the brace system, the beam-column frames and the frame.

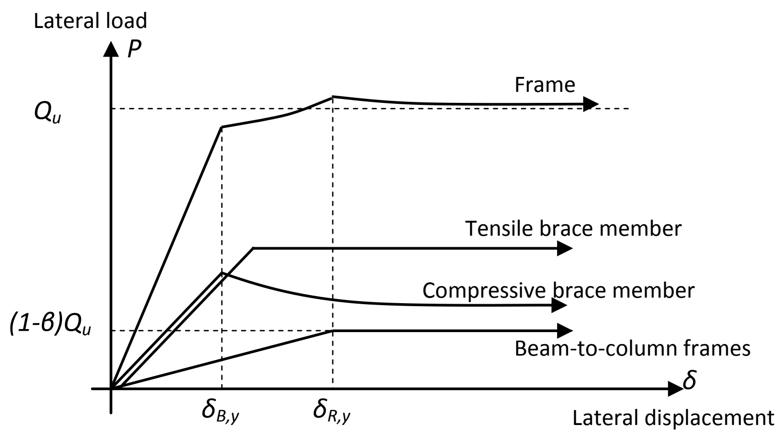


Figure 3.3. Relation of each skeleton curve

Furthermore, in this paper, the Monte Carlo simulations are conducted in order to obtain the histogram of the residual strength and stiffness of the damaged braced structure. Therefore, by the means proposed by Kuwamura H., Iwata Y., (2003), a thousand input motions are generated artificially, which have certain phase characteristics such as epicentral motions and oceanic motions. These motions are adopted for the input motions in the response analysis. Relative to the input motions, the strength of a building is scaled by the coefficient of structural characteristic (*Ds*), and *Ds* is also included in the analytical variables. All of analytical variables are summarized in Table.3.1.

Table 3.1. analytical variables

	Brace's parameter		Input motion's parameter	
Analytical variables	λ	β	*Ds*	*Phase characteristic*
Values	20, 40, 80	0.3, 0.5, 0.7	0.35, 0.45, 0.55	epicentral, oceanic

3.4. Analytical Method

In this study, inelastic response analysis is conducted with respect to the models of each analytical variable. Additionally, the residual strength and stiffness of the damaged frame are calculated from pushover analysis. Furthermore, to obtain the statistical distribution of the residual strength and stiffness of damaged frame, the Monte Carlo simulations are conducted.

4. ANALYTACAL RESULT AND CONSIDERATION

4.1. General Description

Herein, for the histograms of the residual strength and stiffness after inelastic response, the analytical result and consideration are discussed in next four sections. At first, the effects of the input motion's parameters (*Ds,* phase characteristic) on the histograms are studied when brace's parameter are fixed, as shown section 4.2. Second, the effects of the brace's parameters (λ, β) on the histograms are studied when input motion's parameters are fixed, as shown section 4.3. In the section 4.4, on the each distribution, the test of goodness of fit between these distributions and any probability density distribution are conducted by Kolmogorov-Smirnov test, which is statistical nonparametric test for the goodness of fit. Finally, in the section 4.5, based on the result of K-S test, the histograms are simulated for a probability density distribution in order to predict the residual seismic resistant performance after inelastic response at the design process.

4.2. Studying Effect of Input Motion's Parameters (*Ds, phase characteristic*)

In order to study the effects of input motion's parameters on the histograms, here, brace's parameter are fixed (λ=40, β=0.5). Figure 4.1 shows the histograms of residual yield strength ratio (R_{ys}), Figure 4.2 shows that of residual ultimate strength ratio (R_{us}), and Figure 4.3 shows that of residual stiffness ratio (R_k). In addition to these histograms, the average and the standard deviation of each statistical distribution are summarized in Figure 4.4.

4.2.1 Residual yield strength ratio (R_{ys})

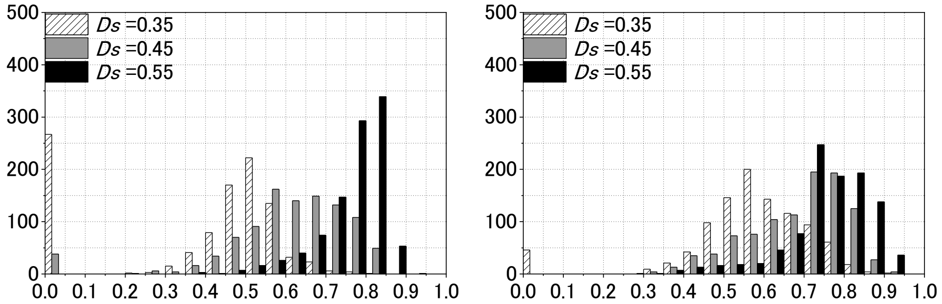
From the results shown in Figure 4.1, when coefficient of structural characteristic (*Ds*) is large, the residual yield strength ratio (R_{ys}) tends to be large. From Figure 4.4 (a), similarly, the trend is confirmed. So, it is clear that the *Ds* have a large effect on the histograms of residual yield strength ratio. Moreover, from the comparison of phase characteristics, it can be said that the R_{ys} of oceanic motions is smaller than that of epicentral motion. As a reason for that, it is considered that the accumulated damage on oceanic motion is larger than that of epicentral motion. In cases that the R_{ys} is 0, it is considered that the tangent stiffness became already less than one-third initial stiffness (K_0) at the initial stage on the pushover curve after response analysis.

4.2.2 Residual yield strength ratio (R_{us})

From the results shown in Figure 4.2 and Figure 4.4 (b), when *Ds* is large, the residual ultimate strength ratio (R_{us}) tends to be large, and the trends is confirmed from Figure 4.4 (b). And then, it is confirmed that the effect of phase characteristics is similar to the consideration described in *4.2.1* Furthermore, from Figure 4.4 (b), in almost cases, it is confirmed that the average of the R_{us} are over 0.8 and that the dispersions of the histograms are relatively small, compared with Figure 4.4 (a), 4.4(c).

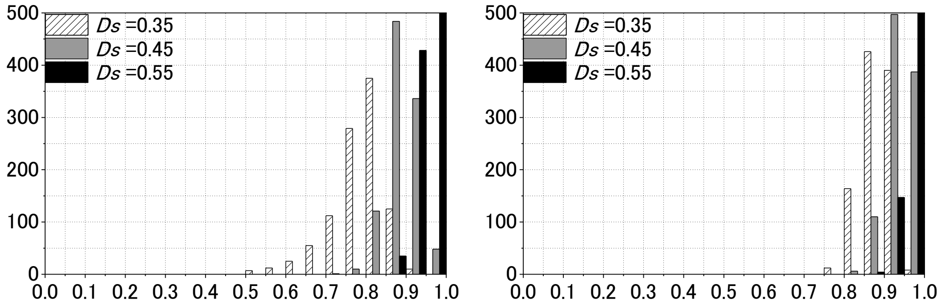
4.2.3 Residual stiffness ratio (R_k)

From the result shown in Figure 4.3 and Figure 4.4 (c), the effect of *Ds* and phase characteristics is similar to the description in *4.2.1 and 4.2.2*. In almost cases, the dispersions of these histograms stay nearly constant 0.081(see Figure 4.4 (c)).



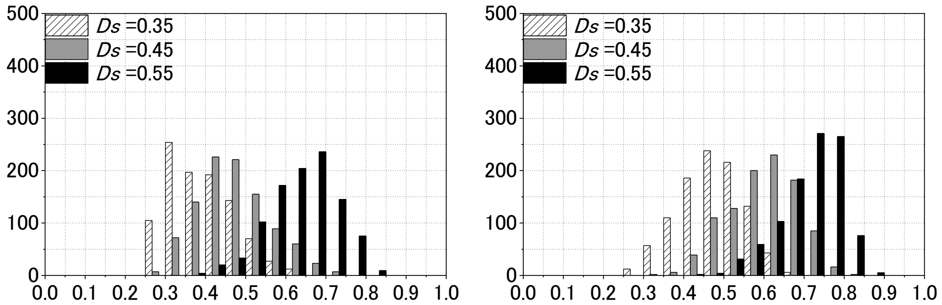
(a) Oceanic motion (b) Epicentral motion

Figure 4.1. Statistical distribution of Residual yield strength ratio (R_{ys})



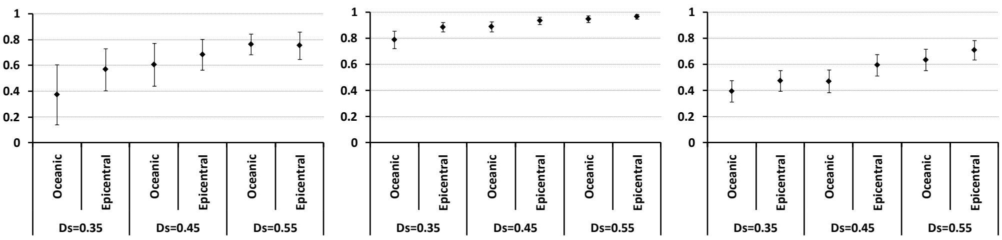
(a) Oceanic motion (b) Epicentral motion

Figure 4.2. Statistical distribution of Residual ultimate strength ratio (R_{us})



(a) Oceanic motion (b) Epicentral motion

Figure 4.3. Statistical distribution of Residual stiffness ratio (R_k)



(a) R_{ys} (b) R_{us} (c) R_k

Figure 4.4. Average and standard deviation of statistical distribution

4.3. Studying the Effects of Brace's Analytical Variables (λ, β)

Herein, in order to study the effects of brace's parameter on the histograms, input motion's parameter are fixed (Ds=0.45, oceanic motion). Figure 4.5 shows the histograms of residual yield strength ratio (R_{ys}), Figure 4.6 shows that of residual ultimate strength ratio (R_{us}), and Figure 4.7 shows that of residual stiffness ratio (R_k). In addition to these histograms, the average and the standard deviation of each statistical distribution are summarized in Figure 4.8.

4.3.1 Residual yield strength ratio (R_{ys})

From the results shown in Figure4.5 and 4.8 (a), as the lateral resistant strength share ratio of braces to the frames (β) are increased, the residual yield strength ratio (R_{ys}) tends to be large except for the case of slenderness ratio (λ) 20. And then, with the λ increased, the R_{ys} decrease. Especially in the case of β= 0.3 (see Figure 4.5 (c)), the R_{ys} are large and the dispersion is small compared with Figure 4.5 (a) and 4.5 (b). As a reason for that, if the β is small, it is considered that the surrounding beam-to-column frames possess enough lateral resistant strength if the brace system lost it after inelastic response.

4.3.2 Residual ultimate strength ratio (R_{us})

From the result shown in Figure 4.8 (b), in all of cases, it is confirmed that the R_{us} are over 0.8 and dispersions are relatively small, and it is indicated that the the R_{us} are little-affected by the β and the λ.

4.3.3 Residual stiffness ratio (R_k)

From the result shown in Figure 4.7 and 4.8 (c), it is observed that the R_k tends to be small with the λ increased, and this trend becomes remarkable as the β is increased. From the trends, it is considered that R_k are strongly affected by the λ.

4.4. Test of Goodness of Fit on Statistical Distribution

Now, in this section, the previously-described histograms are simulated for a probability density distribution. And then, the test of goodness of fit between the histogram and a probability density distribution is conducted by use of one-sample Kolmogorov-Smirnov (K-S) test, which is one of hypothesis test based on statistics. Herein, in the null hypothesis, it is assumed that the histogram is come from a probability density distribution (in this study, beta distribution is assumed), and the significant level is 5%.

Based on the statistical method and these suppositions, the significant probability is calculated as the index of confidence level.

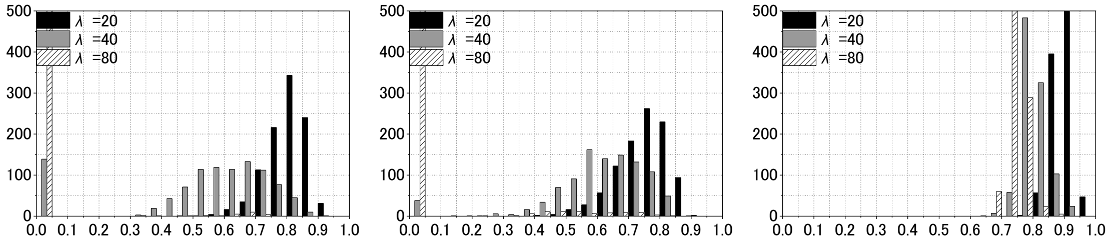
The results of significant probability are summarized in Table.4.1 and Table.4.2

Table 4.1. Significance probability of study on input motion's parameters

	R_{ys}			R_{us}			R_k		
	Ds=0.35	Ds=0.45	Ds=0.45	Ds=0.35	Ds=0.45	Ds=0.55	Ds=0.35	Ds=0.45	Ds=0.55
Oceanic	0	0	0	0	5	8	59	19	45
Epicentral	0	0	0	35	13	0	13	23	3

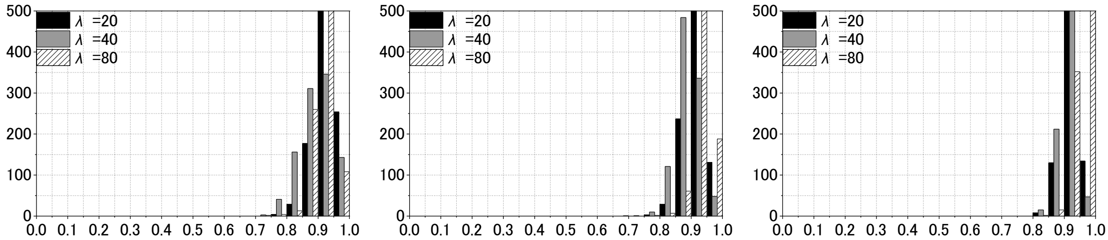
Table 4.2. Significance probability of study on brace's parameters

	R_{ys}			R_{us}			R_k		
	λ=20	λ=40	λ=80	λ=20	λ=40	λ=80	λ=20	λ=40	λ=80
β=0.7	52	-	-	22	44	1	2	55	0
β=0.5	8	0	-	25	5	0	17	18	8
β=0.3	14	0	0	27	3	0	37	8	20



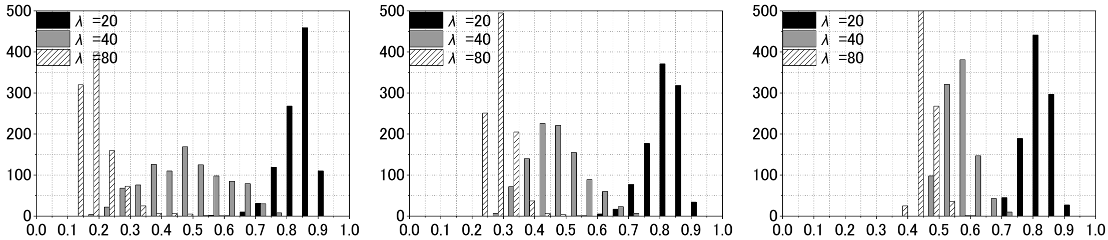
(a) β=0.7 (b) β=0.5 (c) β=0.3

Figure 4.5. Statistical distribution of Residual yield strength ratio (R_{ys})



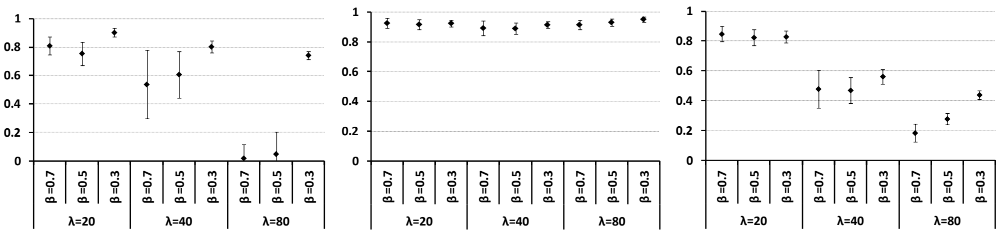
(a) β=0.7 (b) β=0.5 (c) β=0.3

Figure 4.6. Statistical distribution of Residual ultimate strength ratio (R_{us})



(a) β=0.7 (b) β=0.5 (c) β=0.3

Figure 4.7. Statistical distribution of Residual stiffness ratio (R_k)



(a) R_{ys} (b) R_{us} (c) R_k

Figure 4.8. Average and standard deviation of statistical distribution

4.5. Simulation of Histograms for the Probability Density Distribution

In the section 4.4, the test of goodness of fit for the assumption model is conducted. Herein, based on the result of significance probability, residual seismic resistant performance is simulated for the beta distribution. And then, the probability density function of beta distribution is defined as follow.

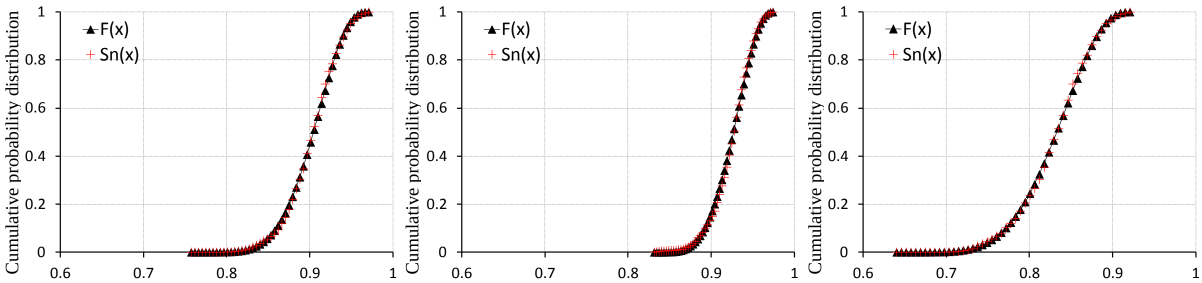
$$f(x)=\frac{1}{B(q,r)}\frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}}, \qquad B(q,r)=\int_0^1 x^{q-1}(1-x)^{r-1}dx \qquad (4.1), (4.2)$$

where, q and r are calculated from blow formula.

$$q=\frac{\mu-a}{b-a}\left[\frac{(b-\mu)(\mu-a)}{\sigma^2}-1\right], \qquad r=\frac{b-\mu}{b-a}\left[\frac{(b-\mu)(\mu-a)}{\sigma^2}-1\right] \qquad (4.3), (4.4)$$

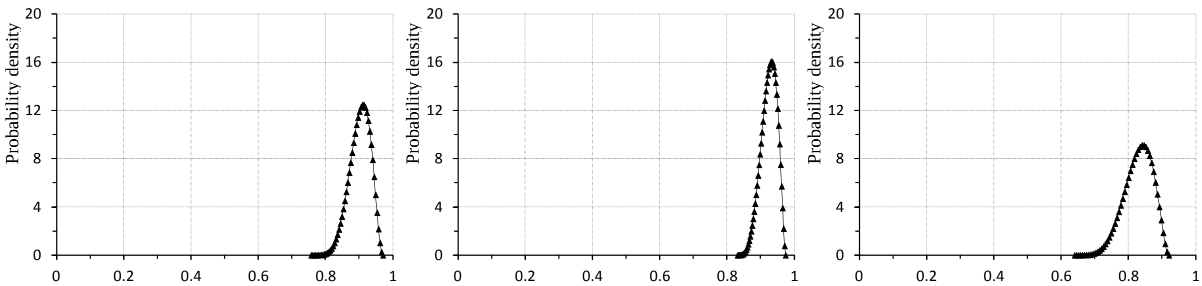
where, a is the minimum sample, b is the maximum sample, μ is the average of the distribution and σ is the standard deviation of the distribution.

In this simulation, the case of best result on the significance probability is picked up, and these analytical parameter of the best result are as follows: the slenderness ratio of brace member (λ) is 20, the lateral resistant strength share ratio of braces to frames (β) is 0.3, the coefficient of structural characteristic (Ds) is 0.45, and the phase characteristic is oceanic motion. Figure4.9 shows the cumulative probability distribution with respect to these analytical parameters.
Each parameter of beta distribution is summarized in Table.4.3. And also, the results of simulation for beta distribution are shown in Figure 4.10.



(a) R_{ys} (b) R_{us} (c) R_k

Figure 4.9. Cumulative probability distribution of the histogram and beta distribution



(a) R_{ys} (b) R_{us} (c) R_k

Figure 4.10. Simulation for the beta distribution

As shown in Figure 4.10, the residual seismic resistant performance, R_{ys}, R_{us} and R_k, were simulated for the each beta distribution, in consequence, it is confirmed that the residual seismic resistant performance after inelastic response can be analytically predicted at the design process.

Table 4.3. Parameters of Beta Distribution

	Prob[%]	a	b	q	r	μ	σ	x_m
R_{ys}	14	0.75	0.97	6.76	3.13	0.90	0.03	0.91
R_{us}	27	0.83	0.97	5.09	2.62	0.92	0.02	0.93
R_k	37	0.63	0.92	6.24	2.94	0.83	0.04	0.84

where, x_m is the mode value of each distribution

5. CONCLUTION

In this study, with regard to the repairability of damaged buildings, the residual seismic resistant performance, R_{ys}, R_{us} and R_k, were analytically calculated for the purpose of making a prediction the residual seismic resistant performance after inelastic response. The following conclusions were drawn:

1. The residual yield strength ratio (R_{ys}), the residual ultimate strength ratio (R_{us}) and the residual stiffness ratio (R_k) were calculated by the pushover analysis after inelastic response analysis. Also, by means of Monte Carlo simulations, the histograms of each R_{ys}, R_{us} and R_k were described.

2. A parametric study of effects on the residual seismic resistant performance was conducted based on these histograms. As a result of parametric study, it is revealed that the trends of the histograms are strongly affected by the slenderness ratio, the share ratio of the lateral strength and the coefficient of structural characteristic.

3. These histograms were simulated for a probability density distribution which is beta distribution in this study. And then, the test of goodness of fit about the simulation was conducted by use of one-sample Kolmogorov-Smirnov (K-S) test. As a result of the test, it is indicated that the distribution of residual seismic resistant performance after inelastic response could be analytically predicted at the design process.

REFERENCES

Zdeněk P. Bažant, Jia-Liang Le, Frank R. Greening, and David B. Benson. (2008). What Did and Did Not Cause Collapse of World Trade Center Twin Towers in New York. *Journal of Structural Engineering, ASCE*. **Volume 134: issue 10** pp.892-906.

Feng, Y. S., Moses, F. (1986). Optimum design, redundancy and reliability of structural systems.*Computers and Structures*. **Volume 24:issue 2**, pp.239-251.

Dan M. Frangopol, James P. Curley. (1987). Effects of Damage and Redundancy on Structural Reliability. *Journal of Structural Engineering, ASCE*. **Volume 113: issue 7**. pp.1533-1549.

Kuo-Wei Liao, Yi-Kwei Wen, D.A. Foutch. (2007). Evaluation of 3D Steel Moment Frames under Earthquake Excitations, Ⅱ: Reliability and Redundancy. *Journal of Structural Engineering, ASCE*. **Volume 113: issue 3** pp.471-480.

Shimoda Y., Ito T. (2010). Database of strength and ductility of steel compression members subjected to cyclic loading, part 1~3. *Research Report in Annual Meeting of Architectural Institute of Japan (in Japanese).* pp.441-452.

Wakabayashi M.,Shibata M. (1982). Mathmatical expression of hysteretic behavior of braces, part 1~2. *Transactions of the Architectural Institute of Japan.* **Volume 316**, pp.18-24.

Iwata Y., Kuwamura H. (2003). Earthquake Input Motion in Performance Evaluation of Steel Building Structures, part 2~3. *Research Report in Annual Meeting of Architectural Institute of Japan (in Japanese).* pp.133-140