

# Seismic performance of semi-continuous steel frames with nonlinear viscous dampers

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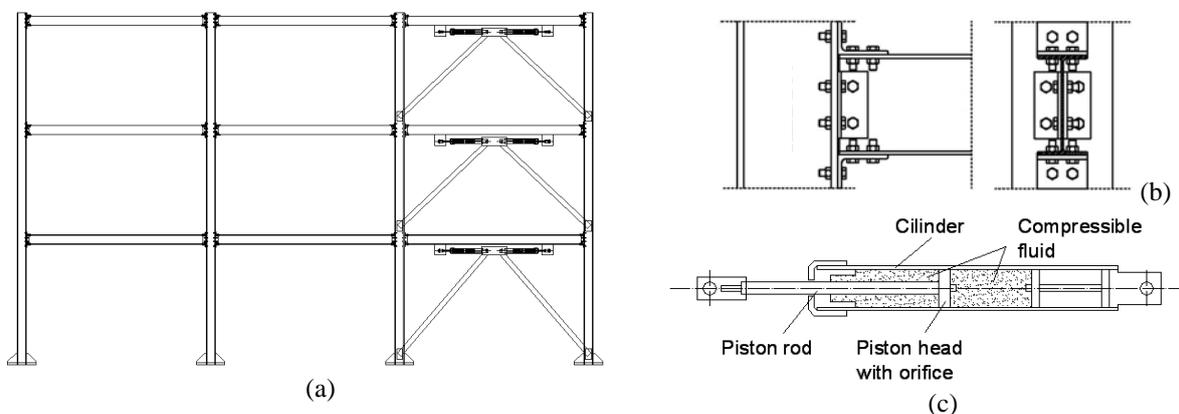
## SUMMARY:

In the paper, the seismic performance of steel frames equipped with nonlinear viscous dampers is analysed and a practical design approach is presented. The proposed strategy is tailored to design new seismic resistant structures or improve the response of existing frames, where the use of nonlinear fluid viscous dampers allows a significant reduction in structural damage and lateral displacements under earthquake loading. The effectiveness of the proposed design procedure is checked considering the results of nonlinear dynamic simulations. In the numerical description for steel frames, beams and columns are modelled using plastic beam elements and nonlinear rotational springs are employed for representing the hysteretic behaviour of beam-to-column connections. The numerical outcomes confirm that the use of nonlinear viscous dampers guarantees not only a limited interstorey drift demand and a reduced plastic damage in the frame but also no increase in base shear.

*Keywords: nonlinear viscous dampers, semi-continuous steel frames, nonlinear analysis, seismic upgrading*

## 1. INTRODUCTION

In the paper the seismic response of hybrid systems made up of semi-continuous steel frames equipped with viscous dampers (Fig. 1.1.a) is analysed. Under earthquake loading, steel frames with semi-rigid joints (Fig. 1.1.b), which are simple and economic structural systems, are characterised by high performance at ultimate limit state (ULS), thanks to the plastic dissipation capacity of the beam-to-column connections in bending. However the large lateral flexibility and joint damage, even under frequent seismic events, prevent their use when the limitation of damage in structural and non-structural components is the main design criterion. Previous research (Amadio et al. 2008, Amadio et al. 2009) has shown that the seismic response of existing steel and steel-composite frames not designed to withstand earthquake loading can be significantly improved using bracings with dissipative devices. In (Amadio et al. 2009), steel X-braces or visco-elastic dampers have been considered for the seismic upgrade of existing semi-continuous frames pointing out the superior performance of the latter solution.



**Figure 1.1.** (a) Viscously damped steel frames. (b) Semi-rigid joints with angles. (c) Viscous fluid device

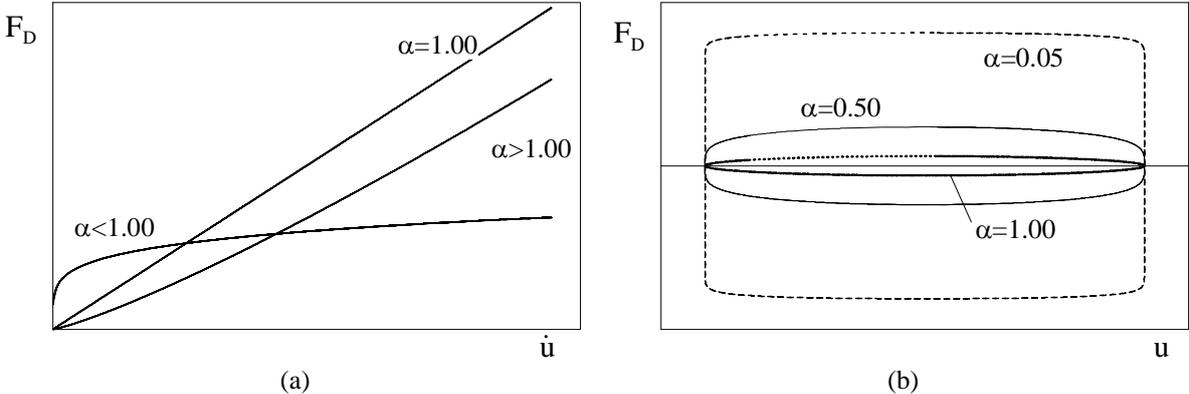
In this research nonlinear fluid viscous dampers (Soong and Dargush 1997), Fig. 1.1.c, are considered. Their behaviour is not significantly affected by temperature and stress frequency and they do not experience damage when in use, thus not requiring any repair or substitution after a seismic event. In general, when using viscous dampers connected to bracing elements to retrofit steel frames, hybrid systems are formed (viscously damped braced frames - VDBF). These are characterised by a seismic response with small interstorey drifts, like in the case of frames strengthened using traditional bracing systems, very limited damage in structural components and no increase in base shear (Amadio et al. 2008). The design of nonlinear viscous dampers is usually carried out using energy considerations and requires some amount of trial and error (Christopoulos and Filiatrault 2006). In particular it is assumed that the earthquake input energy is mainly dissipated by the inherent damping of the frame and the damping provided by the supplemental viscous fluid devices, usually considering that the main structural system (e.g. steel frame) remains elastic under earthquake loading. This becomes impractical and not economic in the case of seismic rehabilitation of existing structures, when a limited amount of structural damage should be acceptable. Thus recent research (Hwang et al. 2008, Goel 2005) and the most advanced codes of practice (FEMA 1997) suggest design strategies based on the use of nonlinear procedures to determine the level of supplemental damping required to satisfied the design objectives, and implicitly allow structural damage in the rehabilitated structure. In the following, the main characteristics of fluid viscous devices are mentioned and the equivalent damping coefficient for a viscous dampers and for MDOF systems with viscous nonlinear devices (e.g. VDBF) are reported. A practical design strategy is then presented, where the Capacity Spectrum Method (Freeman 1998, Fajfar 1999) with damped demand spectra is employed to calculate the required supplemental damping and to estimate the performance of the analysed structure at different limit states. Finally the proposed design strategy is used for the seismic upgrading of a steel frame, and nonlinear dynamic analyses are employed to investigate the response of the frame equipped with nonlinear viscous dampers.

**2. VISCOUS DAMPERS**

Fig. 1.1.c shows a viscous fluid device, where a piston can move under a compression force causing the fluid to flow along the cylinder. The force resisted by the damper is due to the difference in pressure at the two sides of the cylinder and is generally given by the expression (Soong and Dargush 1997):

$$F_D = C|\dot{u}|^\alpha \text{sgn}(\dot{u}) \tag{2.1}$$

where  $C$  is the viscous damping constant,  $\dot{u}$  is the relative velocity between the two ends of the damper,  $\alpha$  is a velocity coefficient in the range of 0.2 to 1, which is characteristic of the device (Soong and Dargush 1997). A device with  $\alpha = 1$  is called linear viscous damper and the force  $F_D$  is proportional to relative velocity. Dampers with  $\alpha < 1$  are named nonlinear viscous dampers and, at low velocity, provide a higher resistance  $F_D$  than linear dampers. Finally dampers with  $\alpha > 1$  are rarely used in practical applications as viscous dissipative devices for seismic upgrading.



**Figure 2.1.** Force-velocity curves (a) and force-displacement curves (b) for viscous dissipation systems

In general, when the velocity coefficient is close to 0, the nonlinear viscous damper provides a nearly constant resistance over a wide speed range (Fig. 2.1.a). Fig. 2.1.b shows the force-displacement curve for devices with different velocity coefficients. It can be observed how dampers with small  $\alpha$  values are characterised by a large energy dissipation capacity, which is represented by the area inside the curves for a force-displacement cycle.

## 2.1. Damping coefficient for VDBF

The main mechanical parameter representing the energy dissipation capacity of the coupled system to be considered in the design procedure is the equivalent damping coefficient  $\xi_{sd}$ . In the following,  $\xi_{sd}$  is firstly calculated for a SDOF system and then the result is extended to MDOF coupled systems representing multi-storey VDBFs. In the latter case, the equivalent damping coefficient is given by the sum of bare frame contribution  $\xi_0$  and the damping coefficient due to the supplemental viscous dampers  $\xi_d$ :

$$\xi_{sd} = \xi_0 + \xi_d \quad (2.2)$$

For a SDOF system,  $\xi_d$  can be determined considering the energy dissipated by the viscous device  $W_D$  under an imposed sinusoidal displacement time history  $u(t)$  (Hwang 2002):

$$u(t) = u_0 \sin(\omega t) \quad (2.3)$$

$$W_D = 2^{2+\alpha} C \omega^\alpha u_0^{1+\alpha} \frac{\Gamma^2(1+\alpha/2)}{\Gamma(2+\alpha)} \quad (2.4)$$

where  $\omega$  is the excitation frequency,  $u_0$  is the displacement amplitude,  $\alpha$  is the velocity coefficient for the viscous damping,  $C$  is the viscous damping constant, while  $\Gamma$  is the function:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt = \frac{\pi}{\Gamma(1-x) \sin(\pi x)} \quad (2.5)$$

Using the relationships above, the equivalent damping for the nonlinear viscous device can then be obtained using the expression (Hwang 2002):

$$\xi_d = \frac{W_D}{2\pi W_K} = \frac{\lambda C \omega^{\alpha-2} u_0^{\alpha-1}}{2\pi m} \quad (2.6)$$

where  $m$  is the mass of the whole system and  $W_K$  is the elastic deformation in a cycle, which can be expressed as:

$$W_K = K u_0^2 \quad (2.7)$$

and

$$\lambda = 2^{2+\alpha} \frac{\Gamma^2(1+\alpha/2)}{\Gamma(2+\alpha)} \quad (2.8)$$

In the case of frames equipped with linear or nonlinear viscous dampers, the damping devices do not change the stiffness of the bare frame. Thus if the MDOF coupled system is characterised by a mass  $m_j$  and a device with viscous coefficient  $C_j$  at floor  $j$ , the relationship for the equivalent damping can be derived from (2.6) considering the first vibration mode of the bare frame with circular frequency  $\omega_0$ .

Thus the strain energy is given by:

$$W_k = \omega_0^2 \sum_j m_j u_j^2 \quad (2.9)$$

Then if all the dampers have the same velocity coefficient  $\alpha$ , the total damping due to the dissipative devices becomes:

$$\xi_d = \frac{\sum W_{D,j}}{2\pi W_k} = \frac{\lambda \sum_j C_j u_{ij}^{1+\alpha} \cos^{1+\alpha} \vartheta_j}{2\pi \omega_0^{2-\alpha} \sum_j m_j u_j^2} \quad (2.10)$$

where  $u_{ij}$  is the relative displacement between the ends of the damper  $j$  and  $\vartheta_j$  represents the damper inclination angle to the horizontal. Considering again the first mode of vibration for the analysed framed structure, the lateral displacement at each floor is given by:

$$u_j = A \phi_j \quad (2.11)$$

with  $\phi_j$  is the normalised displacement at floor  $j$  and  $A$  is the maximum lateral displacement. Thus the damping due to the viscous devices can be expressed as:

$$\xi_d = \frac{\lambda \sum_j C_j \phi_j^{1+\alpha} \cos^{1+\alpha} \vartheta_j}{2\pi A^{1-\alpha} \omega_0^{2-\alpha} \sum_j m_j \phi_j^2} \quad (2.12)$$

where  $\phi_{ij}$  is the normalised interstorey drift modal vector.

### 3. DESIGN PROCEDURE

The suggested design method for hybrid systems with nonlinear viscous dampers is general and can be used for designing new VDBFs or the seismic upgrading of existing steel and reinforced concrete (RC) framed structures. A simple procedure based on the Capacity Spectrum Method is used to determine the amount of supplemental damping required to satisfy the design objectives.

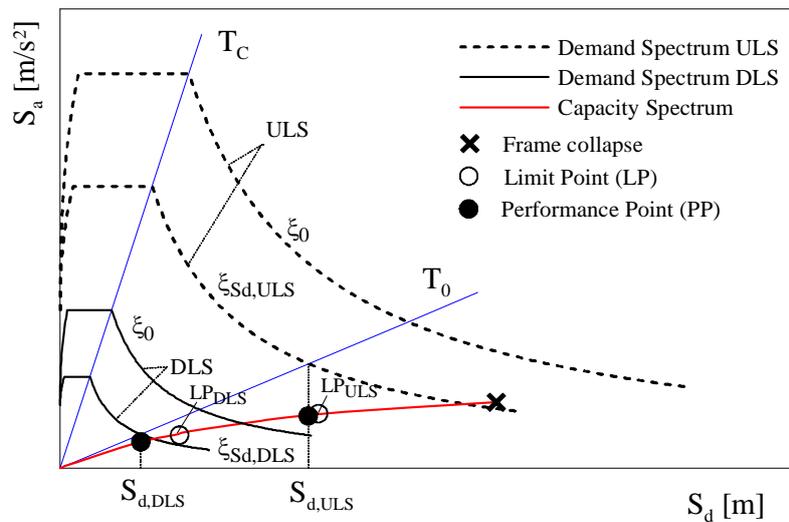


Figure 3.1. ADRS spectra

Depending on the analysed framed structural system (e.g. steel frames, RC frames) a critical design objective is established. This can be associated with a specific limit point (LP) on the capacity curve, which describes the nonlinear static response of the framed structure.

The ADRS format is used, where spectral accelerations are plotted against spectral displacements, with the period of the analysed frame represented by a radial line (Fajfar 1999). In the case of framed structures, the fundamental period of the frame  $T_0$  is usually larger than  $T_C$ , Fig. 3.1., thus the equal displacement assumptions can be used and the nonlinear response of the analysed system compared against an equivalent elastic response. Different damped demand spectra for each limit state are drawn together with the capacity spectrum. This is obtained transforming the coordinate of the nonlinear capacity curve expressed in term of base shear and top displacement into spectral accelerations and spectral displacements of an equivalent SDOF system. Thus the required equivalent damping can be associated with the demand spectrum curve that provides an intersection with the frame period line at a spectral displacement  $S_{d,ULS(DLS)}$  close, but smaller than the displacement at the limit point.

Fig. 3.1 shows a ADRS diagram, where demand curves, capacity spectrum and limit points ( $LP_{DLS}$  and  $LP_{ULS}$ ) are displayed. According to Eurocode 8 (CEN 2004), two limit states, damage (DLS) and ultimate limit state (ULS) are considered with different equivalent damping value  $\xi$ .  $\xi_0$  represent the inherent damping of the bare frame, while  $\xi_{Sd,DLS}$  and  $\xi_{Sd,ULS}$  are the equivalent damping coefficients for the equipped structure, which are different at DLS and ULS. In fact, as the amount of damping supplied by the nonlinear viscous devices depends on the relative displacements at the ends of the dampers, Eqn. 2.10, different equivalent damping values are expected at DLS and ULS for the same VDBF. According to the design procedure, demand spectra can be drawn considering the relationship between spectral acceleration and displacement for a SDOF system:

$$S_d = S_a \cdot \frac{1}{\omega_0^2} = S_a \cdot \left( \frac{T_0}{2\pi} \right)^2 \quad (2.13)$$

while the fundamental period of the analysed frame can be approximated using the empirical expression:

$$T_0 = C_t \cdot H^{3/4} \quad (2.14)$$

where  $C_t$  is a coefficient characteristic of the type of structure (e.g.  $C_t = 0.180$  for steel frames with semi-rigid joints). An accurate numerical description should be employed to determine the nonlinear static response of the frame subjected to a horizontal distribution of lateral forces to collapse. The ability of the nonlinear model to represent the evolution of damage in members and joints allows us to consider specific design objective at DLS and ULS, which are associated with the limitation of structural damage. When using this design strategy for the seismic upgrading of existing structures, the frame characteristics are known. Conversely, in the case of design of a new VDBF, member sizes and connection details can be derived from the structural design based on the non-seismic loading conditions. As mentioned above, when the critical designed objective has been established, the equivalent damping coefficient which provides a demand in displacement close, but smaller than the displacement capacity at limit point (Fig. 3.1.), can be found. Then the damping coefficient  $C$  of the nonlinear viscous devices can be determined using Eqn. 2.12 and considering the first vibration mode of the frame and the displacement demand  $S_{d,DLS(ULS)} = S_d(T_0, \xi_{Sd})$ . Assuming to use devices with the same damping constant  $C$  and with the same inclination angle  $\vartheta_j$ , the damping coefficient can be calculated as:

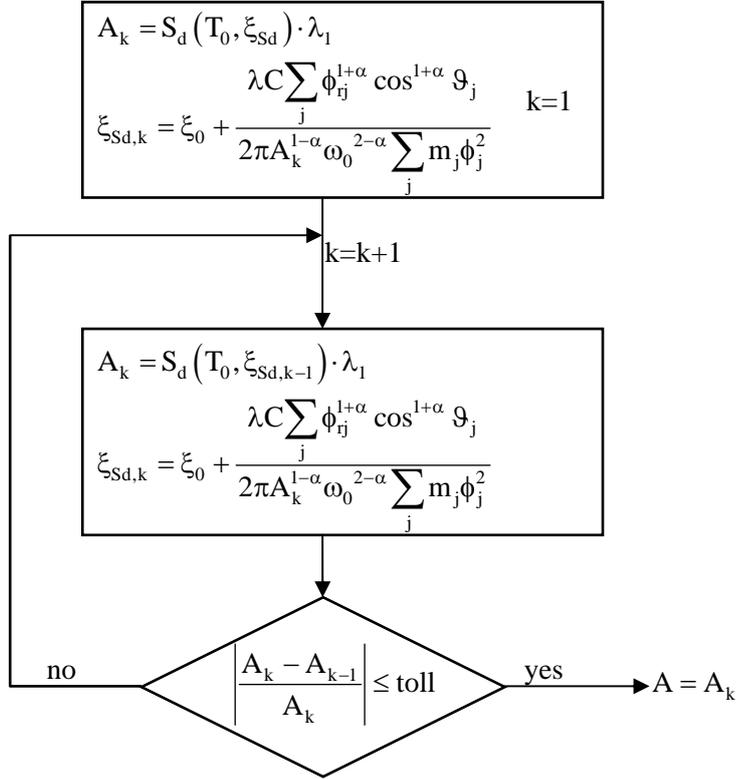
$$C = (\xi_{Sd} - \xi_0) A^{1-\alpha} \frac{2\pi\omega_0^{2-\alpha} \sum_j m_j \phi_j^2}{\lambda \sum_j \phi_{rj}^{1+\alpha} \cos^{1+\alpha} \vartheta_j} \quad (2.15)$$

with

$$A = S_d(T_0, \xi_{Sd}) \cdot \lambda_1 \quad (2.16)$$

where  $\lambda_1$  is the participation factor for the first mode.

Finally, the other design objectives must be checked taking into account the spectral displacements at the associated limit points. At this stage the spectral displacement demand can be found using Eqns. 2.2 and 2.12. Now the viscous damping constant  $C$  is known, as it has already been calculated for the critical design objective (Eqn. 2.15), thus  $\xi_{Sd}$  and  $A$  can be determined using the iterative procedure shown in Fig. 3.2.



**Figure 3.2.** Iterative procedure to calculate displacement demand

The  $A$  value is then used to find the spectral displacement demand  $S_{d, DLS (ULS)}$  (Eqn. 2.16), which must be compared against the spectral displacement capacity at the specific limit point. If the demand value is smaller than the capacity, then the specific performance objective is met, otherwise a higher damping coefficient  $\xi_{Sd}$  leading to a lower demand value must be chosen and the required damping constant  $C$  recalculated (Eqn. 2.15).

#### 4. SEISMIC UPGRADING OF A STEEL FRAME

The performance of VDBF has been investigated considering the seismic upgrading of a steel frame with semi-rigid joints, which has been designed according to current codes of practice. The proposed design strategy is utilized to calculate the required damping coefficient and the damping constant for the nonlinear viscous dampers used for improving the behaviour under earthquake loading. An accurate numerical model is employed to determine the nonlinear dynamic response of the steel frame and the frame equipped with dampers.

##### 4.1. Steel frame with semi-rigid joints

The analysed structure is a three-bay three-storey frame. Column height is 4.0 m at the ground floor and 3.5 m at the upper floors, while the span length is 6 m for each beam. The steel frame has been designed according to Eurocodes (CEN 2003a, CEN 2003b) for non-seismic loading conditions,

taking into account dead, live, wind, snow loads and imperfections. Thus HEB280 section for columns and IPE300 for beams have been determined. The beam-to-column connections are semi-rigid and made up of bolted angles which connect web and flanges of the beam to the column flange (Fig. 1.1.b). In particular, a 100x120x200 mm angle with 12 mm thickness is used for beam web and a 130x130x150 mm angle with 13 mm thickness for beam flanges, while the bolts are high strength M20 10.9 (CEN 2003b).

#### 4.2. Seismic upgrading

The performance of the frame has been checked under earthquake loading, accounting for damage and ultimate limit state (CEN 2004) with  $a_g=0.35g$  and type soil A. It has been found that the interstorey drift demand (0.72% drift) at DLS is higher than the limit value suggested by the code (0.5% drift), thus the structure requires upgrading. The use of nonlinear viscous dampers to form a VDBF has been considered, and the design strategy presented in previous section has been followed to find the amount of viscous damping  $\xi_d$  and the damping coefficient  $C$  for the viscous fluid devices, which lead to a significant improvement in the seismic performance, thus meeting the design objectives. In particular, using  $\alpha = 0.2$  (typical value), it has been calculated that  $\xi_{d,DLS} = 8\%$  is required at DLS. This, for the analysed frame ( $\omega_0 = 5.07$  rad/sec,  $\xi_0 = 2\%$ ) and for two dampers with  $\vartheta = 0$  at each floor, gives  $C = 13.55$  kN/(m/s)<sup>0.2</sup>. According to the design strategy, after determining the number of damper and the viscous coefficient  $C$ , the maximum displacement  $A$  and supplemental damping at ULS can be found using the iterative calculation in Fig. 3.2, which, in this case, gives  $A = 1.31 \cdot 10^{-1}$  [m] and  $\xi_{d,ULS} = 6.73\%$ .

#### 4.3. Nonlinear dynamic analyses

Nonlinear dynamic analyses (NDA), with three different artificial ground motions compatible with the design elastic spectrum, have been carried out to compare the seismic response of the steel frame against the dynamic behavior of the VDBF system. An accurate numerical description has been used for modelling plastic damage in steel member and connections. Plastic beams elements provided by the finite element code ABAQUS (Hibbit et al. 2001) have employed for beams and columns, while nonlinear rotational springs with specific mechanical laws to account for degradation of strength and stiffness have been used for beam-to-column connections (Amadio et al. 2010). In particular the component method has been considered to calculate strength and stiffness for rotational springs. Finally, nonlinear dashpot elements have been used to represent the dynamic behavior of nonlinear viscous dampers.

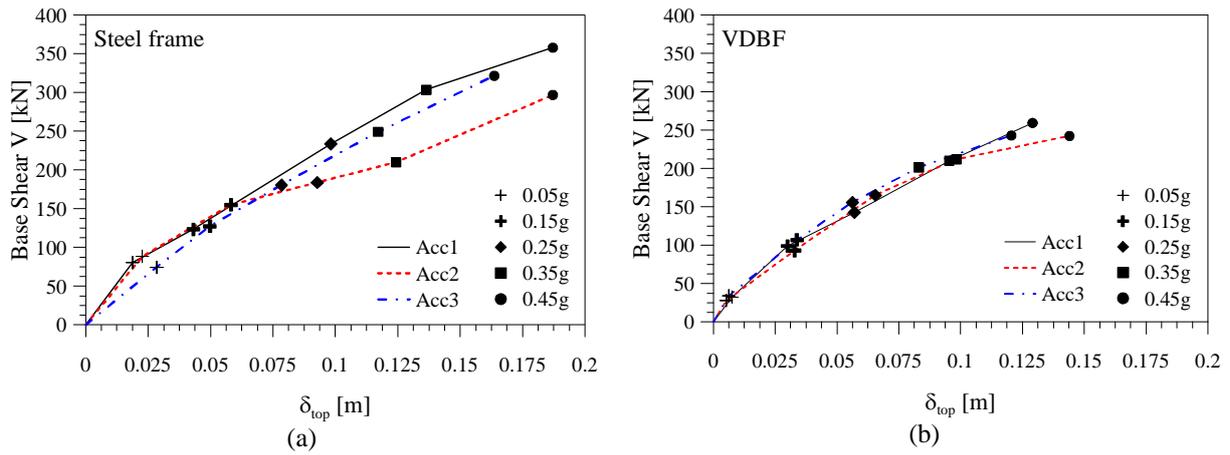
The results of nonlinear dynamic analyses (NDA) allow us to check the accuracy of the designed procedure for VDBF. Table 4.1 reports the maximum top displacements calculated by the numerical model in the three time-history analyses (Acc1, Acc2, Acc3,  $a_g=0.35g$ ) and the value obtained using the ADRS spectra (Fig. 3.1). The design and numerical values are quite close confirming the effectiveness of the proposed approach.

**Table 4.1.** Top displacement at ULS in [m].

	NDA - Acc1	NDA - Acc2	NDA - Acc3	Design value ADRS diagram
Steel Frame	$1.36 \cdot 10^{-1}$	$1.24 \cdot 10^{-1}$	$1.17 \cdot 10^{-1}$	$1.29 \cdot 10^{-1}$
VDBF	$9.54 \cdot 10^{-2}$	$9.82 \cdot 10^{-2}$	$8.31 \cdot 10^{-2}$	$9.96 \cdot 10^{-2}$

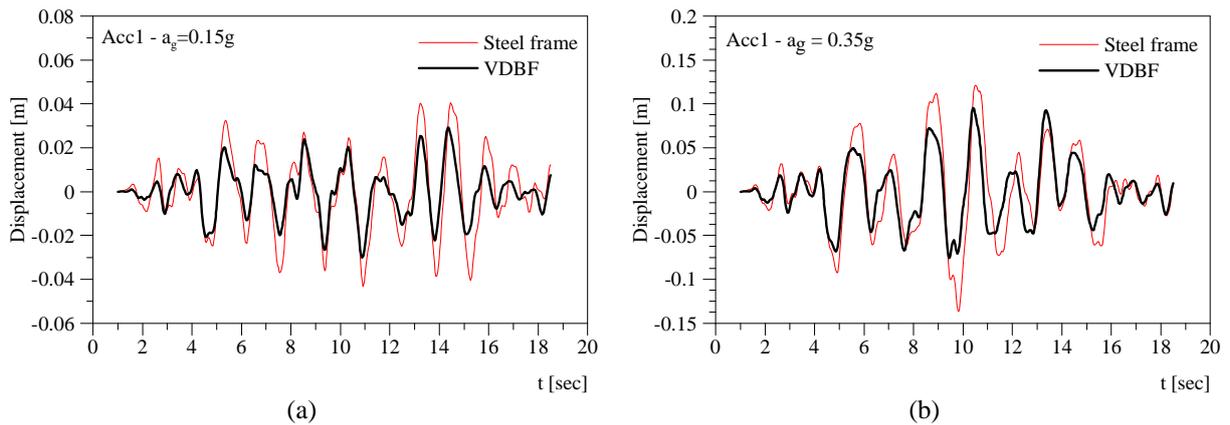
Incremental dynamic analyses (IDA) have been also conducted. Different levels of ground accelerations have been considered:  $a_g = 0.05g, 0.15g, 0.25g, 0.35g, 0.45g$ , where the second value can be assumed as the design value at DLS, while the fourth acceleration as the reference value at ULS.

Fig. 4.1. shows some IDA curves comparing the performance of the bare frame against that of the VDBF system. A specific ground acceleration has been used for each curve, which represents the variation of the base shear against top displacement. It can be observed how the use of dampers leads to a significant reduction in displacements and base shear, and how, in the case of VDBF, the results for different ground motions are very close.

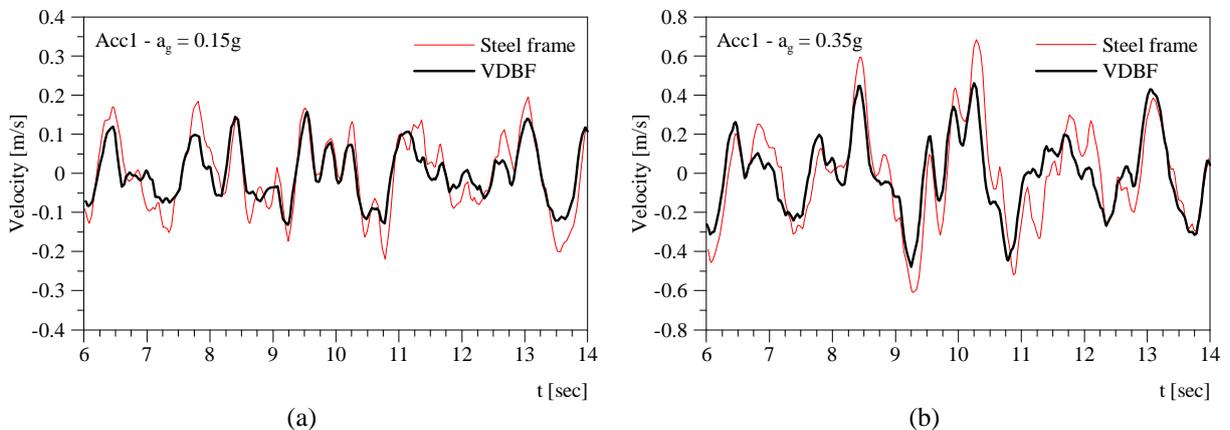


**Figure 4.1.** IDA for bare frame (a) and VDBF (b)

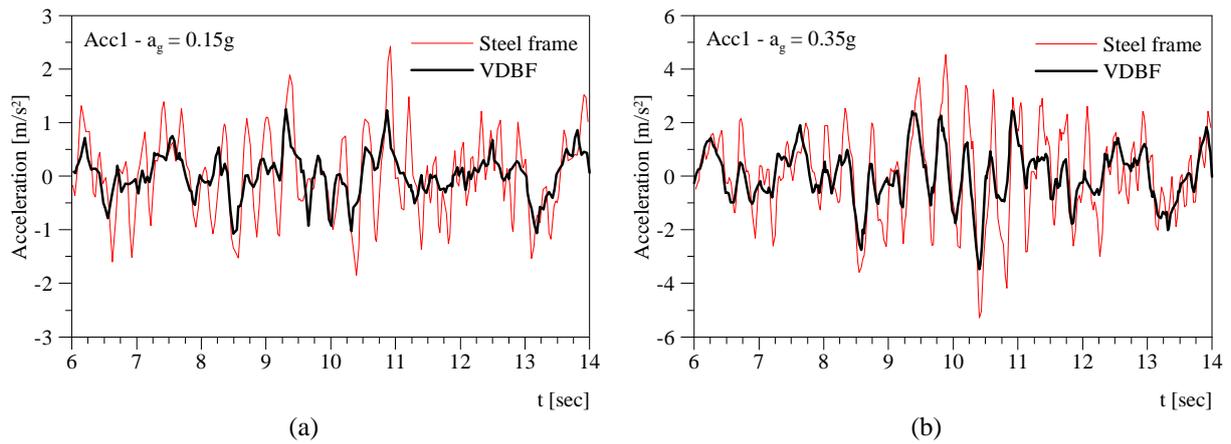
Figs 4.2,4.3,4.4 display the time history of displacement, velocity and acceleration, which allows the performance of the VDBF and the steel frame to be compared. The use of nonlinear viscous dampers leads to a significant reduction not only in maximum displacements, which can be obtained also using traditional steel bracings (Amadio et al. 2009), but also in maximum velocity and acceleration. Moreover the figures show a limited influence of the higher modes contribution in the dynamic response of the frame with nonlinear viscous dampers.



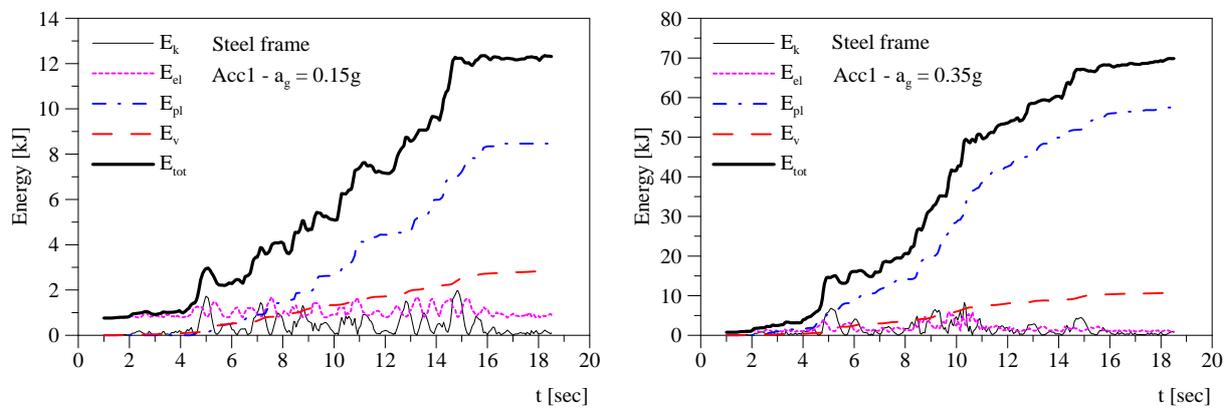
**Figure 4.2.** Time history of displacement at top floor for Acc1 with  $a_g = 0.15g$  and  $a_g = 0.35g$  for steel frame (a) and frame with nonlinear viscous dampers (b)



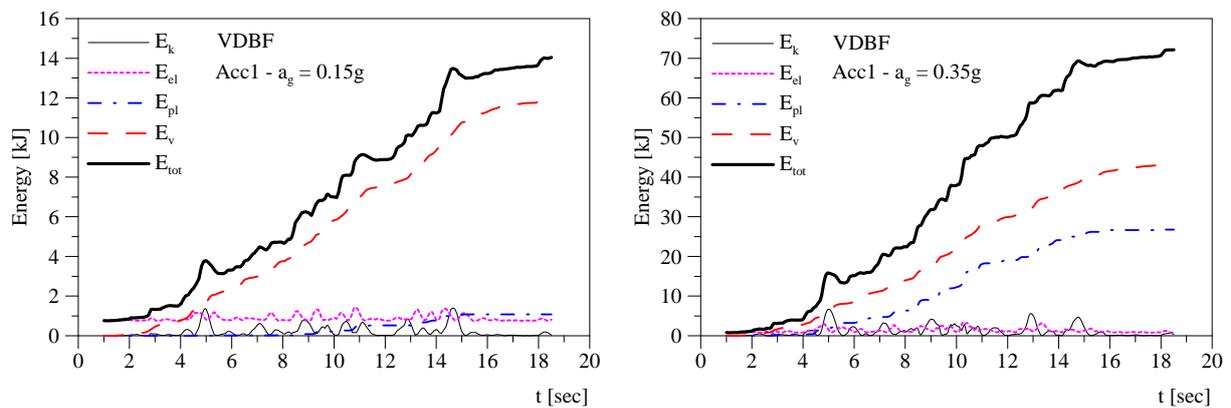
**Figure 4.3.** Time history of velocity at top floor for Acc1 with  $a_g = 0.15g$  and  $a_g = 0.35g$  for steel frame (a) and frame with nonlinear viscous dampers (b)



**Figure 4.4.** Time history of acceleration at top floor for Acc1 with  $a_g = 0.15g$  and  $a_g = 0.35g$  for steel frame (a) and frame with nonlinear viscous dampers (b)



**Figure 4.5.** Time history of Energy for steel frame under Acc1 with  $a_g = 0.15g$  and  $a_g = 0.35g$  for steel frame



**Figure 4.6.** Time history of Energy for VDBF under Acc1 with  $a_g = 0.15g$  and  $a_g = 0.35g$  for frame with nonlinear viscous dampers

Finally, Figs 4.5, 4.6 show the time history of different energy components for the steel frame and the hybrid system with viscous dampers, at different levels of ground acceleration. In particular, the total energy demand ( $E_{tot}$ ) is compared against the kinetic energy ( $E_k$ ), elastic energy ( $E_{el}$ ), viscous energy ( $E_v$ ) and plastic energy ( $E_{pl}$ ). The total energy demand for the two system is very close both at DLS ( $a_g=0.15g$ ) and ULS ( $a_g=0.35g$ ), while there is a significant different in the energy dissipation mechanism. Plastic energy is the more substantial contribution for the bare frame both at DLS and ULS. This points out a considerable plastic damage in the structural system that carries gravitational loading, also in the case of frequent seismic events. Conversely, in the frame upgraded with viscous

dampers, plastic damage is very limited at DLS and plastic deformations are reduced also at ULS. In this case, most of the energy supplied by the earthquake is dissipated by the viscous dampers.

## 5. CONCLUSIONS

In the paper the seismic performance of steel frames with semi-rigid joints equipped with nonlinear viscous dampers has been investigated. A practical strategy for designing new coupled systems or the seismic upgrading of existing steel frames has been suggested. It is based on the use of the Capacity Spectrum Method, where the analysed framed structure is represented by an equivalent SDOF system, and damped spectra are used to find the amount of supplemental damping required to satisfy the design objectives at different limit states. This approach has been checked using nonlinear dynamic analyses. In particular, the seismic response of a steel frame upgraded with nonlinear viscous dampers has been analysed. A detailed nonlinear model for steel frame components has been employed to describe plastic deformations. The numerical results confirm the accuracy of the proposed design procedure and point out the enhanced performance under earthquake loading of the VDBF system, which is characterised by reduced displacements, velocities, accelerations and limited plastic damage and base shear.

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