A Model for Nonlinear Total Stress Analysis With Consistent Stiffness and Damping Variation

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SUMMARY:
The paper discusses modelling of cyclic stress-strain behaviour of soil, in particular a simple model that can produce a desired stiffness and hysteretic damping for a given strain level as observed in laboratory testing is formulated. The unloading-reloading relationship is developed for total stress seismic site response analysis with appropriate damping at large strain. The constitutive model employs a hyperbolic equation as the backbone curve, and uses a modification of the extended Masing unloading-reloading relationship leading to correct measured modulus reduction and damping curves simultaneously. A quasi-static cyclic loading of increasing amplitude is used to demonstrate the model's performance and its capability to allow improved modelling of the magnitude of energy dissipation based on an experimental program on native sandy soils from Christchurch, New Zealand.

Keywords: Masing rule, Hyperbolic equation, Material damping

1. INTRODUCTION

The dynamic response of soil deposits beneath a site, commonly referred to as site response, has a significant influence on the ground motion hazard of engineered structures. The properties that typically need to be determined in order to characterize a particular soil site include shear modulus, G, and material damping ratio, h, amongst others. Shear modulus represents the shear stiffness of the soil and can essentially be considered as the slope of the shear stress - shear strain relationship and is denoted as tangent shear modulus, G_t. It can also be approximated as degree of inclination of a loop in the case of dynamic loadings as illustrated in Figure 1 and in this case, it is known as secant shear modulus, G_s. Damping ratio, h, is a measure of the proportion of dissipated energy to the maximum retained energy during a single cycle of shear deformation or simply a measure of breadth of the loop (Fig. 1).

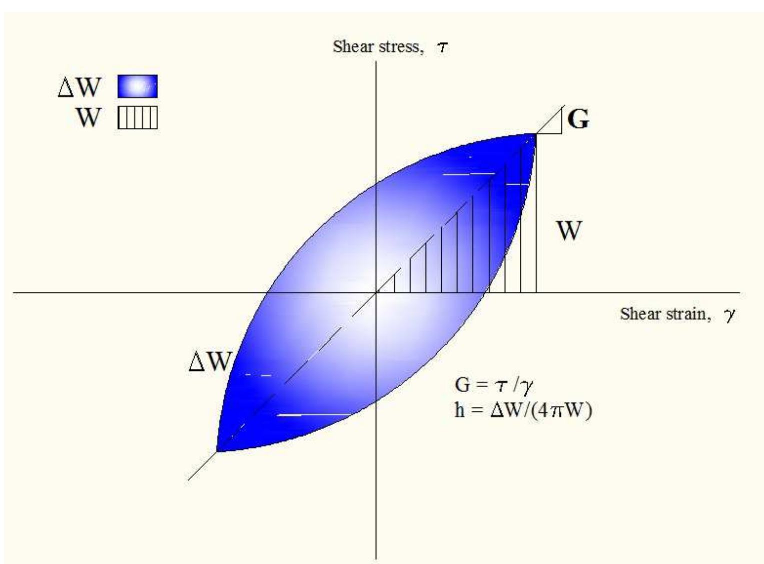

Figure 1. Secant shear modulus, G_s, and material damping ratio, h, during cyclic loading

The relationship between secant shear modulus, G_s, and shear strain amplitude is commonly characterised by shear modulus reduction curves (e.g. Fig. 2a). Also, the nonlinearity in the stress-strain relationship, which leads to energy dissipation per cycle, results in the material damping ratio, *h* increasing with shear strain (Figure 2b).

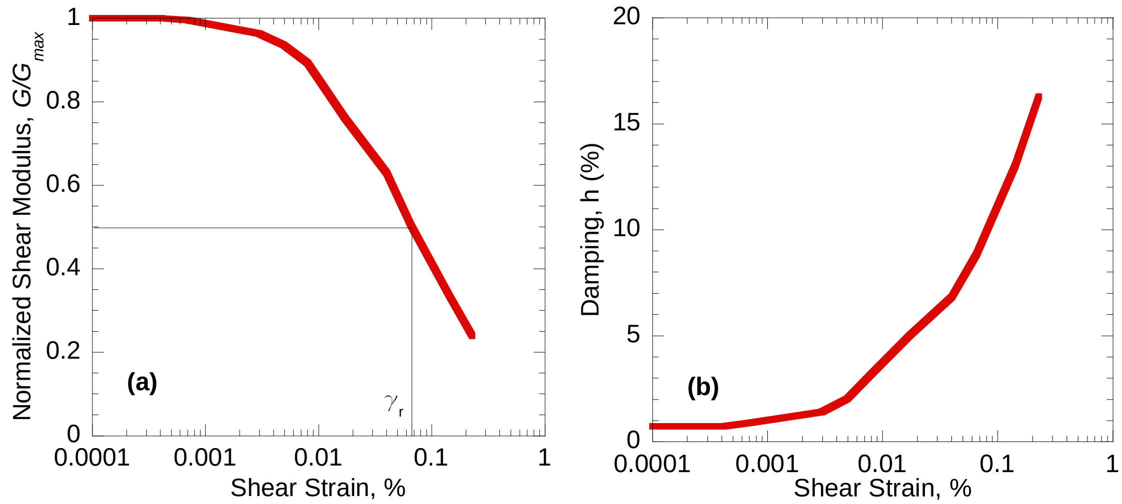

Figure 2. (a) Normalized modulus reduction curve and, (b) nonlinear material damping ratio curve

Mathematical models, which are capable of predicting soil response in future possible earthquakes, are required in order to theoretically understand local site effects. Depending to the desired level of accuracy and simplicity, three general broad classes of soil models have been proposed, namely equivalent linear models, cyclic total stress nonlinear models (Hardin and Drnevich, 1972, Ramberg and Osgood, 1943), and advanced constitutive models which incorporates pore pressure generation (Mroz, 1967, Momen and Ghaboussi, 1982, Dafalias, 1986, Kabilamany and Ishihara, 1990, Gutierrez et al., 1993, Cubrinovski and Ishihara, 1998). Equivalent linear analysis is the simplest and most widely employed scheme but has several important limitations. Whereas, advanced constitutive models can represent many details of dynamic soil behaviour, but numerous parameters which must be determined through laboratory and field tests limit its use for many common practical problems (Kramer, 1996).

Cyclic total stress nonlinear models can approximately simulate the actual stress-strain path during cyclic loading and therefore represent the shear strength of the soil for engineering purposes. These models generally have a backbone curve and a set of unloading-reloading rules which can represent the total stress behaviour of the soil (e.g. Kramer, 1996). Generally, the shape of the backbone curve is determined by the maximum shear modulus, G_{max}, shear strength, τ_{max}, and several curve-fitting parameters. Darendeli (2001) and Phillips and Hashash (2009) proposed a modified hyperbolic equation as a backbone curve based on an earlier work by Hardin and Drnevich (1972):

$$\mathrm{F_{bb}} = \tau = \frac{G_{\max}\gamma}{1+\beta\left|\frac{G_{\max}\gamma}{\tau_{\max}}\right|^{\alpha}} \tag{1}$$

where τ is shear stress; γ is shear strain, α and β are dimensionless factors. In the original form proposed by Hardin and Drnevich (1972) α = β = 1. In this model, the reference strain is defined as:

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}} \tag{2}$$

Using equations (1) and (2) the normalized modulus reduction curve can be evaluated as:

$$\frac{G}{G_{\max}} = \frac{1}{1+\beta\left|\frac{\gamma}{\gamma_r}\right|^{\alpha}} \tag{3}$$

A practical problem with the definition of reference strain, γ_r, is that the shear strength is often not available. Therefore, a pseudo-reference strain is proposed to be used for low to moderate strain levels (Stewart et al., 2008). The pseudo-reference strain is defined from a laboratory modulus reduction curve as the shear strain at which $G/G_{max} = 0.5$ (Figure 2.a). This definition is resulted from hyperbolic fits of modulus reduction curve according to equation (3). The advantage of using the pseudo-reference strain is that in the absence of material-specific tests, empirical relationships exist to predict it from other state parameters (Darendeli, 2001). α and β are fitting parameters generally taken as ≤ 1 and 1, respectively. If α value is adopted as greater than one, the shear stress reaches a maximum value at $\gamma_m = \sqrt[\alpha]{\frac{\gamma_r^{\alpha}}{\alpha-1}}$ (Fig. 1a).

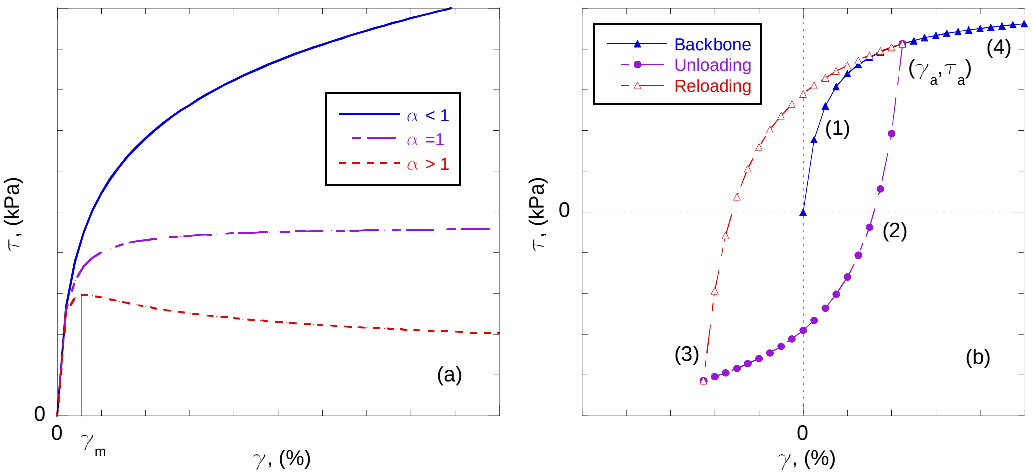

Figure 3. (a) Effect of α on hyperbolic curve, (b) a hyperbolic backbone curve and Masing unloading-reloading branches

Masing rules or modified Masing rules are often used in conjunction with the backbone curve to describe the unloading-reloading behaviour of soil. In the modified Masing rule, if a stress reversal occurs at a point defined by (γ_a, τ_a), the stress-strain curve is identical to the shape of backbone curve but enlarged by a factor of $n = 2$ (Fig. 3b), which is given by:

$$\frac{\tau-\tau_a}{n} = F_{bb}\left(\frac{\gamma-\gamma_a}{n}\right) \tag{4}$$

If the unloading or reloading curve exceeds the maximum past strain and intersects the backbone curve, it follows the backbone curve until the next reversal point. If an unloading or reloading curve crosses an unloading or reloading curve from the previous cycle, the path follows that of the previous cycle.

By applying Masing rule to a hyperbolic model, Ishihara (1996) showed that the damping ratio at each strain level can be obtained as:

$$h = \frac{4}{\pi}\left[1+\frac{1}{\frac{\gamma_a}{\gamma_r}}\right]\left[1-\frac{\ln\left(1+\frac{\gamma_a}{\gamma_r}\right)}{\frac{\gamma_a}{\gamma_r}}\right]-\frac{2}{\pi} \tag{5}$$

It can be seen that damping ratio converges to $2/\pi = 0.637$ when shear strain amplitude becomes infinitely large. The magnitude of damping predicted by Masing rule is not supported by experimental test results observed in the course of this study and values reported by others (Hardin and Drnevich, 1972, Ishihara, 1996, Darendeli, 2001, Stewart et al., 2008). Figure 4.a illustrates that using Masing criteria the area of the hysteresis loop is greater than that measured by experimental test data resulting in overestimation of damping ratio especially for higher shear strain level (Fig. 4b). The overestimation of hysteretic damping induced by employing Masing criteria can unconservatively lead to underestimation of some of the seismic response parameters especially in the higher frequency range (Stewart et al., 2008, Silva et al., 2000).

A solution of the aforementioned damping problem with Masing criteria has been proposed by Phillips and Hashash (2009). Based on an earlier work by Darendeli (2001), Phillips and Hashash (2009) developed a reduction factor to modify the unloading-reloading equations from those of the Masing criterion. Equation 6 presents the functional form for the damping reduction factor proposed by Phillips and Hashash (2009):

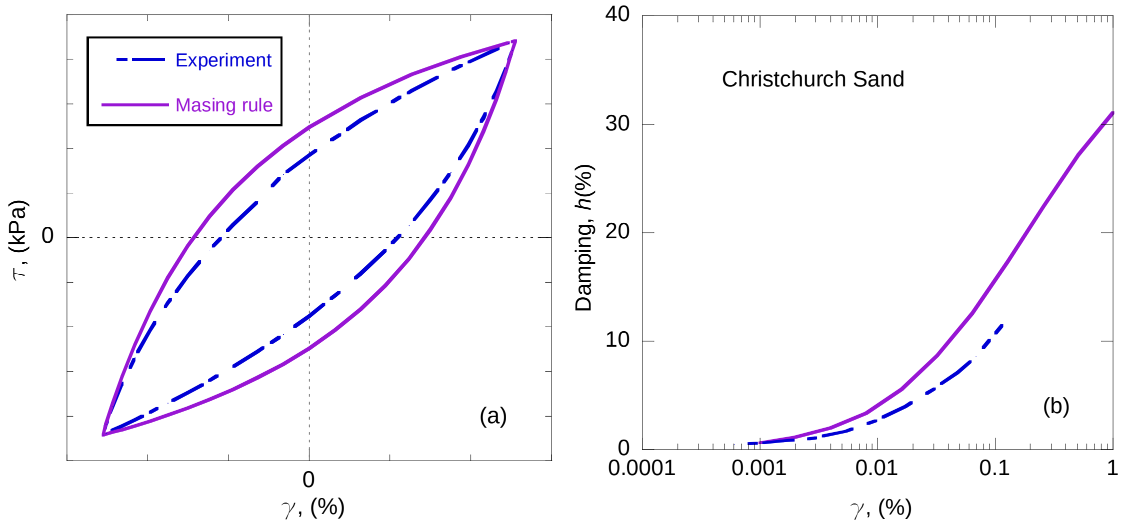

Figure 4. Comparison of experimental and Masing-based calculated hysteresis loop (a), damping ratio curve (b)

$$F(\gamma_m) = p_1 - p_2\left(1 - \frac{G_{\gamma_m}}{G_{\max}}\right)^{p_3} \tag{6}$$

where $G_{\gamma m}$ is secant modulus corresponding to the maximum shear strain level, γ_m, and p_1, p_2 and p_3 are non-dimensional coefficients selected to obtain the best possible fit with the target damping curve. This damping reduction factor given by equation (6) is used in the unloading-reloading relationship given by equation (7) (Hashash et al., 2010):

$$\tau = F(\gamma_m)\left[2\frac{G_{\max}\dfrac{\gamma-\gamma_a}{2}}{1+\beta\left(\dfrac{\gamma-\gamma_a}{2\gamma_a}\right)^a} - \frac{G_{\max}(\gamma-\gamma_a)}{1+\beta\left(\dfrac{\gamma_m}{\gamma_a}\right)^a}\right] + \frac{G_{\max}(\gamma-\gamma_a)}{1+\beta\left(\dfrac{\gamma_m}{\gamma_a}\right)^a} + \tau_a \tag{7}$$

While functional form in equations (6) and (7) provide an improved match between the experimental and mathematical damping curves, the tangent modulus at the point of reversal reduces with the reduction factor and therefore is not equal to G_{max}. This is inconsistent with experimental evidence for sand behaviour under cyclic loading (Hardin and Drnevich, 1972, Hardin, 1978).

Given a soil model for symmetrical loadings, Pyke (1979) proposed an alternative unloading-reloading rule in which the Masing coefficient n can deviate from two, in order to extend the Masing model for use with irregular loadings. A factor n greater than two allows simulation of cyclic hardening, while cyclic softening can be modelled by assuming a value of n less than two (Lo Presti et al., 2006). The objective of this paper is to illustrate that the same idea can be employed similarly, to simulate any target damping ratio curve by modifying the Masing criterion.

2. HYSTERETIC DAMPING FORMULATION

The backbone describing the monotonic stress-strain curve is modelled using the modified hyperbolic relationship stated in equation (1) in which β is assumed to be equal to 1. The cyclic behaviour, or unloading-reloading branches, has been modelled using a modified version of Masing criterion. A parameter, ϕ, is introduced for the unload-reload curves. Moreover, the parameter, n, is allowed to vary depending on the desired level of hysteretic damping.

To preserve the simplicity of the solution proposed by Masing (1926), as well as achieving a better agreement between the experimental and modelled hysteretic damping, two conditions need to be satisfied. First, the unloading-reloading curves for symmetrical periodic and cyclic loadings should form a closed loop for any level of shear strain and moreover, be similar in shape to that of the initial loading curve. Second, the tangent shear modulus on each reversal point should assume a value equal to the initial tangent modulus for the initial loading curve, G_{max}.

To meet the first condition, unload-reload equations are required in which the shear stress at reversal points be equal but with opposite signs; in other words, two points A(γ_a , τ_a) and B($-\gamma_a$, $-\tau_a$) should fall on the unloading and reloading branches. This can be confirmed by expanding equation (1.4) and using equation (1) to obtain:

$$\frac{\tau-\tau_a}{n}=\frac{G_{\max}\dfrac{\gamma-\gamma_a}{n}}{\left(1+\left|\dfrac{\gamma-\gamma_a}{n\gamma_r}\right|^a\right)^{\phi}} \tag{8}$$

It is obvious that points A and B both fall on the above curve considering a Masing coefficient n, to be equal to two. However, this may not be true for an arbitrary value of n. Solving equation (8b) for ϕ, and for a general n-value, and entering point B in the equation yields:

$$\phi=\frac{\ln\left(1+\left|\dfrac{\gamma_a}{\gamma_r}\right|^a\right)}{\ln\left(1+\left|\dfrac{2\gamma_a}{n\gamma_r}\right|^a\right)} \tag{9}$$

Therefore, any adopted combination of ϕ and n which satisfies equation (9) will result to a closed loop hysteresis (Fig. 5). The next step would involve the area of the loop to be in agreement with material damping curve, the former representing a measure of the hysteretic damping. A best n value can readily be obtained by iteration, matching the damping ratio from experimental test results and the one calculated by unloading-reloading rule. Once n is adopted, curvature variable ϕ, can be obtained using equation (9).

It can be shown that the derivative of the unload-reload equation at the reversal points is equal to the initial tangent modulus and hence the second condition remains valid.

For shear strain levels larger than reference shear strain γ_r, coefficient α in the backbone curve can be shown as an alternative parameter to be considered along with Masing coefficient n in order to match the experimental damping curve with the mathematical model.

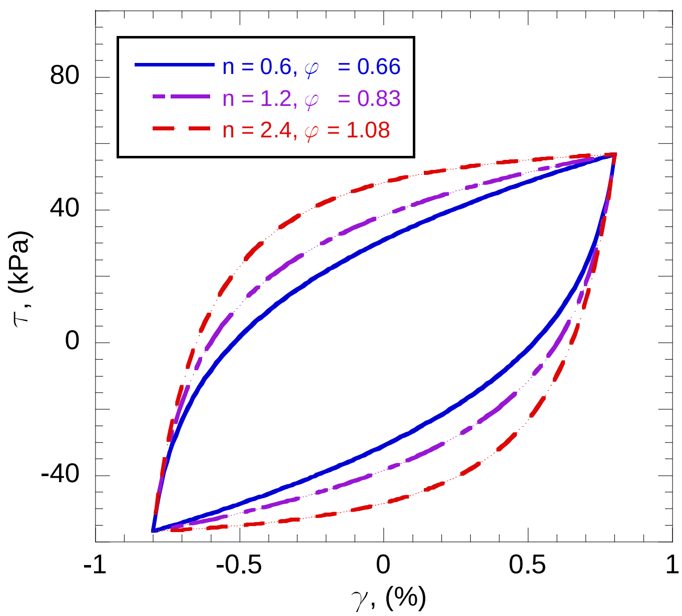

Figure 5. Varying Masing coefficient, *n*, and curvature parameter, ϕ

Hence to summarise, the proposed constitutive model utilizes equation (1.1) for the monotonic behaviour and equations (1.8) and (1.9) for the unloading-reloading equations in order to improve the match between the experimental and modelled hysteretic damping.

3. COMPARISON OF THE PROPOSED RELATIONSHIP WITH EXPERIMENT

A series of cyclic drained triaxial tests have been conducted on sand obtained from Fitzgerald Avenue Site in Christchurch, New Zealand. The confining pressure was kept constant at a value of 100 kPa for all tests examined below. Compression-extension loading cycles were imposed by a Servo controller at a constant frequency under drained conditions. The force applied to the specimen and the displacement at the top of the specimen was recorded in addition to cell pressure and volume change.

The moist tamping method was used for specimen preparation. A total of 10 layers of predetermined quantities of moist soil were worked into a prescribed thickness. CO_2 was percolated up through the specimen using pressure to promote full specimen saturation. De-aerated water was flushed through the sample and the sample was then saturated using a 100 kPa back pressure and left overnight. The soil sample was considered saturated when the Skempton *B*-value was equal to or more than 0.96. Each specimen was consolidated isotropically under confining pressure σ'_c = 100 kPa. The specimens were then loaded with a 0.10 Hz sinusoidal cyclic loading. Standards of Japanese geotechnical society (2000) for laboratory shear tests are employed as guideline to measure the dynamic properties of tested material.

Figure 6 illustrates the normalized modulus reduction and material damping curves are plotted only for clean sands with varying relative density. To obtain the G/G_{max} results from experimental results the secant Young's moduli obtained from cyclic triaxial tests were converted to secant shear moduli assuming Poisson's ratio, ν = 0.1.

In order to simulate the experimental modulus reduction curves shown in Figure (6a) with the hyperbolic equation given by equation (3), least square error method is employed to estimate the required parameters in the hyperbolic model. Each curve presented in Figure (6a) is considered separately in this case; therefore a unique set of parameters (α, γ_r) for hyperbolic equation is defined for each test.

Hardin and Drnevich (1972) illustrated that considering some assumptions, a relationship between shear strain and damping can be derived. Hence a similar hyperbolic equation can be established for material damping curve:

$$\frac{h}{h_{\max}} = \frac{\left(\dfrac{\gamma}{\gamma_r}\right)^a}{1+\left(\dfrac{\gamma}{\gamma_r}\right)^a} \tag{10}$$

where h_{max}, is the maximum value of the damping ratio which is suggested as between 25% and 33% for clean sands. A good fit between the equation (10) and material damping curve requires evaluating a different set of curvature coefficient, α and reference strain γ_r. Therefore, a similar approach to the modulus reduction curve is taken to fit equation (10) to experimental damping results. It is to be noted that equation (10) implies that at a very low strain range the damping ratio is close to zero, hence small strain damping must be separately accounted for in the viscous damping matrix for time domain solutions; this is shown with the solid line in Fig. 7b. This is in contrary to laboratory findings as illustrated by dotted points in Fig. 7b and also shown by Stokoe et al. (1999); but since the overdamping due to employing Masing's rules occurs at higher strain ranges, it is sufficient for the purpose of this work to use the above equation.

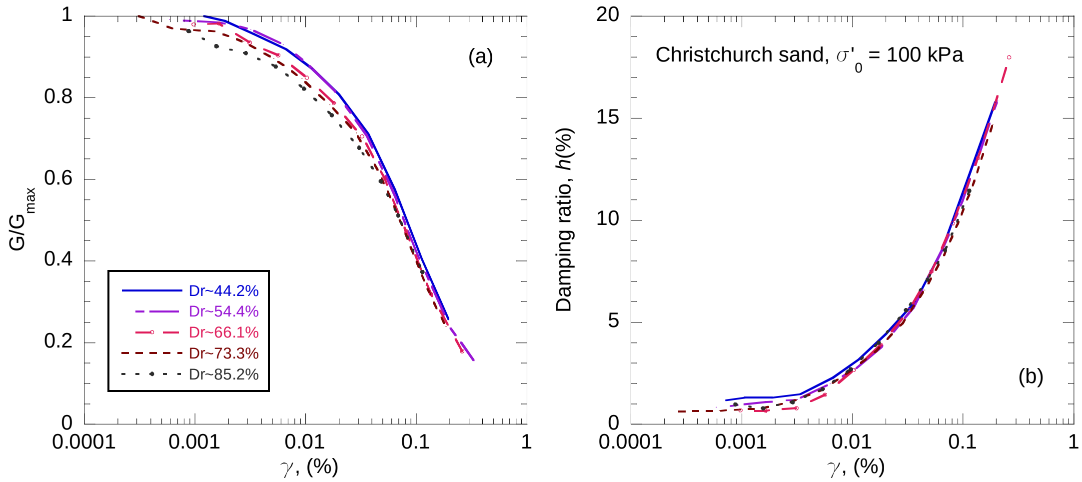

Figure 6. (a) Normalized modulus reduction, and (b) material damping curves for Christchurch sand

Figure 7 illustrates a comparison between the stress-strain and damping ratio-strain obtained using the aforementioned relationships in comparison to experimental results for a particular specimen, FB8.

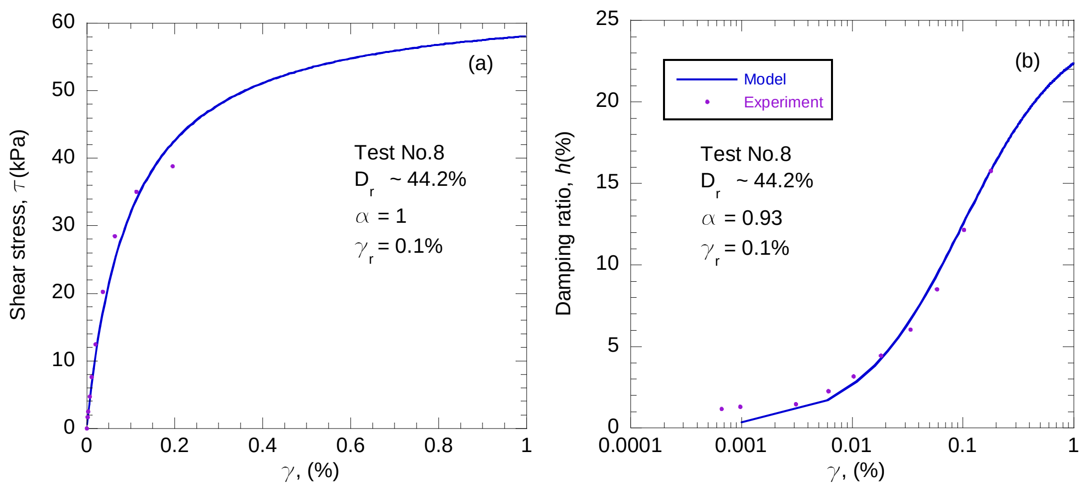

Figure 7. Comparison of hyperbolic stress-strain back-bone curve (a) and material damping curve (b) with experimental test results for test FB-8

4. RESULTS

A symmetrical cyclic shear strain time series was employed to evaluate the performance of the proposed model. Subsequent strain cycles are obtained as twice the amplitude of the previous cycle (Fig. 8a). Based on the imposed shear strain time series in Fig 8a, the shear stress response in Fig 8.b was computed using the procedure explained in section 2.

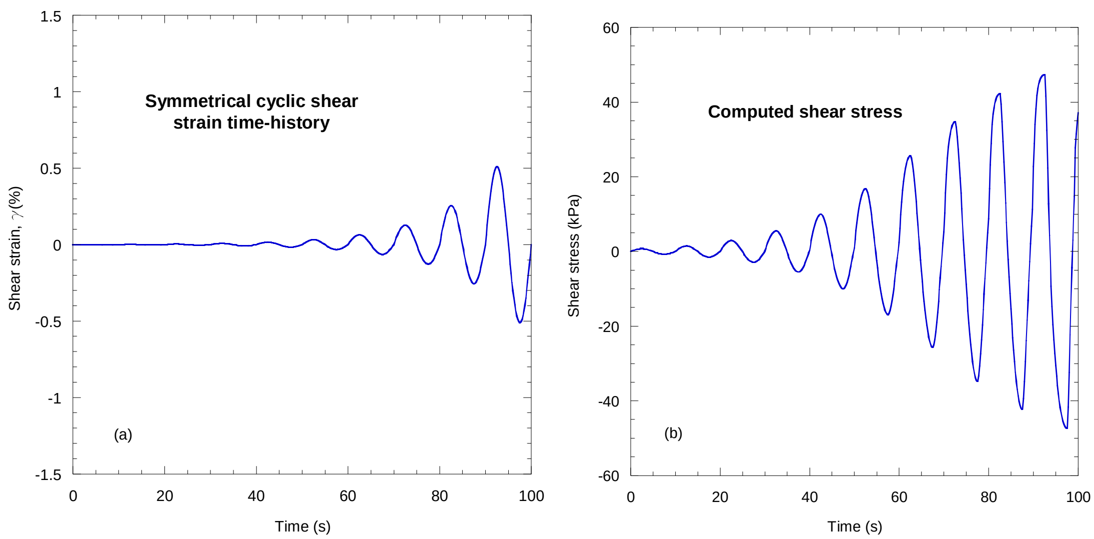

Figure 8. (a) Input shear-strain time history, (b) computed shear stress time history for test FB-5

Fig. 9 presents stress-strain hysteresis loops generated by employing either Masing criteria or modified Masing criteria explained above. The solid line represents the modified Masing behaviour developed here and the dashed line of the conventional Masing rule. It is illustrated that the breadth of the loops for higher strain levels are smaller for modified relationships implying lower hysteretic damping. This difference between the two methods in damping is explicitly illustrated in Fig. 9b, which also illustrates the experimental test results. Triaxial results for a particular test (FB-5) are represented by open circles. It can be seen that modified Masing rule is capable of capturing experimental damping ratios quite well.

It is to be noted that the tangent shear modulus at the reversal points are equal to initial shear modulus, G_{max}. This is not affected by increase of cyclic shear strain amplitude.

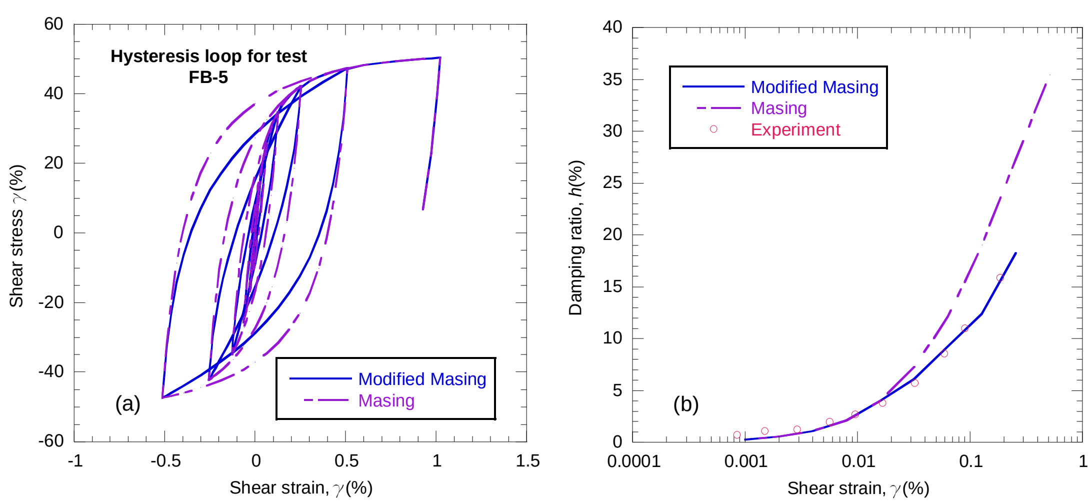

Figure 9. (a) Comparison of stress-strain hysteresis loops, and (b) damping ratio curves using Masing and modified Masing criterion

The Masing coefficient, *n*, is not constant and therefore varies with increase of shear strain level. In Figure 10, n-values are plotted against shear strain for sands of variable relative densities. A smoother plot can be obtained assuming smaller shear strain increments in the input time history introduced in Figure 8a. An n-value smaller than two is required to circumvent the overdamping issue introduced by employing Masing criterion.

It is envisaged that a similar procedure can be carried out for different types of soils having varying conditions in order to develop a relationship between *n*-value and shear strain amplitude. The computed *n*-values can then be employed directly in 1-D site response analyses to better simulate the experimental modulus and damping curves for a wide strain range. A functional form can be obtained for *n*-value in terms of shear strain, relative density, fines content etc. carrying out similar procedures.

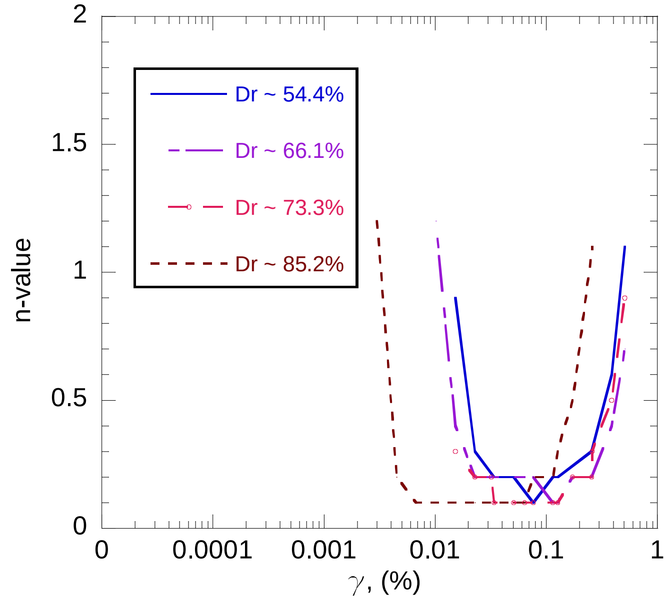

Figure 10. Masing factor versus shear strain level for different clean sand with variable relative density

5. CONCLUSIONS

A new simple equation was proposed for modelling of unloading-reloading branches of cyclic stress-strain hysteresis loops for sandy soils. The proposed model uses the hyperbolic model as the backbone to represent the modulus reduction curve. A simple cyclic shear strain time series was employed to compute the shear stresses and evaluate the performance of the model.

It was shown that the equation is capable of capturing any desired level of energy dissipation as a function of shear strain in contrast to conventional models which tend to overestimate damping. A parameter ϕ, can be obtained for any given shear strain amplitude in order to match the damping produced by the numerical model with the observed behavior in the laboratory. Therefore, both the modulus reduction and damping curves can be simulated simultaneously.

It was illustrated that Masing coefficient *n*, should be smaller than two, in order to match the hysteretic damping with the measured strain dependent damping.

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