Influence of 1D and 2D Spatial Variability on Site Response Analysis

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SUMMARY:
The main soil properties required for site response analysis are the shear-wave velocity (Vs) profile and the nonlinear modulus reduction and damping curves. Spatial variability and uncertainties in these properties across a site are often taken into account by modeling multiple one-dimensional (1D) profiles in 1D site response analyses. However, this approach assumes that analyzing multiple 1D profiles captures accurately the effects of the true multi-dimensional spatial variability of the soil properties. This study compares the results of 1D and two-dimensional (2D) site response analyses that incorporate spatial variability in the Vs profile through Monte Carlo simulation. The surface response spectra computed by the 2D analysis along a 100-m wide horizontal segment of the 2D random field were compared with the surface response spectra computed by multiple 1D analyses each using a different velocity profile. The results indicate that 1D analyses of multiple 1D velocity profiles can provide a similar median response as 2D analyses. However, the 1D analyses display more variability in the responses, particularly when the horizontal correlation between velocities is ignored.

Keywords: One-dimensional site response analysis, two-dimensional site response analysis, Monte Carlo simulation, spatial variability of shear wave velocity.

1. INTRODUCTION

Predicting site response is an important part of geotechnical earthquake engineering. Site response analyses are performed to evaluate the influence of the local soil deposit on strong ground shaking, with the resulting surface response spectrum being used in seismic design. The main soil properties required for site response analysis are the shear-wave velocity (Vs) profile and the nonlinear modulus reduction and damping curves. Evaluating design response spectra at a site requires considering the spatial variability and the uncertainties in these properties across a site. The traditional approach is to perform one-dimensional (1D) site response analyses for multiple 1D profiles. Thus, this approach assumes that the two-dimensional (2D) and three-dimensional (3D) spatial variability of soil properties can be modeled by analyzing multiple 1D profiles.

The goal of this study is to compare the results of 1D and 2D site response analyses that incorporate spatial variability in the Vs profile through Monte Carlo simulation. The analyses are performed for a 2D random field of shear-wave velocity developed for a 100-m deep alluvium site. Two dimensional equivalent-linear site response analyses are performed for a generated 2D random field and 1D equivalent-linear site response analyses are performed for the vertical 1D profiles within the generated 2D random field. Additionally, 1D analyses are performed for a 1D random field and compared to the results from a 2D random field. Specifically, this study compares median predictions of the surface response spectra across a 100-m wide region of interest and the associated standard deviation from 1D and 2D analyses.

2. AVAILABLE TECHNIQUES TO PREDICT SITE RESPONSE

The characteristics of ground shaking during earthquakes are influenced by the earthquake the source, the propagation of stress waves from the source through the earth, and the local soil deposits that overlie the bedrock (Kramer, 1996). In engineering practice, the earthquake source and propagation path are simply modeled by earthquake magnitude (M) and site-to-source distance (R), respectively, and these values are used within an empirical ground motion prediction equation (GMPE) to predict ground shaking at a site. Using the GMPE to predict ground shaking for rock conditions, the remaining task is to evaluate the response of the soil deposit due to this bedrock motion. Researchers have generated various numerical approaches to predict the surface response of a soil deposit due to strong ground motion. These approaches model the nonlinear soil response through fully nonlinear constitutive models or through the equivalent-linear (EQL) approximation, in which a linear elastic analysis is performed with the soil properties (i.e., shear modulus and damping ratio) selected to be compatible with the induced shear strain. In addition to the approach used to model nonlinear soil behavior, the site response methodologies are categorized based on the dimensionality of the problem they can address. Even though the true problem is three-dimensional (3D), researchers often focus on two-dimensional (2D) or one-dimensional (1D) models because of the complexity of 3D analysis. In practice 1D analyses are almost always performed for site response analysis.

The use of 1D site response analysis is often justified by considering the direction of wave propagation from the source to the site. Initially inclined waves become predominantly vertical as they propagate through the lower velocity layers encountered closer to the ground surface. As a result, 1D analyses that model only vertically propagating shear waves as they propagate from the underlying bedrock through the overlying soil layers has been deemed appropriate for site response studies. The layers in a 1D analysis extend infinitely in the horizontal direction such that horizontal variations in the soil properties cannot be taken into account. Yet, soil properties may vary significantly in the horizontal direction. The conventional approach to account for the spatial variability in site properties is to run multiple 1D site response analyses, each with a different velocity profile. However, multiple 1D analyses may not be sufficient to reflect the effect on the predicted site response of the existing adjacent soil elements with different material properties. Two-dimensional site response analyses in which the properties vary both vertically and horizontally can be used to evaluate the effect of the horizontal soil property variability on the ground response.

One-dimensional, EQL site response analysis is commonly performed in the frequency domain because of the computational efficiencies gained in the frequency domain and because the damping can be modeled as frequency-independent. Two-dimensional site response analyses are generally solved by dynamic finite-element analysis in the frequency domain or time domain. The finite element method assumes the 2D field is composed of discrete quadrilateral elements whose corners are defined by nodal points. The 2D EQL approach is similar to the 1D EQL approach in that the modulus and damping ratio for each element are iteratively modified until they are compatible with the induced shear strains for that element. When performed in the time domain 2D EQL analysis incorporates Rayleigh damping to model the viscous damping, which results in frequency-dependent damping. In this study, the computer program Strata

(Kottke and Rathje 2008) is used to perform 1D site response analyses and QUAD4M (Hudson et al., 1994) is used to perform 2D site response analyses, as well as some 1D analyses.

3. INPUT SITE AND GROUND MOTION CHARACTERISTICS

The ground response analyses were performed for the Sylmar County Hospital site located in the San Fernando Valley of Southern California. The general soil profile and shear wave velocity profile of the site (Kottke 2010) are shown in Figure 1. The site has 100 m of alluvium above bedrock, with the Vs ranging from about 250 m/s at the surface and increasing to above 700 m/s at 60 m (Kottke 2010). The nonlinear modulus reduction and damping curves were assigned to the four main velocity layers based on the empirical model of Darendeli (2001). Damping of the bedrock was assumed to be 1 %. The small strain natural period of the site is about 0.6 s. The shear wave velocity profile presented in Figure 1 was used as the baseline profile to generate a 2D random field and multiple 1D profiles of Vs.

The region of interest over which the ground surface response is investigated is assumed to be 100 m wide. Therefore, the site response of a 100 m x 100 m field with spatially varying soil profiles is considered. The geometry of the 2D finite element mesh is illustrated in Figure 2. The area shown by the red square is the 100-m wide region of interest and the 2D mesh extends a significant distance outside the region of interest (ROI) to minimize the possible effects of the boundary conditions. It was found that extending the mesh 250 m to either side of the ROI (i.e., 2.5 times ROI_{width}) resulted in a stable response across the ROI without boundary effects. The mesh is formed by 10 m by 10 m 4-node elements. The size of the elements is sufficient to capture frequencies up to about 10 Hz for the lowest velocity layers.

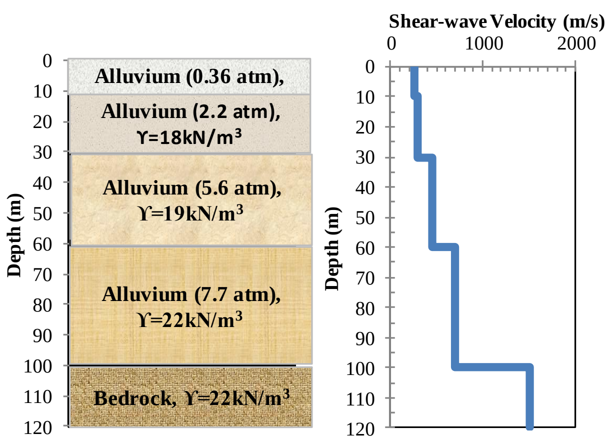

Figure 1. Site profile with corresponding baseline shear-wave velocity profile

Monte Carlo simulations are used to generate the 2D random fields used in the 2D site response analyses. A 2D random field of Vs is generated by developing a covariance matrix that represents the strength of correlation between velocities at different locations. This covariance matrix is defined by a separable model with exponential correlation functions in the x and z directions (e.g., Vanmareke 1983):

$$COV(i,j) = \sigma_{lnVs}^2 \cdot \left[\exp\left(\frac{-2|x_i - x_j|}{\theta_x}\right)\right] \cdot \left[\exp\left(\frac{-2|z_i - z_j|}{\theta_z}\right)\right] \qquad (1)$$

in which i and j represent the average shear wave velocities in 10 m x 10 m elements i and j, located respectively at (x_i, z_i) and (x_j, z_j). In equation (1), σ_{lnVs} is the standard deviation of the natural logarithm of

Vs (averaged over each 10 m x 10 m element), while θ_x and θ_z are the horizontal and vertical correlation distances, respectively for Vs. The standard deviation of ln(Vs) is used because shear wave velocities have been shown to be reasonably represented by a log-normally distributed (Toro 1995). The exponential terms in (1) represent the correlation between velocities. The correlation model varies from 1.0 when considering the velocities of adjacent locations (i.e, $x_i \sim x_j$, $z_i \sim z_j$) and tends towards zero as distances exceed the correlation distances. The covariance matrix along with a median velocity profile and a vector of independent random variables randomly generated from a standard normal distribution are used to generate the 2D velocity field (Vanmarcke 1983).

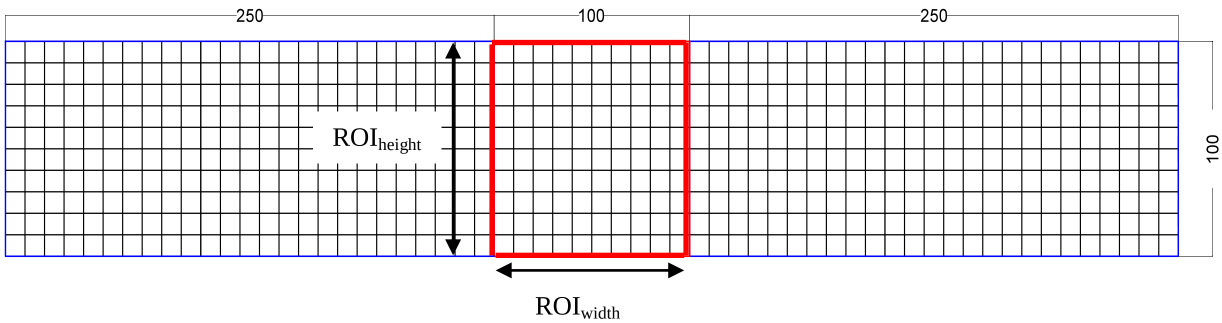

Figure 2. 2D finite element mesh used for 2D site response analyses (all dimensions are in m). Region of interest shown in red.

The Vs profile presented in Figure 1 is used as the baseline profile and a 2D random field is generated with a logarithmic Vs standard deviation (σ_{lnVs}) of 0.2, a vertical correlation distance (θ_z) of 80 m (i.e., θ_z=0.8ROI$_{width}$), and horizontal correlation distance (θ_x) of 100 m (i.e., θ_z=ROI$_{height}$). Two-dimensional site response analyses were performed for 20 generated realizations of the 2D random field to accurately represent the effect of spatial variability in the Vs profile.

Two sets of randomized profiles are used for the 1D site response analyses. The first set of profiles corresponds with the previously generated 2D random fields. These Vs profiles represent the columns of elements within the region of interest shown within the red square in Figure 2. 1D site response analyses with these Vs profiles allow for the direct comparison with the 2D site response predictions for the same velocity profiles. The second set of 1D profiles are obtained from multiple 1D Vs profile realizations generated through Monte Carlo simulation of a 1D random field. The 1D random fields were created using the baseline Vs profile in Figure 1 and statistical parameters of σ_{lnVs} = 0.2 and θ_z = 80 m. Note that there is no horizontal correlation distance within the 1D random field because the horizontal correlation is not modeled. Two hundred 1D profiles were generated such that there are 20 realizations of groups of ten 1D profiles, similar to the 20 realizations of the 2D random field across the ROI.

Site response analyses were performed using the input bedrock motion recorded during the 1985 Michoacan, Mexico earthquake (M_w=8.1) at Caleta de Campos Station. The station was located 38 km away from the source and the recorded peak ground acceleration for the motion is about 0.14 g. The 5% damped acceleration response spectrum of this rock motion is shown in Figure 3.

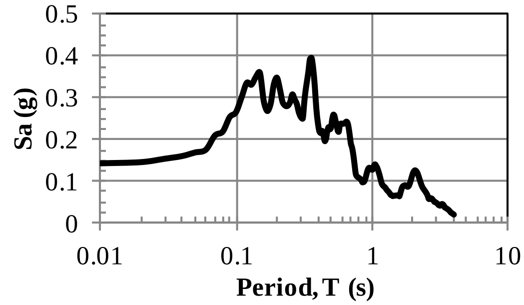

Figure 3. Response spectrum of the Caleta de Campos motion recorded during 1985 Michoacan earthquake

4. SITE RESPONSE ANALYSES

4.1. 2D Site Response Analysis of 2D Random Fields

The shear wave velocity distribution across the ROI is influenced by the horizontal and vertical correlation distances. As the correlation distance approaches zero the velocity field becomes more random (i.e., uncorrelated), while larger correlation distances result in more correlated velocities within the ROI. Figure 4(a) plots the velocity profiles versus depth for the 10 columns of elements across the ROI for a single 2D realization. Because of correlation distances used to generate the velocity fields (θ_z = 80 m, θ_x = 100 m) are similar in size to the ROI, the velocities tend to assemble on one side of the baseline profile, such that the median velocity profile across the ROI deviates from the baseline. However, over all 20 realizations (Figure 4(b)) the median velocity profile across the ROI is similar to the baseline.

Site response analyses were performed for each generated 2D velocity field. The ground surface motion was calculated at the surface along the ROI (at 11 nodes). For each realization the computed response spectra at the 11 nodes along the surface of the ROI were used to calculate the average of the natural log of the spectral acceleration (Sa) across the surface of the ROI, the median value for Sa across the surface of the ROI where $(\text{Median Sa})_{ROI}=e^{(\mu Sa)ROI}$ and the standard deviation of lnSa (σ_{lnSa}) across the surface of the ROI.$(\text{Median Sa})_{ROI}$ is intended to simplistically represent the response of a single 100 m (ROI_{width}) wide structural system . The variability in the average lnSa from realization to realization (Fig 5a), which is presented as a standard deviation $(\sigma_{\mu lnSa})_{ROI}$, is intended to represent the uncertainty in the response of a single 100 m wide structural system due to 2D spatial variability in Vs. The variability in lnSa for each 10 m wide element across the ROI, quantified as $(\sigma_{lnSa})_{ROI}$ is intended to represent the variability in the response of small-scale (10 m wide) structural elements within the ROI.

Figure 5 illustrates the median Sa and σ_{lnSa} versus period for each realization, as well as the average values over the 20 velocity realizations. The median surface responses generally show a major peak at a period of around 0.36 s and subsequent peaks are observable around 0.6 s and 1.0 s. However, the amplitudes of the response spectra from realization to realization vary considerably, with the maximum Sa ranging from 0.6 g to more than 1.2 g over the 20 different realizations. The variability in the median Sa is largest at periods less than 1.0 s. Longer periods are not influenced by variations in shear wave velocity because these periods are longer than the first mode period of the site. The σ_{lnSa} across the ROI is quite variable from realization to realization, but on average is 0.1 at shorter periods and peaks at a value of 0.25 at a period of 0.3 s.

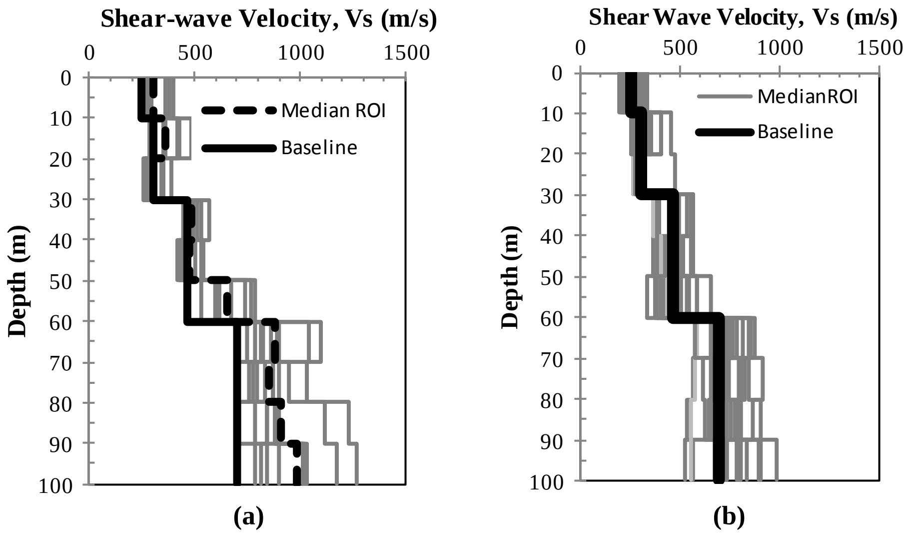

Figure 4. (a) Vs profiles across the ROI for one realization of the 2D random field, (b) median Vs profile across ROI for each realization of the 2D random field.

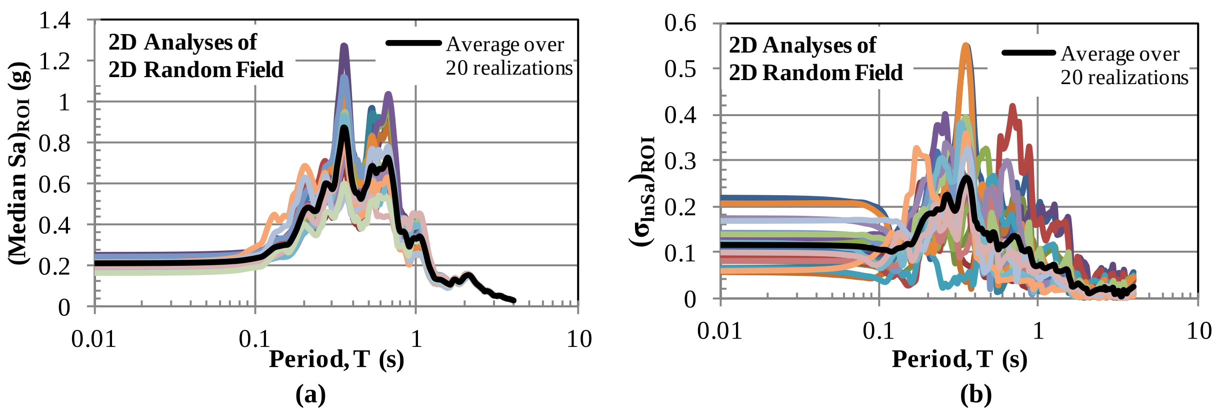

Figure 5. Results of the 2D site response analyses for each realization of the 2D random field: (a) the average spectral acceleration across the ROI, and (b) the logarithmic standard deviation of the spectral accelerations across the ROI(θ_z = 80 m, θ_x = 100 m)

4.2. 1D Site Response Analysis of 2D Random Fields

To explore the ability of multiple 1D site response analyses to accurately model the effects of 2D variations in shear wave velocity, 1D site response analyses were performed for the Vs profiles of the 10 columns of elements across the ROI of each realization of the 2D random field. Here, the 1D site response analyses were performed with QUAD4M (Hudson et al., 1994) for a single column, such that no differences in the solution procedure (i.e., time domain vs. frequency domain) are introduced. The 1D site response results are processed in a similar way as the 2D analyses; the median Sa and σ_{lnSa} of the 10 1D Vs profiles across the ROI are computed, as well as the average of these values across the 20 realizations. The 1D results are shown in Figure 6. The average response obtained from the 1D analyses of the 2D velocity fields (Figure 6(a)) is within ±10% of the 2D analyses of the 2D velocity field (Figure 5(a)), although the variability from randomization to randomization is smaller for the 1D analyses. The σ_{lnSa} across the ROI for the 1D analyses (Figure 6(b)) is about 0.1 at shorts periods and peaks at values of about 0.2 at periods of around 0.2 s and 1.0 s. Compared with the results for the 2D analyses (Figure 5(b)), the maximum σ_{lnSa} is similar between the 1D and 2D results, but the peaks occur at completely different periods.

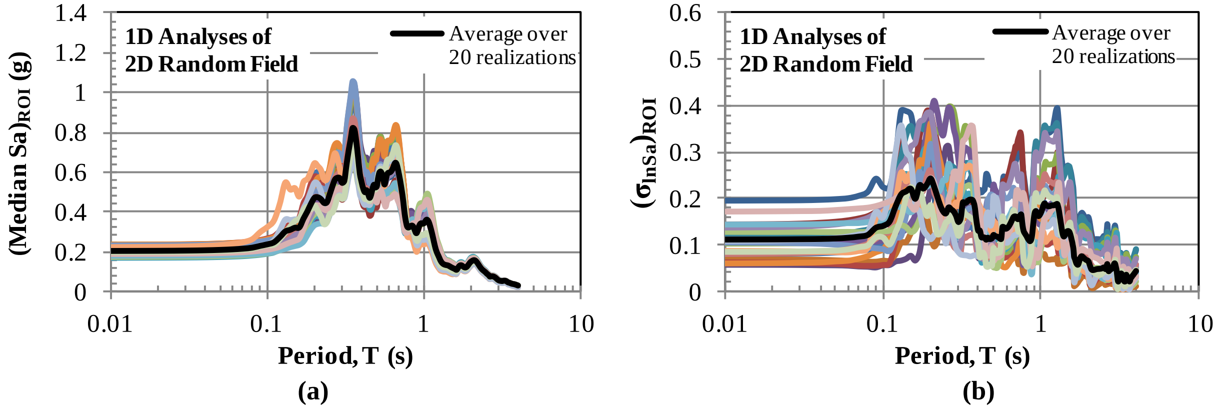

Figure 6. Results of the 1D site response analyses for each realization of the 2D random field: : (a) the median spectral acceleration across the ROI, and (b) σ_{lnSa} across the ROI(θ_z = 80 m, θ_x = 100 m)

4.3. 1D Site Response Analysis of 1D Random Field

In practice, spatial variability in Vs is modeled in 1D site response analyses by generating multiple 1D Vs profiles without any consideration of the horizontal correlation between velocities. The effect of this approach is investigated by generating multiple 1D Vs profiles using the Monte Carlo Simulation technique with σ_{lnVs} of 0.2, a vertical correlation distance (θ_z) of 80 m, and a horizontal correlation distance (θ_x) of 0 m (i.e., velocities are uncorrelated horizontally). Two hundred 1D Vs profiles were generated and 1D equivalent-linear site response analyses were performed for each profile using software program Strata (Kottke and Rathje, 2008). The results of the analyses are binned into 20 realizations of 10 profiles to have a comparable number of realizations as the 2D analyses (i.e., there are 20 realizations of each 10 columns in the ROI). Figure 7 presents the results in terms of the median Sa and σ_{lnSa} of each of the realizations of 10 1D Vs profiles (i.e., ROI), as well as the average of these values across the 20 realizations. The average response is again similar to the previous analyses (Figure 7(a)), but for these analyses the variability from realization to realization is significantly reduced. This reduced variability between realizations can be explained by considering the variability in the Vs profiles within each realization when the horizontal correlation distance is set to zero. Figure 8(a) plots the velocity profiles for one realization of 10 profiles. This plot shows that while there is variability between the profiles, the median profile is close to the baseline profile. This observation is true for each realization (Figure 8(b)), while for the profiles developed with θ_x = 100 m (Figure 4(b)) the variability between the median velocities in each realization is more significant. This leads to less variability between the median responses computed for the different velocity field realizations when θ_x = 0 m. The σ_{lnSa} values for the 1D analyses of the 1D random field range from 0.15 at short periods to peaks of 0.25 to 0.3 at periods of 0.15 s and 1.2 s (Figure 7(b)).

4.4. Comparisons

Figure 9 compares the results from the 2D and 1D analyses. The results are compared with respect to the median Sa across the ROI averaged over 20 realizations and the σ_{lnSa} across the ROI averaged over 20 realizations. The median responses are very similar for all analyses (Figure 9(a), ± 5 to 10%), showing that multiple 1D analyses can represent the median response of a 2D random field, even if the horizontal correlation distance is ignored in the 1D analysis. The biggest difference is between periods of 0.6 and 0.8 s where the 2D analysis is 10% larger than either of the 1D analyses. Even though the differences between

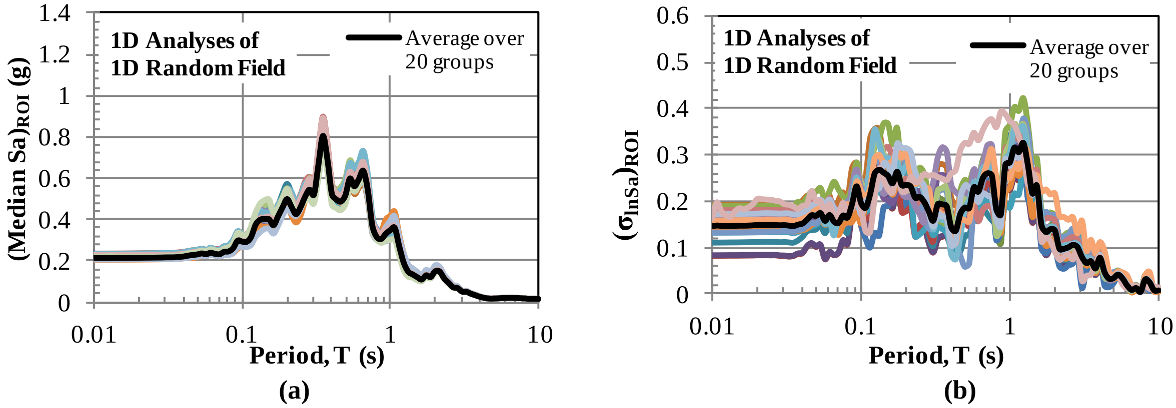

Figure 7. Results of the 1D site response analyses of each realization of the 1D random field : (a) the median spectral acceleration across the ROI, and (b) $\sigma_{\ln Sa}$ across the ROI(θ_z = 80 m, θ_x = 0 m)

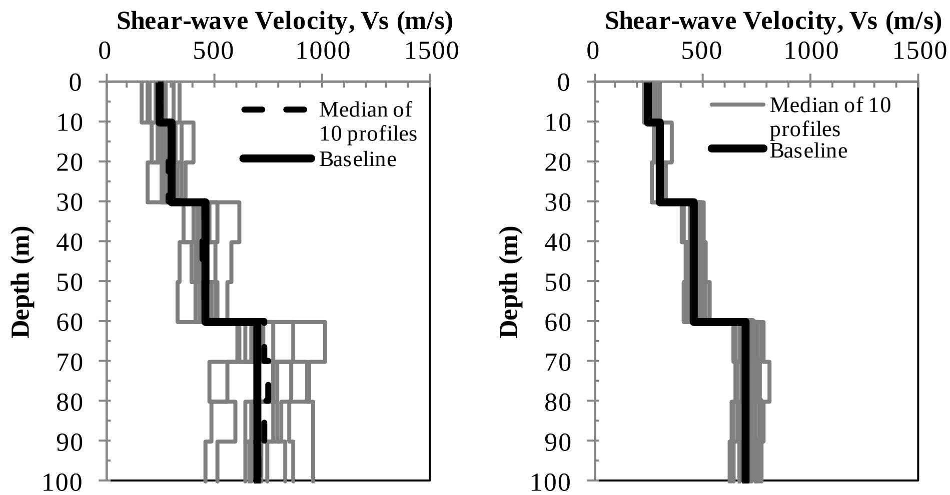

Figure 8. (a) Vs profiles across the ROI for one realization of the 1D random field, (b) median Vs profile across ROI for each realization of the 1D random field.

the median responses are not very significant, they are still noticeable. These differences observed between the 1D and the 2D analyses of the 2D random field are not systematic differences due to randomization, they reflect the real difference between the 2D and the 1D site response. The difference becomes more noticeable around the peak responses observed around the typical periods of a structure (0.3 s to 1.0 s), which might play an important role in structural design. The median response of the1D analyses of the 1D field and the 1D analyses of the 2D field matched quite well. This result verifies the randomization theory and provides a baseline check.

The variability of lnSa across an ROI differs substantially between 1D and 2D analyses (Figure 9(b)). In the 2D analyses the variability peaks at the period of maximum Sa in the input motion (~ 0.35 s in this case), while for the 1D analyses the variability peaks at the degraded site period (~1.0 s for this site) and some higher modes (~0.15 s for this site). The $\sigma_{\ln Sa}$ from the 1D analyses of 1D random fields can be as much as 3 times as large as for 2D analyses of a 2D random field. In the 1D analyses the variability is the largest when the horizontal correlation distance is ignored (i.e., 1D field) because this leads to the most variable Vs values across the ROI. The large variability observed for the 1D analyses of 1D field verifies the theory instead of providing an insight on the effect of the horizontal correlation distance because the observed difference between the 1D analyses of the 2D and the 1D field is relative to the ROI. If the ROI is infinitely wide, the observed variability would have been similar.

Figure 9(b) interprets the multiple 1D profiles as representing spatial variability across an ROI. In practice, the results from multiple 1D profiles are typically interpreted to represent the uncertainty in the median response across an ROI (i.e., each 1D profile represents an estimate of the median surface response across the ROI in the same way that one 2D realization represents an estimate of the median surface response across the ROI). Within this interpretation, the relevant standard deviation for the 2D analyses is the standard deviation of the median surface response spectra across the ROI for the 20 2D realizations (i.e., the σ_{lnSa} of the 20 spectra in Figure 6(a)). This standard deviation is compared with the standard deviation of the median response spectra across the ROI from the corresponding 1D realizations (i.e., the σ_{lnSa} of the 20 spectra in Figure 7(a)), as well as with the standard deviation of the median response spectra from 20 1D realizations of the 1D random field. These results are plotted in Figure 9(c) and generally show that the 1D analyses of a 1D random field (i.e., horizontal correlation distance equal to 0) produces the largest variability in the median surface response spectrum, similar to Figure 9(b) if the ROI is infinitely wide the observed variation between the 1D analyses of the 1D and 2D field would have been similar because observed variability is quantified relative to the width of ROI in the 1D analyses of 2D field. Within this interpretation, the difference with the 2D analyses of 2D random fields is not as large as in Figure 9(b).

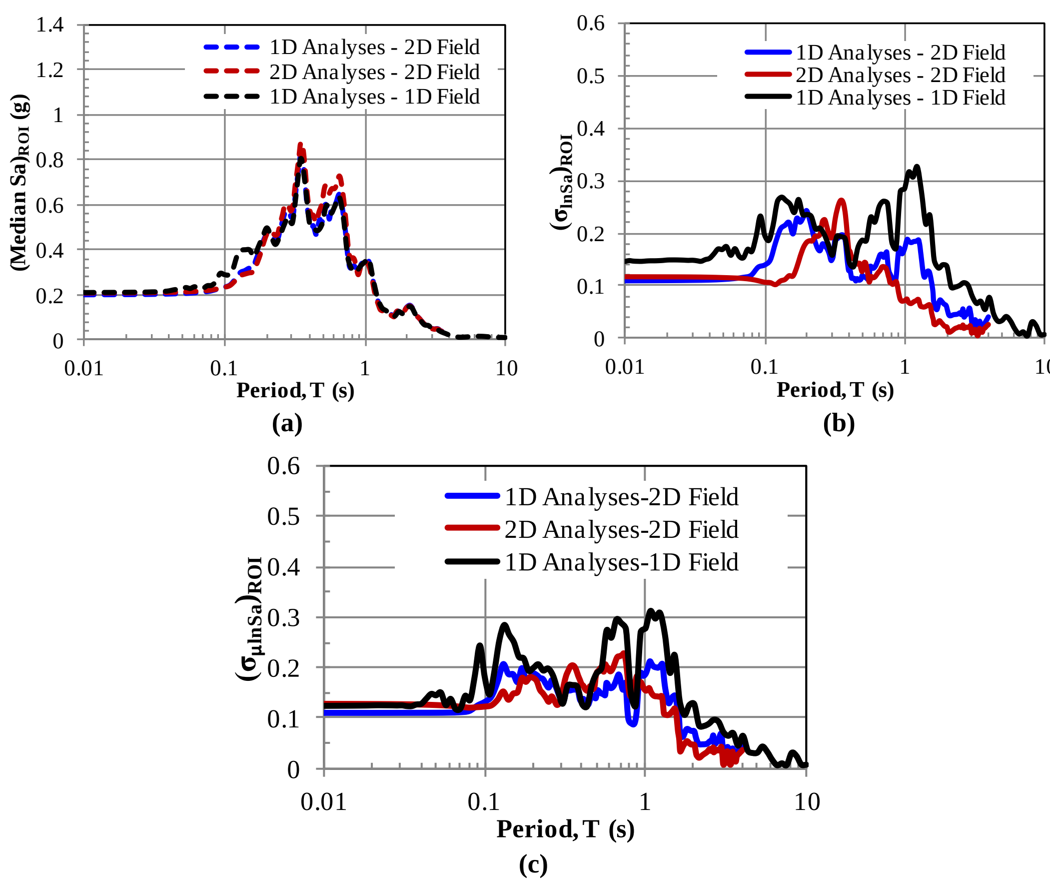

Figure 9. Comparison of the results of 2D and 1D analyses: (a) the average median spectral acceleration for 20 realizations, (b) the average σ_{lnSa} within the ROI for 20 realizations, (c) the σ of the natural log of the median Sa (i.e., μ_{lnSa}) across 20 realizations

5. CONCLUSIONS

This study investigates the use of 1D and 2D site response analyses to model the spatial variability in the shear wave velocity through Monte Carlo simulation. Two-dimensional and one-dimensional random fields of Vs were generated for a 100 m deep alluvium site. The region of interest was a region 100 m wide. 2D realizations of the velocity field were generated and each analyzed using the 2D dynamic finite element code QUAD4M (Hudson et al. 1993). The median surface response spectra across the 100-m wide region of interest were investigated along with the variability in these spectra. 1D site response analyses were performed using the 1D profiles within the ROI of the 2D random fields. Additionally, 1D analyses were performed for 1D Vs profiles generated as 1D random fields in which the horizontal correlation between velocities is ignored.

The results show that multiple 1D analyses can generate median response spectra across at 100-m wide region of interest that are similar to 2D analyses. The largest observed difference was about 10%. The differences in the variability in the computed responses by 1D and 2D analyses were more significant. Here, the 1D analyses tended to predict larger standard deviations, particularly when the horizontal correlation between velocities is ignored. Additional work is required to explore how these differences are influenced by the size of the region of interest, the modeled correlation distances, the intensity of the input motion, and the characteristics of the input motion.

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